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Two Definitions of Contingency and the Concept of Knowledge

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ABSTRACT: This paper analyses two definitions of contingency. Both definitions have been widely accepted and used as to identify contingent events. One of them is primarily of a philosophical character, whereas the other is more commonly used in mathematics. Evidently, these two definitions do not describe the same set of phenomena, and neither of them determines the completely intuitive notion of contingency. Namely, carefully selected examples testify that the first definition is too narrow and the second too wide. These facts have certain epistemological consequences. They must, therefore, serve as a warning for using the definitions only in a restrictive and cautious way when detecting random beliefs – those we cannot identify as knowledge.

KEYWORDS: Contingency, definition, knowledge, mathematics, philosophy.

1. Introductory remarks

Each scholarly discipline has a certain conceptual apparatus, a number of terms that are used as theory-making tools. Since it is natural to expect deductive correctness in the construction of theory, each of these terms would have to be defined by using the other, already defined terms, or terms that will in a sense be considered as clear and comprehensible without the need to be further determined.

A number of terms belong exclusively to certain scholarly fields, while most of other terms shall be used and interpreted in not just one area. The concept of contiguity or a random event belongs to the second group of terms. For example, this term is frequently utilised in mathematics and physics, but also in philosophy and psychology. The fact is, however, that only sometimes in the areas of philosophy and mathematics, as opposed to other scholarly fields and the lay everyday usage of the term, it is not used as absolutely obvious. Instead, attempts are made at explaining this term and making it comprehensible by means of other terms. This paper is another modest attempt at this.[1]

The concept of coincidence can, then, be viewed from different perspectives: mathematical, philosophical, physical or psychological, for example. It is natural that the selection of the point of view determines the approach itself. We could say that the angle from which this paper analyzes the notion of contingency is philosophical and mathematical. More specifically, the concept of contingency will be addressed in the epistemic context. Once we have proposed two definitions of contingency, we will be interested to know how much they are useful in connection with the concept of knowledge. Why connect these two concepts exactly? Although there are different views of the concept of knowledge in the modern literature, this article was inspired, among others, by the thought of Duncan Prichard and his views of the so called *anti-luck epistemology*. He notes that the role of the concepts of contingency and luck in terms of the concepts of knowledge is very complex. In fact, on the one hand,

> [a] common intuition often expressed regarding knowledge is that it is true belief that has been formed in a non-lucky or non-accidental fashion. Indeed, this is often thought to be the proper moral to be drawn from the Gettier counterexamples to the classical tripartite account of knowledge – that the classical account left knowledge possession unduly exposed to the vagaries of luck. It is not difficult to see the attraction of such a view, since knowledge is clearly a cognitive achievement of some sort and cognitive achievements are not naturally of as being due (either in whole or part) to serendipity.[2]

And on the other,

> Nevertheless, this cannot be the full story because it does seem that all knowledge must be, to some degree, dependent upon luck. After all, knowledge involves a kind of union of agent and world, and thus is ineliminably dependent upon the co-operation of that world.[3]

This author distinguishes between several types of luck-accident, noting the fact that some of them are "compatible" with knowledge and do not question it, while the appearance of others excludes speaking about knowledge.

Encouraged by the previous considerations, we will try to do the following: we will propose two definitions of the term "contingency", one of which

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[1] A large number of authors feel there is no need to define this term in philosophical discussions. Such are, for instance, Gjelsvik (1991), Hall (1994), Heller (1999), Vahid (2001).

[2] Pritchard (2004: 193).

[3] Pritchard (2004: 194).

will be fully formalized and represented mathematically, the way it is normally done in the theory of probability. Then, we will analyze their relationship and their (in)compatibility with the concept of knowledge. We will be interested to know what the relationship is between sets of the phenomena that the two definitions describe, that is, whether these sets are potentially equal, or they may have at least some common elements, or are, however, disjoint. At the same time, by using the relevant examples, we will also try to observe to what extent each of these definitions corresponds to our "pre-philosophical" understanding of the concept of contingency. Based on this, we will finally try to draw a conclusion as to whether and how these definitions can be used as tools for identification of contingently truthful beliefs, those that we can classify as knowledge.

2. Two definitions of contingency

Let's try to analyze the concept of a contingent event. What is most often demanded from such an event is that it is under a control or "conduction" of some sort or other which is driven by one's intention or will.[4] If such a control was present, it would be clear that, with regard to our intuitive understanding of the concept of contingency[5], we could not speak of a contingent event, because its outcome is in a way determined by the intentions and actions of some entity. Let us take a few typical examples of events that are considered to be contingent, which would satisfy the previous condition.

[4] Pritchard (2005: 127).

[5] At this point it is necessary to give a formal-logical explanation. This paper is an attempt of defining the concept of contingency. Defining a concept is a process of a more precise explanation of the term by means of the terms that have already been defined or are in a sense clearer than the new term. On the other hand, the effort to define a term also implies that we do not have its precise terminological and conceptual clarification. Still, what is a useful defining step is some sort of an intuitive idea of what we understand by that term. When we say "intuitive idea", we think of an idea that has not necessarily been translated into a precise formal definition. Thus, it is clear that the notion of intuitive idea does not imply the existence of its precise definition. Let us explain this by an analogous situation we have in mathematics. For example, according to one of several existing axioms of Euclidean geometry, the concepts of point, line and plane are considered as basic terms. This means that they feel intuitively clear, so we do not need to define them. A mathematician may say that these are clear terms, though he will not be able to tell us much about them. Using these and some derived concepts we could define, for example, the concept of a circle, of which we also have an intuitive idea, but whose formalized and ready-to-use idea we can have only after creating a potential definition. When we propose a definition of the circle, we are able to talk about how it fits our intuitive idea of this concept, although we do not say anything about the concept itself. Specifically, the very attempt at describing the idea would be an attempt at defining it. In the same manner we will proceed in this paper when speaking about the intuitive idea of contingency.

Example 1. Walking down the street I found a 10-Euro note.

Example 2. On the New Year's lottery my number was drawn as the jackpot.

Example 3. Flipping a coin, I got tails.

Among other things, indeed, all three cases have one thing in common; the final outcome of neither of them is affected by a subject's will. In the first example I found the note, but at the same time, neither I nor anyone else has "wished" for something like that to happen. No one's hidden intention is behind it. In another example, I am a lottery winner and no one has willingly contributed to the fact that exactly my lottery ticket was drawn. Finally, in the case of a coin toss, which is often cited in the theory of probability as a typical example of an accidental event, no one's desire or control has affected the result of the throw as to be exactly tails.

Once we have realized that the absence of one's control and "directing" of the outcome of the event should be a necessary condition for an accident, it is natural to wonder about the sufficiency of such conditions. The question is whether it is possible to find examples of events which will meet this condition, about which, nevertheless, we will not be inclined to talk as about contingent event. Take the example of the next day's event of sunrise. If we accept that this is an event which happens without control and is unaffected by the will of any subject, we should in accordance with the proposed conditions state that it is a contingent event. This, however, will not be easily said of the event of sunrise. We will also face problems when acquiring certain perceptual knowledge. When we talk about the knowledge that, according to the traditional interpretation involves a belief, we already take certain reasons for granted, a justification for this belief, therefore, the absence of a contingent, reasonable and random belief. However, there are different perceptual beliefs and types of knowledge which are often acquired outside of any control and intentions of the subject. If we define a contingent event by means of lack of any sort of or anyone's control, we would not allow a significant number of perceptual beliefs to become knowledge, which is not an acceptable result.

A significant attempt at defining contingent events is the one that involves the use of the concept of *possible worlds*.[6] In fact, by using this con-

[6] Theorists who use the concept of possible worlds consider the *actual world*, the world in which we live, as one of the possible worlds. Amongst the authors who use the term, there are disagreements about the nature of possible worlds. Their ontological status is not clear, particularly the existence of differences, if any, between the ontological status of the actual world and other possible worlds. The very idea can be traced back to Leibniz (2006), in the context of proving that God created our world as *the best of all possible worlds.* Contemporary authors who introduced the concept to the modal context are Kripke and Lewis. In this sense, the link

cept, we can say that an event is *contingent* if it occurs in our real world, but does not occur in *the general class of the nearest possible worlds* in which the *relevant* initial conditions for that event are the same as in our world.[7] Let us, for easier further reference, denote this definition by D1. To examine the appropriateness of the proposed definition, it is necessary to make a few preliminary clarifications.

When we speak about the *relevant initial conditions*, we refer to those conditions that do not directly determine the occurrence of the event itself, but are assumed regardless of whether the event occurred or not. In the first example, such conditions would be: making the decision to take a walk, the lack of any kind of suggestions and hints to the man who walks where he "ought to" take the walk, of what could be found in the street, etc. In the second example, such conditions would be: buying a lottery ticket, the fair organization of the draw without any rigging or fraud, inserting a ball into the machine under special circumstances that do not place it in "advantage" over others, and so on. In the third example such conditions would be: flipping the coin without any special control (for example, attempting to affect the outcome by shorter throws or fewer revolutions of the coin), the approximate uniformity in the distribution of mass on both sides of the coin, etc.

Now let's explain what we mean by the phrase *broad class* of the proposed definition. When you play the lottery, in almost all other nearby worlds, you will not get the jackpot. This means that there are many more close worlds in which the described event, under the same relevant initial conditions, will not happen, that is it will not appear in a larger, wider class of close worlds. This sort of condition seems natural if we want to talk about trying to define the concept of a contingent event, because if the event appears in the majority of close worlds, then the question is whether following our intuition we could consider such an event a contingent one. However, following the mathematical manner, it is natural to ask where the limit is of a class which we consider to be a wider class. Let us explain the motivation for the previous question by one example. Let's imagine a quiz show participant who randomly chooses one of the two offered answers to a question, of which one is correct whereas the other is false. He will lose (gain) in approximately half of close possible worlds. However, if he has to choose one out of five-choice answers, of which three are correct and two are false, although it is a random choice, he will get the correct option in a greater number of close possible worlds. Our intention was to detect the events that do not occur in most close possible worlds by

between the question of determining the accuracy of modal statements (the possible, necessary ones) and the idea of possible worlds are being discussed. We will use this term for description of events which occur in our world which, mathematically speaking, are likely to occur.

[7] Pritchard (2005: 128).

means of the last definition of contingency, which means that the selection of a correct response, regardless of the randomness in the last example, we could not consider a "complete" coincidence.[8] It might naturally be expected that at least half of the times an event is realised in such a way that it is impossible to give it the status of a contingent one, and this sort of limitation could be motivated exactly by the example of a coin toss.

Let us consider how in accordance with definition D1 we can observe previously mentioned characteristic examples. When it comes to the first example, in the majority of cases, and therefore in the wider class of close possible worlds, walking down the street we will not find a 10-Euro bill, so in this case the definition of contingency is appropriate. Looking at the second example, it is clear that in majority of cases, which means in the wider class of close possible worlds, playing the New Year's lottery we will not get the prize. Finally, when tossing a coin, the number of cases in which we got tails, will approximately[9] be the same as the number of cases in which we got heads. Also, in accordance with the proposed definition, the sunrise case will not be seen as a contingent event, as in all close possible worlds and therefore in the wider class of close possible worlds the event of sunrise will occur.[10]

Let us now see how the concept of contingent events can be defined in mathematics. Namely, the basic model in the theory of probability is experiment (phenomenon) in which the realisation of certain conditions does not lead to the unequivocal (deterministic) result.

Example 4. A roll of the dice. In this experiment the conditions are "throwing the dice" and the result is "the number of points displayed on the upper side of the dice once it is rolled".

The set of all possible outcomes of the experiment we will mark by Ω. Its elements (individual, possible outcomes) we can name elementary events. We can mark them by ω. When describing an experiment, in addition to specifying the conditions, it is necessary to thoroughly explain what is observed as its result.

Example 5. We flip a coin five times and record the number of times we got tails. In this case, $\Omega = \{0,1,2,3,4,5\}$. Possible elementary outcomes are the numbers zero to five.

[8] We could possibly state that this case is "less" contingent than others.

[9] As the number of throws increases, the ratio of the cases in which we get tails (heads) and the total number of throws tends to 0,5.

[10] The exception would be a close world in which, for example, there was a cosmic catastrophe that would disrupt the movement of certain celestial bodies within the solar system.

Now we can define the (contingent) event A as an *arbitrary subset of the set* Ω.[11] This definition of contingent events we can denote by D2. We will say that an event A is realized if and only if it achieved an outcome ω which is an element of the set A.

Example 4a. Assuming the conditions from example 4 are satisfied, we obtain that Ω = {1,2,3,4,5,6}. Consider, for example, the event B, where B = {3,6}, that the number of points obtained by throwing the dice will be divisible by 3.

Example 5a. Assuming the conditions from example 5 are met, we can observe, for example, the event C = {4,5}, that tails appear more than three times.

According to definition D2, Ω is too a contingent event[12], and one that is always realized, and it can be considered to be a certain or secure event. In our example 5, this would be the event in which tails appear 0, 1, 2, 3, 4, or 5 times in a total of five coin flips. On the other hand, the empty set Ø, as a subset of every set, including the set Ω, we would consider an impossible event. In our example, this would be an event in which after flipping the coin five times, we do not get tails neither 0 or 1 or 2 or 3 or 4 or 5 times.

Earlier we saw that in the first three examples, contingent events were described according to definition D1. The same can be said of the examples 4a and 5a. Indeed, in the majority of close possible worlds it would not happen that when rolling the dice once we get a number that is divisible by three, nor will it happen that when flipping a coin five times we get the tails on more than three occasions. Let's look at how things stand with the mathematical definition of D2. Examples 4a and 5a represent random events which are presented to illustrate how definition D2 works. How do things stand with the first three examples? Can we "fit" them too into the terms of the definition D2, that is, can we accurately determine their objects – a set Ω and some of its subsets A, B or C which were determined by the definition?

Let's start with example 3, in which a coin is tossed once and we get tails. Here, Ω = {tails, heads}, and the event A = {tails}. Therefore, formal requirements of definition D2 have been satisfied, so we can say that it is also an example of a contingent event. In Example 2, in which my ticket was drawn as the jackpot on New Year's lottery, in a similar way, we can identify the sets Ω and A.

In this case, the set Ω is the set of all ticket numbers that were sold by the New Year's lottery[13], and A is a single-element set whose only element is my ticket number. So, recognizing the terms of the definition D2, in accord-

[11] Ivković (1980: 9).

[12] Each set is its own subset.

[13] In this example we take that the lottery was organized in such a manner that the jackpot had to be drawn, i.e. that it could not happen that the unsold ticket number was drawn.

ance with it, we can say that by giving the example 2 we have illustrated a contingent event. Is it so with example 1?

What can possibly complicate the answer to the previous question is the process of determining the set Ω. If we have determined the set, we could easily determine its arbitrary subset, that is, a random event and elementary events ω as elements of the set Ω. Example 1 is different from the other examples in that it is not "strictly" standardized in relation to the initial conditions that occur in it. The remaining examples are typical examples of mathematical contingent events, for which it is possible to very accurately determine the relevant initial conditions. The nature of these events does not allow too much variation in relation to the initial conditions that appear in them (registration of the number of points on top of the dice, determining the outcome of the flipped coin, specifying how many times certain side of a repeatedly thrown coin has occurred, checking the geometric regularity of the dice and the coin which are being used, the draw of balls on the New Year's lottery, their equality in weight and shape, etc.).

In such standardized experiments it is not particularly difficult to determine the set of all possible outcomes. In fact, when describing the random event of finding banknotes, it is not clear what in this case is the "experiment" and what its possible outcomes, which is a necessary thing to do if we want to make sure that the described event can be included under definition D2. However, it seems that with a little more detail and precision in the formulation of examples, this too would be possible. In this sense, let us make a possible illustration. For example, we could say that the initial conditions in this example assume that I am going for a walk at a specific time and along the specific streets, without any indication as to whether I can find something on the way. In other words, the experiment would be an individual's trip along a specified itinerary, during which the person has no idea whether he/she could find anything. What are the results of such an experiment which are reasonable to expect, given the outcome described in Example 1? One and at the same time the most trivial possibility[14] of setting up the set Ω is the one in which we would consider two possible disjunctive outcomes[15]: to find a 10-Euro note

[14] There is an infinite number of ways in which we can describe this event as a result of a certain experiment, and this depends on how we will determine the set Ω. For instance, we could say that Ω = {finding a 10-Euro note, finding a lost ID, finding a lost ring, not finding any of these}, or Ω = {finding a 10-Euro note, finding a 20-Euro note, finding a 50-Euro note, not finding any of the given bank notes}, etc. It is important that the set Ω has to exhaust by its elements the set of all possibilities which are the likely results of the experiment.

[15] The terminology used here is related primarily to the set theory. Two sets are considered to be disjoint if their intersection is an empty set. In the domain of logic in this situation we can use an equivalent exclusive disjunction which does not allow both of the elements to be accurate.

while walking and not to find a 10-Euro note while walking. This is how we found a way to describe the event from Example 1 as a result of the relevant experiment, in which the set Ω is a two-item set, and the contingent event described in the example is one of its subsets. Thus, we have simultaneously shown that random events, such as, for example, is the one described in Example 1, which, perhaps, are not typical examples of mathematical random events, can, by acquiring appropriate details, still be given in this form and can be considered analogously as any typical example of a random event from the theory of probability.

It would be natural to consider the relationship between definitions D1 and D2, and in connection with the question which one is more appropriate for what we intuitively consider a contingent event.[16] The main difference that is immediately noted in the above remarks is that the set of contingent events S1 which falls under the definition D1 is not the same as the set of contingent events S2 determined by the definition D2. More specifically, S1 is a real subset of S2. Indeed, all contingent events, as defined by definition D1, are events that would never happen in the broader class of worlds that are close to ours. All these events, even if they are not given in a precise formal way (accurately described experiment, the initial conditions, the complete set of outcomes) can, as we have seen in the example, be presented in such a way as to allow us to consider them contingent according to definition D2. On the other hand, all contingent events, as defined by definition D2, we can consider contingent according to definition D1. In order to explain this, we offer the following illustration:

Example 6. Assuming the conditions of Example 4 have been met, we have $\Omega = \{1,2,3,4,5,6\}$. Let us consider the event $C = \{1,2,3,4,5\}$ that the number of points obtained by rolling the dice once will be smaller than 6.

Event C in the example above is random according to the definition D2, because it meets all the formal conditions required by the definition. On the other hand, C cannot be considered a random event according to definition D1 because it is not true that it will not occur in a wider class of close possible worlds. On the contrary, it is highly unlikely that it will not happen in a world that is close to our own. In this way we have shown that there is a relationship of strict inclusion between the sets S1 and S2.

Since we have seen that the two definitions do not define the same set of cases, it would be natural to ask ourselves which definition to choose in order

[16] We will here ignore the fact that definitions D1 and D2, by their form, application and the nature of the notions they utilize, belong, conditionally speaking, to different scholarly areas (D1 belongs to philosophy, whereas D2 belongs to mathematics). We will, first of all, be interested in the types of events which are determined by them.

to define the concept of contingent events. We will not make this judgement simply by considering the definitions and the space of contingent events that each of them determines. We will say that event C in the last example is contingent according to the definition D2, although it will not be marked as such on the basis of definition D1.

On the other hand, the idea of defining the notion of contingency by means of the notion of close possible worlds also seems natural. The question of acceptance of the definition we can set a little differently, in what seems to be a clearer form. Since everything that we accept as a contingent event according to definition D1 we can present in such a way as to accept it as a random event according to definition D2 as well, while the reverse is not true, we only need to decide whether all the random events as defined by definition D2 are forms of contingency as we intuitively, at least functionally, understand them within a given theory. We will try to make a judgement about this by observing an important epistemological context of contingency – the existence of knowledge implies the absence of an accidentally true belief.

3. Two Definitions of Contingency and Knowledge

It is widely accepted among epistemologists that a belief, even if true, cannot be understood as knowledge if we obtained it due to pure chance. In order to prevent such beliefs as candidates for knowledge, we introduce different conditions related to the acquired beliefs, such as requirements for truthful beliefs, their sensitivity to changes that may occur in connection with the subject of belief, and so on. The majority of epistemologists have focused their attention exactly on the specific technical requirements that would exclude consideration of beliefs whose truthfulness has been obtained by accident. These conditions are not the focus of our attention, but we focus on the question of defining the concept of contingency which is important for the definition of knowledge. We have considered two options of this determination above. The decision on whether it is appropriate to accept the formulation D1 or D2 as a definition of contingency (randomness) we will bring precisely within the epistemological framework of the concept of contingency. To be more precise, as previously mentioned, D1 is a variant definition which has already been proposed within this framework in a relevant way. D1 has already been accepted and is well-established as a possible definition within the epistemological framework of the concept of contingency, and this is nothing new. What would be interesting is to check whether D2 as well can be seen in these terms. If the answer to the previous question was positive, it would not necessarily mean that D2 is more appropriate than D1 in the theory of knowledge, but it would certainly mean that D1 is too narrow and inadequate. This is exactly what we are going to illustrate further on. In order to do this, it is enough to give an example which is a contingent event according to definition D2, but which is not contingent based on D1 and which will intuitively "undermine" knowledge. In this way we will show that when it comes to knowledge, the more formal definition D2 may in some cases be taken as a basis for identification of contingency, even before definition D1.

Gettier's (1963) examples, as well as all the subsequent Gettier-style examples were created with the aim to show that the traditional tripartite definition of knowledge, as a justified true belief, is not adequate. They provide the cases of beliefs that are in some sense justified and therefore true, but for which intuitively we would not say that they represent knowledge. What we call intuitive and understand as contingent is used in their construction. Recall, for example, the second Gettier's example.

Let's suppose that Smith has a good reason to believe the truthfulness of the following statement:

p: Jones is an owner of a Ford.

The reason is based on the fact that Smith has got evidence which proves that Jones had a Ford and that he recently offered him a ride. Smith has a friend whose name is Brown. The utterance *p* entails

q: Jones owns a Ford or Brown is in Barcelona.

Based on the two Gettier's assumptions, Smith has full right to believe *q*. Smith, however, does not have adequate information on the whereabouts of Brown. Let's imagine that Jones is the owner of a Ford, and that Brown is, indeed, in Barcelona. *q* is true, Smith believes that *q* is a true statement, and his belief is justified. However, this is not knowledge, says Gettier and concludes: we can have a justified true belief which is not knowledge, and therefore the traditional definition is inadequate.

At this point we are not interested in the role of the example as a means of discrediting a traditional definition. What is important to us is the role of contingency of events, or more precisely, a contingency of truthfulness of a statement as regards a specific event, in the last example with respect to the definitions D1 and D2. Probably it would be difficult to say that Brown is in Barcelona by accident, but we will still say that Smith's believing the statement *q* is accidentally true. Of course, there are probably specific reasons why Brown was in that city, [17] so from his perspective, we could not say that he found himself there randomly. However, from Smith's perspective who can only guess where Brown is since he has no information about his whereabouts, the truth of the utterance *q* is purely coincidental.

[17] For tourism, business, health reasons, etc.

Let's see if the veracity of q can be understood as a contingent event according to the two definitions.

According to D1, the truth of q would be, of course, a contingent event. In fact, in most of the worlds which are close to our own, the truth of q cannot be realised because Smith chose Barcelona without any information about where Brown really is. In the majority of worlds which are close to our own, that statement would be false, because there is a very small chance that a randomly selected city would be the same one as the city in which Brown is actually located.

In the general discussion on the contingent events, we previously showed that every contingent event explained in the spirit of definition D1, with appropriate formal refinement we can also understand as a contingency according to D2. Let us show that when it comes to the contingency in the context of knowledge it is no different. To see whether the veracity of q is a contingent event according to D2 as well, it is necessary to describe the event in accordance with this definition, that is, to describe the experiment, its terms, the set Ω and to see if the event in question can be described as a subset of Ω. We examine the veracity of Smith's statement q by means of an experiment. In addition, when producing the utterance, Smith has no clue as to where Brown could be located. When producing the utterance, Smith can choose any city in the world and Brown is a person who often travels and is rarely found in the city where he, otherwise, lives. So in this case we have the following situation:

$$\Omega = \{\text{“true”}, \text{“false”}\}.$$

If we denote by A the fact that the utterance q is true, than we have that

$$A = \{\text{“true”}\},$$

by which we confirm the earlier consideration that every contingent event as defined by definition D1, even the one from the second Gettier's example, can be viewed as contingent according to definition D2.

Also, in an earlier general review of contingency we have seen that there are contingent events according to D2, which, however, are not contingent according to D1. For us, the important question is whether such an example can be found when speaking about contingent events in the context of knowledge. If the answer to that question was yes, then it would mean that D2 can also, perhaps by introducing to it appropriate additional conditions, be used as a means for recognizing contingency in the context of knowledge. It would also mean that D1 does not recognize all forms of contingency which exist in relation to knowledge. In order to show that this is the case, let's consider the already mentioned, but somewhat changed second example of Gettier's.

Let's suppose that Smith again has a good reason to believe the truth of the following statement:

p: Jones is an owner of a Ford.

The reason is based, as before, on the fact that Smith has evidence which proves that Jones had previously had a Ford that he recently offered him a ride. Smith also has a friend Brown. The utterance *p* entails the utterance

q: Jones owns a Ford or Brown is the father of fewer than four children.

Smith has information that Brown is a father, that the birth rate in their area is low, that very few couples in their country have more than three children, the small family tradition was followed by Brown's parents and his wife's parents, but there is no precise information on how many children Brown, in fact, has. Imagine that Jones is the owner of the Ford, and Brown is the father of two children. *q* is true, Smith believes that *q* is true and his belief is justified.

Let's see if the truth of *q* is a contingent event. According to D1, we cannot say this is the case because in the majority of close worlds that statement would also be true, that is, the event would be realized. In this case, when we say close worlds, we think of the worlds in which Smith has similar information about Brown in connection with his family situation, family traditions as regards the number of children people have, as well as information on the average birth rate in their area.

In order to possibly determine that this is a contingent event according to D2, it is necessary to, as we did before, describe the event in accordance with this definition, that is, to describe the experiment, its terms, the set Ω and the corresponding event. We examine the veracity of Smith's statement *q* by means of an experiment. In addition to this, when Smith states the utterance, he has no clue as to whether Brown has got children. When uttering the statement, Smith can choose any natural number, or any subset of natural numbers.[18] So in this case we have that

Ω = {"true", "false"}.

If we denote by B the fact that the statement *q* is true, then we have that

B = {"true"},

by which we have proved that the truth (falsity) of the utterance *q* we can regard as a contingent event according to definition D2.

So, by providing the last example we have found a contingent event which is contingent according to D2, but which cannot be considered so

[18] He can state: "Brown has got two children", or "Brown has got more than four children", etc.

based on D1. It remains to be seen whether we can say that such an event "undermines" knowledge.[19] Can we say that Smith, whose statement *q* is true, *knows* that Brown has fewer than four children? His testimony is very likely to be true, given the relevant information that he owns, but intuitively, it would be very difficult for us to say that he *knows* something about the numerical state of Brown's family. One could rather say that his statement *q* is *guesswork* which is very likely to be true.[20]

What did we show by the previous example? We found an event that, intuitively, we consider being contingent, and that it is such according to D2 but not according to D1. This means that the definition of contingency by means of the notion of close worlds is too narrow, that is, the events that we intuitively see as contingent cannot all fall under this definition. We have demonstrated that the quoted event can be considered as contingent according to D2, which is what is expected from a definition of contingency. Of course, what we have just said does not allow us to say that D2 is the right definition. We can say this if we determine that it is not too narrow or too loose, i.e. that there are no intuitively random events that cannot be subsumed under D2, and that there are no examples of events for which we will intuitively not say that are contingent, but which, however, still fall under this class by D2.

The first condition that we have illustrated by the above examples has always been there, i.e. definition D2 meets this condition. Indeed, whenever we have an appropriate event whose outcome is not absolutely determined, we can talk about the description of the event, circumstances under which it happens as well as possible outcomes. These are precisely the formal prerequisites for proclaiming something as a random event according to D2. However, it is uncertain whether definition D2 meets the above mentioned second condition. To be more precise, we can certainly come up with certain events which intuitively we cannot consider contingent, but they will still be contingent according to D2. How can we be sure of something like that before finding an example of it? Relying on D2, we will consider that a contingent event is any set A, which is a subset of the set of all elementary outcomes Ω. It is, at the same time, theoretically possible that the set Ω is "very large", infinite and even uncountable[21] and that the set A is described as, for

[19] We will consider knowledge a justified true belief, as determined by the so called traditional definition.

[20] A change in the subject of belief, i.e. the number of Brown's children (let's suppose he has just got twins) would not in any way affect Smith's belief and the validity of his utterance having in mind the available information he has based it on.

[21] We will state that an infinite set is uncountable (i.e. it is of larger cardinality than the set of natural numbers N), if there is no bijection between that set and the set of natural numbers. Such is, for instance, the set of real numbers R.

example, the singleton, subset of Ω. For our purposes, the event B = A^c would be an interesting one, the complement of A, that is, the event – a set which is disjoint from the set A, and which in a union with the set A gives a set of elementary outcomes Ω. In this case, the probability of occurrence of an accidental event A would be rather low, close to a zero, while the probability of occurrence of an accidental event B would be very high, close to one.

Let us illustrate the above with two examples. Let the experiment consist of a random selection of balls, numbered 1 through 100, from a lotto machine. After the selection, we record the result as an exact number of the selected balls. The set Ω is in this case a set of natural numbers on a scale of 1 to 100. Let A be the event of selecting a particular number, for example, the number 4, and let B = A^c, the event that we have not selected the number 4. The probability that B will be realised is 0.99 and something like that we cannot intuitively understand as a contingent event. Or let us take even a more extreme example. Let the experiment consist of a random selection of points from the interval (0,1) on a real line. After the selection, we register the result as a specific number from the interval. The set Ω in this case is an infinite, uncountable set that has the same number of elements as the whole real line. Let A be the event of selecting a particular number, for example, the number 0,37, and let B = A^c, the event that we have chosen a number which is not 0,37. Likelihood that B will be achieved is 1, that is, we can say that the event will certainly come true, which we particularly cannot intuitively consider a random event.

Let's summarize the above. We have shown that definition D1 is too narrow because we have been able to find examples which intuitively we can understand as contingent events, but not according to this definition. Such examples, with certain adjustments, we can understand as descriptions of random events as defined by D2. However, we have seen too that there are examples which were described as contingent by D2, but we, intuitively, are not able to accept them as such. In other words, definition D2 is too wide and it too is not adequate.

Previous considerations naturally open a final question. Namely, can definitions D1 and D2 be appropriately adapted so that they are completely relevant when talking about the notion of contingency in the context of the concept of knowledge or is their "falsity" hidden in their very design and idea, without any hope that improvement can be made? In our article, we thus focus on definition D2.

The main objection that we specified in connection to definition D2 was linked to a range of events that we have, based on it, considered to be contingent. To be more precise, the definition was too broad. It allowed us to mark as contingent even those events the achievement of which was highly probable, such as event B in the last two examples. Inspired by the problems that we have encountered in relation to these examples, we can look at whether some details in definition D2 could be changed, supplemented or specified as to make the definition more appropriate. At the same time, we must avoid the danger that often lurks in these situations. We have to avoid an *ad hoc* change of the definition which would solve the problem related to the specific examples or classes of examples that have inspired us to change, but which would not solve the difficulties associated with a number of other examples, or even create new difficulties in connection with the examples which had no problems prior to the change of the definition.

As noted above, what does not allow us to accept events such as event B in the last two examples as contingent, is that they display probability which is close to one, or is exactly one. The discussion, of course, no longer refers only to the aforementioned events, but all events whose probability is expressed in large values. In this sense, we should think about the possible adjustment of D2. The task should be reduced to the following: can definition D2 be changed in such a way that it does not cover the events which have a high likelihood of being achieved? The task that stands before us is a difficult one. Namely, in order to answer this question we would have to determine what "high" probability is. Are these only the events whose probability is close to one? What measure is to be used and how to express the closeness to number one?

The standard example of the coin toss has, perhaps, partially motivated us to give answers to the above questions. Once we have flipped a coin the probability of getting tails is 0,5. This is an example of an event that is typically considered contingent. Its probability is a number that is equidistant from both the values zero and one. If the number was closer to zero than to one, the event would still intuitively be understood as contingent, as something that does not happen for certain, but under special, extraordinary circumstances. If the number was closer to one than to zero, its understanding as a contingent event would be brought into question, to the extent to which the value was closer to the number 1. That is, if we think that the possibility that an event will happen is higher rather than it not happening, intuitively, we could not easily say that such an event is contingent. So, maybe someone would suggest that the bordering value for determining the status of an intuitively contingent event that would be relevant in the context of knowledge should be the likelihood of 0,5. However, we would not allow then some of the events to be contingent, although we consider them as such.

The principal problem with the mathematical definition D2 is that according to it we will consider all the events contingent, no matter how high their probability is or whether it is close to one. A mathematician will, regardless of the criticism, offer his intuition regarding the status of a contingent event in this situation as well. He will simply say that although the probability of event A is very high, he considers the probability of occurrence of event A or its complement possibility of it not occurring, thus putting aside the size of the likely alternatives. Pursuant to such an intuition, we can say that neither the realization of event A nor achievement of its complement is determined or predetermined by anything. There is a possibility that both events will be achieved. The probability rate of either event does not give us the right to declare one of them as more contingent than the other. Both events are contingent, although they have different probability. The realisation of certain events can be graded according to the likelihood of their occurrence, but that does not mean that we will consider one of them as more or less contingent. A mathematician will thus finish his explanation. Any attempt at changing the definition D2 by which we would try to preserve the "non-mathematic" intuition about contingency, essential to the concept of knowledge, would upset the whole basic mathematical idea of a contingent event as a non-deterministic outcome, that is, formally expressed, as a kind of a collection of basic elements that are selected from a set of all possibilities. On the other hand, if we accept the idea presented by D2, we would also think that a contingent event is the realization of the event of selecting a white ball out of a machine containing 999 blue and one white, which also will not be easy to accept.

The final and the most commonplace attempt at "improving" D2 as we have just indicated, would be to limit the probability of occurrence in D2 by selecting a constant from an interval (0,1), for example the value of 0,5 or some other value which is close to that one. So, following this idea, we would consider that an event is contingent if its probability is smaller than a fixed constant from the interval (0,1). Only then would we run into problems and see how powerful a mathematical idea of contingency is. In fact, if we chose, for example, the constant of 0,5, then we would not consider, for instance, that the statement of the aforementioned converted second Gettier's example[22] was accidentally true, as well as a number of other events that we can construct and that we would intuitively accept as contingent. The selection of any of the constants from the interval (0,1) as a boundary that would determine the status of contingency would inevitably pose the question to which we cannot give reasonable/justified answers: why was precisely this constant chosen, and not any other, and can we guarantee that one cannot construct an example which will portray an event that can be intuitively considered as accidental, and whose probability of realization would be greater than the selected constant?

[22] Providing the exact statistical data on the country's natality, the data on the number of members of Brown's parents' family and his wife's parents' family, as well as many other important details that can affect the number of family members, we could speak of a concrete probability, in this case, which would surely be higher than 0,5.

Definition D1 carries a natural and expected idea that we should consider as contingent all those events that will happen quite rarely and by chance, under certain special and unusual circumstances. The condition "quite rarely" is expressed through the requirement that the event does not get to be realized in the "majority of close possible worlds". However, from each definition and this one as well we expect accuracy when it comes to the given conditions, the condition which is already not met when we speak about the "majority of the worlds". Such a blanket approach, unfortunately, creates a problem that we cannot allow in a valid definition. We are not able to clearly draw a line between the contingent events and the ones that are not contingent. The fact that D1 gives us an opportunity to recognize a wide class of random events under certain circumstances, it is not enough, as we have shown above, for it to be declared correct and proper. On the other hand, an attempt to overcome the imprecision that exists in D1 by giving it quite precise conditions given by D2 creates a new problem that is not easily solved without completely destroying the mathematical idea of coincidence. The formalism of D2 would have paid the price of too wide a definition. This definition would allow us to consider as contingent even those events which we intuitively do not accept as such.

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