

MULTI-INFLUENCE FACTORS PREDICTION FOR WATER BLOOM BASED ON MULTI-SENSOR SYSTEM

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Abstract:

This paper proposes a new multi-influence factors prediction method for water bloom prediction based on a remote monitor system and multi-sensor data taking into account the integrated effect of multiple influential factors along with the periodicity and random effect of environmental variables. Valid and accurate water-bloom prediction can be obtained by combining various multidimensional time series methods. Comparing the proposed model based on multi-sensors data to a traditional one-dimensional time series model based on one-sensor data, it has been found that a multidimensional model is a useful and accurate model for establishing multiple influential factors time series of water bloom. The optimum model can be used not only to predict water bloom but also to determine the period and random change rule of multiple influential factors.

1 Introduction

The formation of water bloom is a typical water pollution problem in water environment. In view of the regional differences in large outbreaks of water bloom as well as strong dependence of water bloom on various environmental factors and on different kinds of substances contained in water, the problem of water bloom occurrence is an uncertain non-linear problem. At present, critical factors and mechanisms of water bloom outbreak are not clear. Moreover, control methods are not efficient enough and water bloom outbreak cannot be accurately predicted.

In recent years, many scholars have made a great success in water bloom prediction. However, most of them have focused on the use of one-sensor data [1-5], which are in such a short supply that they only emphasize a single influential prediction factor.

A few scholars predict the occurrence of water bloom using multi-sensor data [6-15], however, they analyze them separately and give little consideration to the integrated effect of multiple influential factors and prediction ability of the model.

Multidimensional time series analysis based on multi-sensor data is a method for establishing a multidimensional stochastic model used for predicting multi-influence factors on the basis of their autocorrelation and cross correlation. Subsequently, this stochastic model is to predict the long term trend. Traditional time series analysis methods usually ignore the integrated and the periodicity effect of multi-influence factors based on multi-sensors data [16-20]. Therefore, this paper proposes a new water bloom prediction model/method based on multi-influence factors, i.e., multi-sensors data to take into account the integrated effect of multiple influential factors along with the

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periodicity and random effect of environmental variables

2 Designing of water bloom remote monitor and prediction system

There are three sections in the water bloom remote monitor and prediction system: multi-sensor on-site data sampling module, GPRS network communication module and monitor center module, which are shown in Fig. 1.

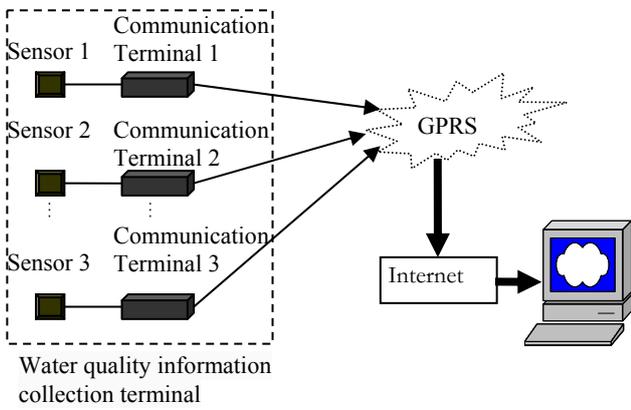


Figure 1. Structural diagram of an overall system.

Multi-sensor on-site unit module is used for data sampling, packing, transmitting, and responding to the commands from the monitor center and processing according to those commands. Data will be transmitted to Internet by GPRS network. The monitor center can be used to accept, display, store and process data, and even predict the influence factors of water bloom and transmit control command to on-site unit.

3 Multidimensional time series modeling

In water environmental monitoring, multiple influential factors monitoring data is usually collected at defined consistent intervals that produce a homogeneous variance.

Let Y_t denote the multiple influential factor measurement at time t . Based on the Cramer Decomposition Theorem, any multidimensional time series $\{Y_t\}$ can be decomposed into two components: a deterministic component and a stationary random component.

In water bloom predictions, due to the periodicity effect of environmental variables, deterministic components of influential factors are usually

periodicity rather than aperiodicity. Hence, deterministic component is the equal of seasonal component. Y_t could be expressed as,

$$Y_t = S_t + R_t, \quad t = 1, 2, \dots, m \quad (1)$$

Here, Y_t , S_t and R_t are n -dimensional vectors. S_t is a multidimensional seasonal component. R_t is a multidimensional stationary random component, n is the total number of influential factors and m is total sampling time.

3.1 Multidimensional seasonal component modeling

Most traditional seasonal component model, such as Hidden Periodic model, is one-dimensional model [21]. This paper puts forward a multidimensional seasonal component model, called multi-dependent Hidden Periodic model, which takes into account the periodicity effect of environmental multivariable.

S_t is modeled by using a multi-dependent Hidden Periodic model,

$$S_t = S(t) = \begin{pmatrix} S_{1t} \\ S_{2t} \\ \vdots \\ S_{nt} \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^q a_{1j} \cos(\omega_j t + \varphi_{1j}) \\ \sum_{j=1}^q a_{2j} \cos(\omega_j t + \varphi_{2j}) \\ \vdots \\ \sum_{j=1}^q a_{nj} \cos(\omega_j t + \varphi_{nj}) \end{pmatrix} \quad (2)$$

Here S_t is an n -dependent variable hidden periodic function, which effectively fits the period of the monitoring data, s_{kt} is the seasonal component of k^{th} influential factor, a_{kj} is the amplitude of k^{th} influential factor, q is the total number of angular frequency, ω_j is the j^{th} angular frequency, φ_{kj} is the j^{th} phase, $k=1, 2, \dots, n$.

3.2 Multidimensional random component modeling

The multidimensional seasonal component S_t is extracted from multiple influential factors. Then the multidimensional random component R_t is modeled using the multidimensional AutoRegressive model,

$$\mathbf{R}_t = \sum_{j=1}^p \mathbf{H}_j \mathbf{R}_{t-j} + \mathbf{E}_t, \quad \mathbf{H}_j = \begin{pmatrix} \eta_{11j} & \eta_{12j} & \cdots & \eta_{1nj} \\ \eta_{21j} & \eta_{22j} & \cdots & \eta_{2nj} \\ \vdots & \vdots & \ddots & \vdots \\ \eta_{n1j} & \eta_{n2j} & \cdots & \eta_{nnj} \end{pmatrix}, \mathbf{E}_t = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{nt} \end{pmatrix} \quad (3)$$

Here p is the order of multidimensional autoregressive model; \mathbf{H}_j is an $n \times n$ multidimensional autoregressive coefficient matrix; η_{klj} is the k^{th} influential factor multidimensional autoregressive coefficient from l^{th} influential factor; \mathbf{E}_t is a n -dimensional white noise vector which obeys $N[0, \mathbf{Q}]$; ε_{kt} is the white noise of k^{th} influential factor, $k=1, 2, \dots, n, l=1, 2, \dots, n$.

3.3 Multiple influential factor time series modeling

The multi-dependent Hidden Periodic model for the multidimensional seasonal component \mathbf{S}_t and the multidimensional AR model of the multidimensional stationary random component \mathbf{R}_t are combined into \mathbf{Y}_t . Hence, the influential factor measurement \mathbf{Y}_t is obtained as,

$$\mathbf{Y}_t = \mathbf{S}_t + \mathbf{R}_t = \mathbf{S}(t) + \sum_{j=1}^p \mathbf{H}_j \mathbf{R}_{t-j} + \mathbf{E}_t \quad (4)$$

Eq.4 is a new multidimensional composite time series model, and it is also called Multidimensional Hidden Periodic-Auto Regression (MHPAR) model in this paper.

4 MPHAR model parameters estimation and prediction

4.1 Seasonal component model parameters estimation

This paper puts forward a new parameter estimation method for multi-dependent Hidden Periodic modeling as follows.

Suppose $\{\mathbf{Y}_t\}$ is an n -dimensional variance stationary time series, and its observed data is

$$\mathbf{Y} = (\mathbf{Y}_1, \mathbf{Y}_2, \dots, \mathbf{Y}_m) = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1m} \\ y_{21} & y_{22} & \cdots & y_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{nm} \end{pmatrix} \quad (5)$$

To determine frequencies of the multi-dependent Hidden Periodic model, Eq. 2 is transformed into a complex domain form, that is

$$\mathbf{S}_t = \mathbf{S}(t) = \begin{pmatrix} S_{1t} \\ S_{2t} \\ \vdots \\ S_{nt} \end{pmatrix} = \begin{pmatrix} \sum_{j=1}^q b_{1j} \exp(it\lambda_j) \\ \sum_{j=1}^q b_{2j} \exp(it\lambda_j) \\ \vdots \\ \sum_{j=1}^q b_{nj} \exp(it\lambda_j) \end{pmatrix} \quad (6)$$

Here, $b_{kj} = \frac{1}{2} a_{kj} \exp(i\phi_{kj}), j = 1, 2, \dots, q,$

$k = 1, 2, \dots, n, \lambda_j$ is the j^{th} angular frequency in a complex domain form.

Then, this paper introduces an exponential function, that is

$$\mathbf{S}_m(\lambda) = \sum_{t=1}^m \mathbf{Y}_t e^{-i\lambda t} = \begin{pmatrix} S_{1m}(\lambda) \\ S_{2m}(\lambda) \\ \vdots \\ S_{nm}(\lambda) \end{pmatrix} = \sum_{t=1}^m \begin{pmatrix} y_{1t} \\ y_{2t} \\ \vdots \\ y_{nt} \end{pmatrix} e^{-i\lambda t} \quad (7)$$

It can be proved [22] that when

$$m \rightarrow \infty, \sup |\mathbf{S}_m(\lambda)| = O(\sqrt{m \ln m}). \quad (8)$$

Then, set

$$A_m > 0 \quad (9)$$

to satisfy that when

$$m \rightarrow \infty, A_m = O(\sqrt{m \ln m}) \quad (10)$$

Firstly, define

$$\mu(j) = j\pi / (2m) - \pi, j = 1, 2, \dots, 2m, \quad (11)$$

calculate

$$\begin{pmatrix} d_1(\mu(j)) \\ d_2(\mu(j)) \\ \vdots \\ d_n(\mu(j)) \end{pmatrix} = \begin{pmatrix} |S_{1m}(\mu(j))| \\ |S_{2m}(\mu(j))| \\ \vdots \\ |S_{nm}(\mu(j))| \end{pmatrix}. \quad (12)$$

In the following calculation, if there is not only one maximum value, any maximum value can be taken. Secondly, calculate

$$d_k(\mu(j_{k1})) = \max_{1 < j < 2m} \{d_k(\mu(j))\}, k = 1, 2, \dots, n. \quad (13)$$

When

$$\min_{1 < k < n} \{d_k(\mu(j_{k1}))\} \leq A_m, \quad (14)$$

define

$$\hat{q} = 0, \quad (15)$$

stop calculating. Here \cap is used as notation for estimates of values. Otherwise, define

$$\mathcal{A}_k(1) = \left\{ \lambda \in (-\pi, \pi] : 1/\sqrt{m} \leq |\lambda - \mu(j_{k1})| \leq 2\pi - 1/\sqrt{m} \right\} \quad (16)$$

In $\mathcal{A}_k(1)$, calculate

$$d_k(\mu(j_{k2})) = \max_{\mu(j)=\lambda} \{d_k(\mu(j))\}, k = 1, 2, \dots, n. \quad (17)$$

When

$$\min_{1 < k < n} \{d_k(\mu(j_{k2}))\} \leq A_m, \quad (18)$$

define

$$\hat{q} = n. \quad (19)$$

Rearrange values

$$\mu(j_{11}), \mu(j_{21}), \dots, \mu(j_{n1}) \quad (20)$$

by size, which is the estimate value of angular frequency

$$(\hat{\lambda}_1, \hat{\lambda}_2, \dots, \hat{\lambda}_n)^T, \quad (21)$$

stop calculating. Otherwise, define

$\mathcal{A}_k(2), \mathcal{A}_k(3), \dots$, and so on until in

$$\begin{aligned} \mathcal{A}_k(h-1) &= \left\{ \lambda \in (-\pi, \pi] : 1/\sqrt{m} \leq |\lambda - \mu(j_{kl})| \right. \\ &\left. \leq 2\pi - 1/\sqrt{m}, l = 1, 2, \dots, h-1 \right\}, k = 1, 2, \dots, n \end{aligned} \quad (22)$$

there is

$$\begin{aligned} d_k(\mu(j_{kh})) &= \max_{\mu(j)=\lambda} \{d_k(\mu(j))\}, \\ \min \{d_k(\mu(j_{kh}))\} &> A_m, \end{aligned} \quad (23)$$

and in

$$\begin{aligned} \mathcal{A}_k(h) &= \left\{ \lambda \in (-\pi, \pi] : 1/\sqrt{m} \leq |\lambda - \mu(j_{kl})| \right. \\ &\left. \leq 2\pi - 1/\sqrt{m}, l = 1, 2, \dots, h \right\}, k = 1, 2, \dots, n \end{aligned} \quad (24)$$

there is

$$\begin{aligned} d_k(\mu(j_{k(h+1)})) &= \max_{\mu(j)=\lambda} \{d_k(\mu(j))\}, \\ \min \{d_k(\mu(j_{k(h+1)}))\} &\leq A_m, \end{aligned} \quad (25)$$

then, the defined and estimated value of q is

$$\hat{q} = nh. \quad (26)$$

Rearrange values

$$\mu(j_{k1}), \mu(j_{k2}), \dots, \mu(j_{kh}), k = 1, 2, \dots, n \quad (27)$$

by size, which is the estimated value of angular frequency

$$(\hat{\lambda}_1, \hat{\lambda}_2, \dots, \hat{\lambda}_{\hat{q}})^T. \quad (28)$$

Set

$$\hat{\mathbf{c}} = \begin{pmatrix} \exp(i1\hat{\lambda}_1) & \exp(i2\hat{\lambda}_1) & \cdots & \exp(im\hat{\lambda}_1) \\ \exp(i1\hat{\lambda}_2) & \exp(i2\hat{\lambda}_2) & \cdots & \exp(im\hat{\lambda}_2) \\ \vdots & \vdots & \ddots & \vdots \\ \exp(i1\hat{\lambda}_q) & \exp(i2\hat{\lambda}_q) & \cdots & \exp(im\hat{\lambda}_q) \end{pmatrix} \quad (29)$$

By using least square method, estimation of the amplitude of multi-dependent hidden periodic model, $\mathbf{b}=\{b_{kj}\}$, $k=1,2,\dots,n, j=1,2,\dots,q$, is obtained

$$\hat{\mathbf{b}} = \begin{pmatrix} \hat{b}_{11} & \hat{b}_{12} & \cdots & \hat{b}_{1q} \\ \hat{b}_{21} & \hat{b}_{22} & \cdots & \hat{b}_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{b}_{n1} & \hat{b}_{n2} & \cdots & \hat{b}_{nq} \end{pmatrix} = \mathbf{L}_{cc}^{-1} \mathbf{L}_{cy} \quad (30)$$

Here,

$$\begin{aligned} \mathbf{L}_{cc} &= \hat{\mathbf{c}}^T \left(\mathbf{I} - \frac{1}{n} \mathbf{1}^T \mathbf{1} \right) \hat{\mathbf{c}} \\ \mathbf{L}_{cy} &= \hat{\mathbf{c}}^T \left(\mathbf{I} - \frac{1}{n} \mathbf{1}^T \mathbf{1} \right) \mathbf{Y} \\ \mathbf{1} &= (1, 1, \dots, 1) \\ \mathbf{I} &= \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \end{aligned} \quad (31)$$

Then, the estimation of seasonal component $\mathbf{S}=\{\mathbf{S}_t\}$, $t=1,2,\dots,m$, is

$$\hat{\mathbf{S}} = \hat{\mathbf{b}} \hat{\mathbf{c}} \quad (32)$$

4.2 Random component model parameter estimation and MPHAR model prediction

Parameters of the multidimensional autoregressive model are estimated by applying the Yule-Walker estimation method.

The l^{th} step prediction of \mathbf{Y}_t is obtained from the best linear unbiased prediction of Eq. 4. The prediction formula is

$$\begin{aligned} \mathbf{Y}_{m+l} &= \mathbf{S}_{m+l} + \mathbf{R}_{m+l} \\ &= \mathbf{S}(m+l) + \sum_{j=1}^p \mathbf{H}_j \mathbf{R}_{m+l-j}, l=1,2,\dots \end{aligned} \quad (33)$$

5 Validation of water bloom multi-influence factors prediction

A lot of research papers indicate that the growth of phycophyta is influenced by many kinds of factors. Among these factors, the most important restricted one is nitrogen which is a necessary nutrient source for the growth of hydrophytes. Additionally, the growth of phycophyta is also affected by environmental factors, such as the water temperature.

Meanwhile, mass propagation of algae will also have adverse effect on water environment, which will change the pH value, chemical oxygen demand and so on. Water body chlorophyll concentration is an important reference index for measurement of water body primary productivity and eutrophication situation and it is also the ultimate index of water body algae stock on hand and judgment of water bloom.

Hence, total nitrogen, pH value, temperature, oxygen demand, chlorophyll and algae cell density are 6 water bloom influential factors in this paper, and chlorophyll and algae cell density are 2 reference indexes for judgment of water bloom.

1104 days monitoring data of 6 influential factors gained by Jiangsu Jinshu hydrological station from June in 2009 to June in 2012 are selected to validate MPHAR water bloom prediction model in the rivers and lakes of Jiangsu.

In this paper, 1094 days monitoring data of 6 influential factors are used for MPHAR modeling and predicting monitoring data of 6 influential factors from 1094 to 1104 days. The real monitoring data of 6 influential factors from 1094 to 1104 days are showed as blue curves in Fig. 2 to Fig. 7.

Prediction result is showed in Table 1 and as red curves in Fig. 2 to Fig. 7. Prediction error is a relative error, which is obtained by

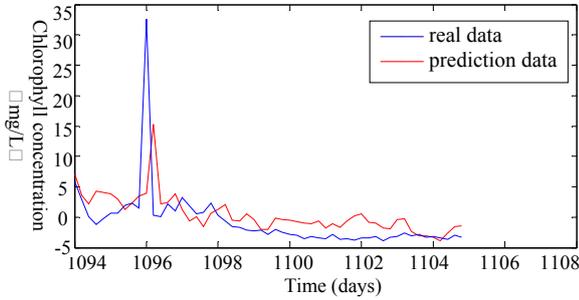
$$\text{error} = \left| \frac{\text{prediction data} - \text{real data}}{\text{real data}} \right| \quad (34)$$

A comparison between the prediction data by 1-Dimensional method and the prediction data by 6-

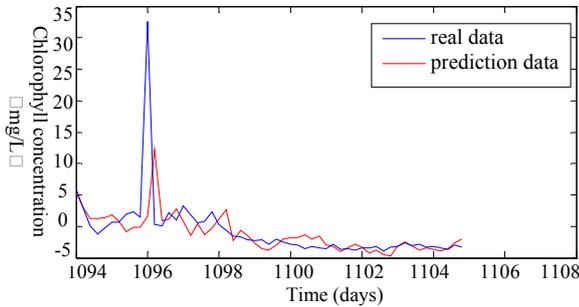
Dimensional method is showed in Table 1 and Fig. 2 to Fig. 7.

Table 1. Prediction Error

Influence Factor	Error by 1D method	Error by 6D method
chlorophyll	0.1914	0.1743
algae cell density	1.3528	0.6984
total nitrogen	0.1668	0.1302
temperature	0.2092	0.0676
oxygen demand	0.3055	0.1380
pH value	0.6315	0.5785

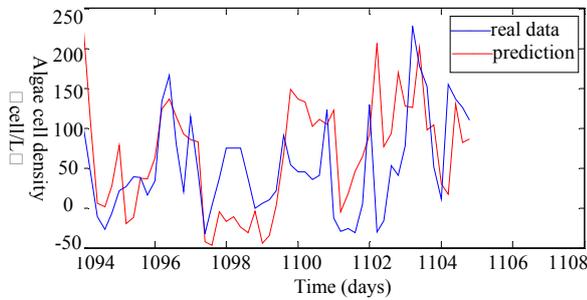


a) Prediction by 1-Dimensional method

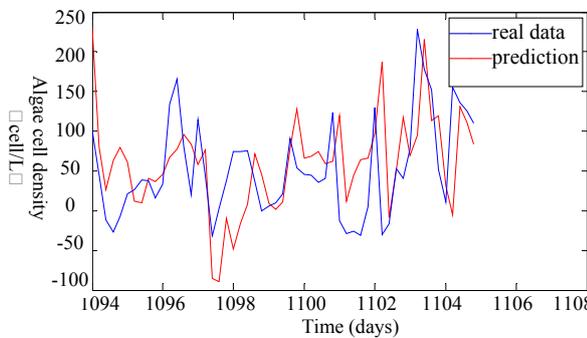


b) Prediction by 6-Dimensional method

Figure 2. Prediction of chlorophyll concentration.



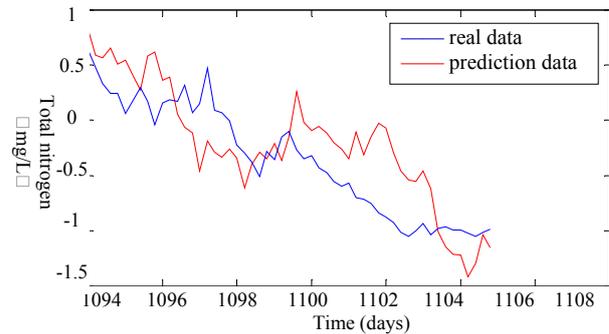
a) Prediction by 1-Dimensional method



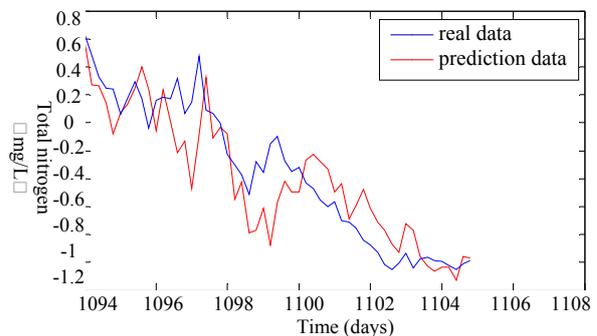
b) Prediction by 6-Dimensional method

Figure 3. Prediction of algae cell density.

From Table 1 and Fig. 2 to Fig. 7, compared with the prediction accuracy of 1-Dimensional method, prediction accuracy of MPHAR, which is 6-Dimensional method, is very satisfactory. MPHAR has improved both on the basis of a one-dimensional Hidden Periodic-Auto Regression model taking into account the integrated effect of multiple influential factors, and on the basis of a Multidimensional Auto Regression model taking into account the periodicity effect of environmental variables.



a) Prediction by 1-Dimensional method



b) Prediction by 6-Dimensional method

Figure 4. Prediction of total nitrogen.

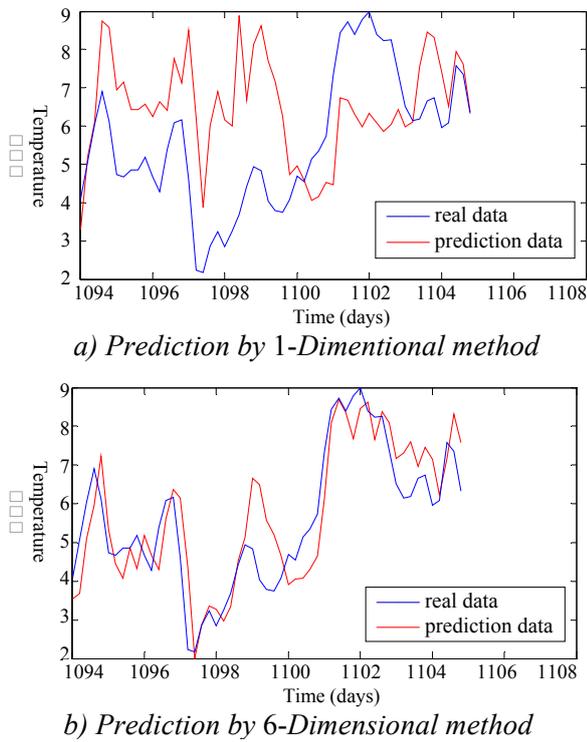


Figure 5. Prediction of temperature.

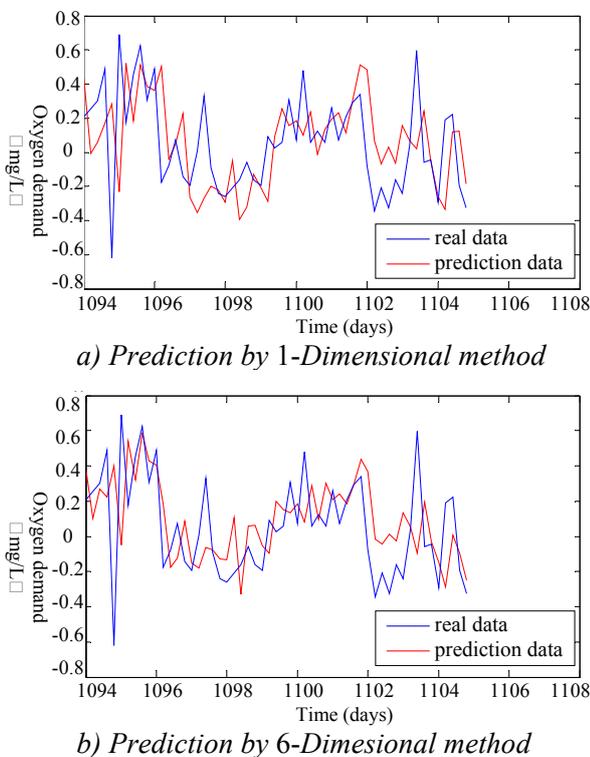


Figure 6. Prediction of oxygen demand.

6 Conclusions

In this paper, a new MPHAR model is proposed for time series analysis of multiple influence factors employed in the prediction of water bloom. Very accurate and valid water-bloom prediction is obtained by combining time series methods. According to the occurrence of the principle of water bloom, total nitrogen, pH value, temperature, oxygen demand, chlorophyll and algae cell density are used as influence prediction factors. The Chlorophyll and algae cell density are thus predicted as 2 reference indexes for judgment of water bloom. The prediction results show the effectiveness of the proposed method.

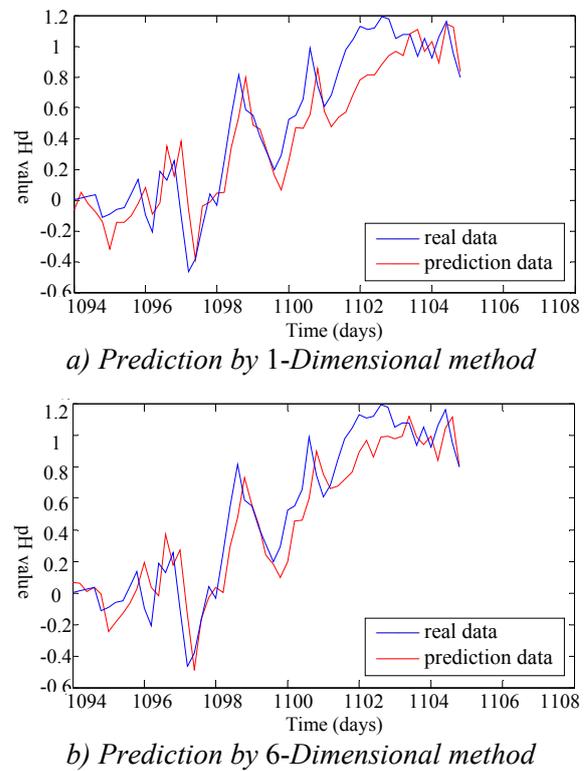


Figure 7. Prediction of pH value.

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