STRESS AND DYNAMIC ANALYSIS OF OPTIMIZED TRAILER CHASSIS

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This paper discusses the stress and dynamic analysis of truck ladder chassis. At the first stage, in order to design a chassis for self-weight reduction, material type and cross section profiles of chassis are selected according to a maximum normal stress and maximum strain theories. Then, the stress analysis of truck chassis has been carried out by ABAQUS software to determine maximum transverse deflection and stress distribution. Results show that open U-shaped profiles are sufficient for weight reduction which can endure loads. In the next stage, the prediction of the vibrational properties of the chassis which is of great significance in determining the natural frequencies of the structure, are considered. For this purpose, the modal analysis has been accomplished by the finite element packaged ANSYS software, and natural frequencies and mode shapes have been determined. In the final stage, vehicle dynamic model is used to compare vehicle dynamic parameters and ride response in optimized and not optimized chassis conditions. Simulation results clarified that the optimized chassis increases the roll stability and improves ride conditions. Also, in order to avoid matching the natural frequency of the chassis with the excitation frequency, natural frequencies and vibration modes have been analysed during the loading conditions.

Keywords: *dynamic analysis, finite element, ride and handling, truck chassis, vibration analysis*

Analiza naprezanja i dinamička analiza optimirane šasije prikolice

Članak se bavi dinamičkom i analizom naprezanja lančane sheme šasije kamiona. Najprije se, kako bi se konstruirala šasija najmanje težine, odabiru vrsta materijala i profili poprečnog presjeka šasije u skladu s maksimalnim normalnim naprezanjem i teorijama maksimalne deformacije. Zatim je provedena analiza naprezanja šasije kamiona primjenom ABAQUS softvera u svrhu određivanja maksimalnog poprečnog progiba i raspodjele naprezanja. Rezultati pokazuju da su U-profili dovoljni za smanjenje težine koja može izdržati opterećenja. U sljedećem su koraku razmatrana vibracijska svojstva šasije, što je od velike važnosti za određivanje prirodnih frekvencija konstrukcije. U tu se svrhu provela modalna analiza metodom konačnih elemenata pomoću ANSYS softvera te su određene prirodne frekvencije i oblici vibriranja. U završnom se stupnju dinamički model vozila iskoristio za usporedbu dinamičkih parametara vozila i voznih karakteristika kod optimalizirane i neoptimalizirane šasije. Rezultati simulacije su pokazali da optimalizirana šasija povećava stabilnost kod ljuljanja i poboljšava uvjete u vožnji. Također, da bi se izbjeglo poklapanje frekvencije pobude (ekscitacije) s prirodnom frekvencijom šasije, prirodne frekvencije i oblici vibriranja analizirali su se uz promjene uvjeta opterećenja.

Ključne riječi: *analiza vibracija, dinamička analiza, konačni element, šasija kamiona, vožnja i rukovanje*

1 Introduction

A truck chassis is a structural pillar of a commercial vehicle. As shown in Fig. 1, the main task of a trailer chassis is withstanding the components and the loads placed on it. The stress and modal analysis techniques are significantly essential for automotive chassis structure design. It is important to study the dynamic characteristic of the car chassis so that resonance and structure failure does not occur on the chassis in working condition.
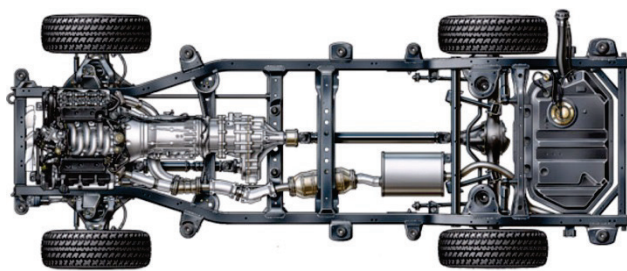

Figure 1 Ladder chassis and component configuration [1]

Resonance is often the cause of or at least a contributing factor for too many of the vibration and noise related problems that occur in structures and operating machinery. Therefore, investigation on dynamic characteristic is very significant in order to control vibration and noise. Suhir and Burke in [2] stated that finite element analysis using finite element method is a very useful tool to identify dynamic characteristics such as natural frequencies and mode shapes. A good chassis shall also meet precise stiffness requirements in order to

allow a safe drive under the most diverse traffic conditions. FE method is a powerful numerical engineering analysis tool, and widely used in static and dynamic stress analyses of vehicles [3 ÷ 8]. In [4] the results of the numerical analysis revealed that the location maximum deflection and maximum stress agrees well with theoretical maximum location of simple beam loaded by uniform force. Xu and et al. [8] proposed a new finite element (FE) solver based on the solid-shell element dynamic explicit algorithm and implicit algorithm is developed for the simulation of medium thick plate metal forming and springback. Results demonstrated that the solid-shell element model is competitive enough in the simulation of medium thick plate metal forming and springback. Taheri-Behrooz et al. in [9] presented a comprehensive numerical investigation, using the finite element method (FEM), to simulate the instability response of the intact and perforated composite tubes under compressive axial loading. In [10] the crashworthiness of the intermediate rail vehicle was assessed by simulating an impact of the vehicle with a rigid wall, then, the structural characteristics of the vehicle were analysed based on the structural behaviour during this impact. Conle and Mousseau in [11] reported on an analytical study of the fatigue life of automobile chassis components using automotive proving ground load history results combined with recent computational advances.

Sane et al. [12] performed stress analysis on a light commercial vehicle chassis using iterative procedure for reduction of stress level at critical locations. Guo and

Chen [13], research into dynamic and modal analysis of a space chassis (complex 3-dimensional chassis) and analyse transient response using the principal of superposition. Most dynamic models for the engine mount systems have been based on isolation theory, and vibration of the foundations has been neglected. It is necessary to model engine mounting systems with flexible foundations in order to capture these vibration coupled problems [14]. In this previous research, it was found that the coupling effects were substantial for frequencies lower than idle speed but negligible for frequencies higher than the idle speed. Kotari and Gopinath [15] present the analysis of chassis frame for improving its payload by adding stiffener and *c* channel at maximum stress region of chassis frame, which results in increasing pay load capacity by about 20 %. Marzban et al. in [16] presented the most important parameters including material, thickness, shape and impact condition and studied them for design and analysis of an automotive front bumper beam to improve the crashworthiness design in low-velocity impact. For this purpose bumper beams in passenger cars as a key structure for absorbing impact forces during collisions, were characterized by FEM modelling in accordance to low-velocity impact standards.

As a car travels along the road, depending on the road profile, the car chassis is excited by dynamic forces induced by the road roughness, engine, transmission and more. Under such various dynamic excitations, the car chassis vibrates. Whenever the natural frequency of vibration of a machine or structure coincides with the frequency of the external excitation, there occurs a phenomenon known as resonance, which leads to excessive deflections and failure [17].

In [18] the modal analysis was performed to determine the mode shapes and the natural frequencies of the car chassis (Uni body chassis) from 50 ÷ 99 Hz. Also, it showed that Maximum stress, 45 MPa was determined at each corner at pillar joint and this value is under the allowable stress for steel which is 300 MPa.

Numerical simulation and field test are used to investigate heavy duty vehicle load. The results demonstrate that the proposed vehicle model can offer efficient and realistic simulation for stochastic dynamic loads [19]. In [20] vehicle dynamic response improved by optimization of suspension, but in this paper, effects of chassis optimization in heavy vehicles dynamic is considered. Previous works [21÷ 23] show that optimal design of chassis regarding to weight reduction and mounting place of components on chassis can significantly affect vehicle performance and dynamic

characteristics. Therefore, in this study, numerical results on stress, Dynamic and Vibration analyses of ladder chassis are presented. Finite element (FE) method is used to assess the static and dynamic structural behaviour of chassis.

2   Chassis design

Components of truck, such as engine, transmission system, exhaust, suspension system, fuel tank and luggage are mounted on FH16 trailer chassis as shown in Fig. 2. As shown in Tab. 1, the mentioned equipment imposes a considerable load on the truck and in order for the chassis to survive the loads some precautions must be taken.

Table 1 Loads of vehicle components on chassis

Automotive component	Weight / kg	Position / mm	Label load	Load / N
Front bumper	15	0	P1	147,15
Radiator	6	300	P2	58,86
Engine	100	650	P3	981,00
transmission	90	1100	P4	882,90
Front pass seat	200	2050	P5	1962,00
Exhaust	15	2885	P6	147,15
Rear pass seat	300	3000	P7	2943,00
Fuel tank	70	3100	P8	686,70
Luggage	70	3850	P9	686,70
Rear bumper	15	4200	P10	147,15
Front body in white & trim	-	0 ÷ 1250	W1	1450 N/m
Centre body in white & trim	-	1250 ÷ 3300	W2	1650 N/m
Rear body in white & trim	-	3300 ÷ 4200	W3	850 N/m

Also the chassis is bound to have the necessary rigidity for the loads. Due to uncertainty in estimating the vehicle loads and the dynamic and oscillating nature of it, the quasi-static method is used to conceptual design of ladder frame. In this method instead of dynamic and oscillating loads, coefficient of static loads on the chassis is applied which is called dynamic load factor (DLF). In order to design a chassis at the first step, the force exerted on the chassis and its location is determined. Then, note to the boundary condition, bending moments and shear forces diagrams along the chassis are extracted. Finally, appropriate profile section which reduces the weight and endures loads is selected from handbook [24] and Maximum allowable stress and strain theories are used to design a chassis. In this research the result of two common profiles, open web studs and closed rectangular blank are compared.
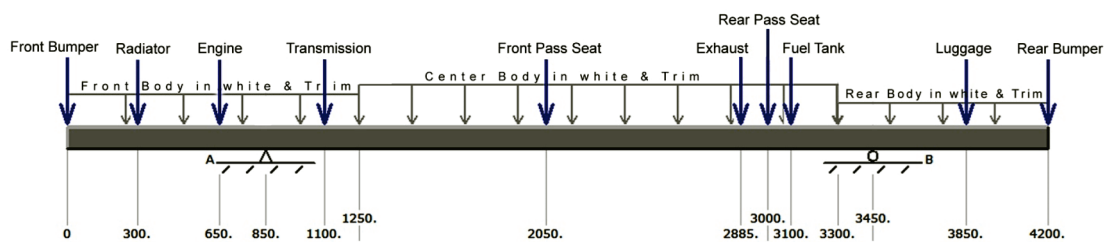

Figure 2 Vehicle configuration load on chassis

Bending moment $M_b(x)$ and shear force $V(x)$ along the chassis are [24]:

$$V(x) = -\int w(x)\mathrm{d}x,$$
$$M_b(x) = \int V(x)\mathrm{d}x, \qquad (1)$$

where $w(x)$ is expanded load on chassis.

2.1 Maximum stress theory

Regarding the bending moment and shear stress diagrams maximum normal stress along the chassis can be evaluated from:

$$\sigma_{\max} = N\frac{M_{b\,\max}}{S_{\min}}, \qquad (2)$$

where $\sigma_{\max}$ is maximum normal stress, N is dynamic factor load (includes safety factor) and $S_{\min}$ is minimum section module. Therefore, according to maximum allowable stress, the section module is determined by considering the dynamic load factor. Optimal section module should be larger than $S_{\min}$ and have minimum density to reduce chassis weight.

2.2 Maximum strain theory

Maximum strain theory, which is based on the maximum allowable chassis deflection, is more conservative than maximum stress theory. The relation between chassis strain and bending moment is described as follows:

$$E \cdot I \cdot y(x) = \int \mathrm{d}x \int M_b(x)\mathrm{d}x + C_1 x + C_2, \qquad (3)$$

where E and I are modulus of elasticity and inertia moment of chassis respectively. $y(x)$ is function of deflection along chassis deflection, and C_1, C_2 are constant coefficients, which are calculated according to boundary conditions.

3 Finite element modelling

The ABAQUS and ANSYS software package is used to build and analyse the solid model and finite element mesh of the truck chassis for dynamic and vibration analysis, respectively. In order to achieve accurate results of FEM an adequate choice of both the finite elements size and the contact elements definition is necessary. In this paper to manage the contact formulation, the Augmented Lagrangian Method has been chosen as it allows, by manually setting the contact stiffness parameters, a better approximation of the interactions occurring within the contact areas of welded structures [25].

The chassis shown in Fig. 3 is modelled taking advantage of symmetry, using shell elements. In the meshing process, the mesh is biased towards the joints where the most deformation occurs. Compared with previous works approximately 7000 elements for the

model is appropriate [23]. In order to perform stress and strain analysis in static and dynamic condition chassis is fixed to the connection point with wheels in vertical direction. A API 5L X65 steel, with a Young's modulus E = 210 GPa, a Poisson's ratio $v = 0,3$, a yield stress R_e = 440 MPa and a mass density $\rho = 7800$ kg/m^3 has been employed.

Also for modal analysis with ANSYS, all degrees of freedom are free to evaluate nature response of chassis.

The chassis is realized by means of rectangular section tubular beams, by a length of 4200 mm, joined to each other by full welding; several added rectangular and c channel at maximum stress region of chassis frame stiffeners increase the overall assembly stiffness.
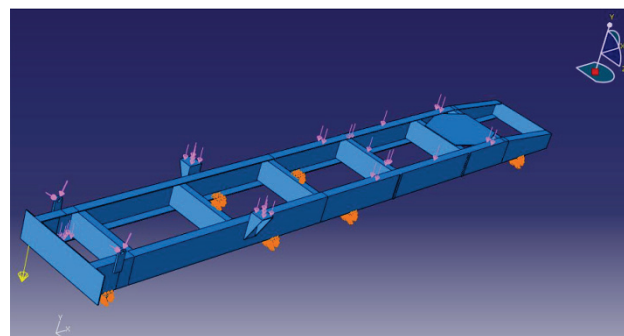

Figure 3 The model of chassis with boundary condition and loads in ABAQUS

To determine the loads on the chassis of the dynamic state, logical and experimental criteria should follow. Therefore dynamic load factor (DLF) is considered.

4 Vibration analysis

Modal analysis, using Frequency Response Function (FRF) measurement techniques based on Fast Fourier Transformation (FFT), has been widely used in automotive industries in determining the dynamic characteristics (natural frequencies, mode shapes and damping) of linearized vehicle model. The equation of motion for an undamped system is:

$$[m]\{\ddot{q}\} + [k]\{q\} = \{0\}, \qquad (4)$$

where k and m stand for stiffness and mass respectively, q is displacement, which for a linear system, will be of harmonic form. Thus,

$$\{q\} = \{Q_i\}\sin(\omega_i t), \qquad (5)$$

where Q_i is amplitude of mode shape of the ith natural frequency ω_i. Substituting in Eq. (6):

$$-\omega^2[m] + [k] = \{0\}. \qquad (6)$$

The solution to Eq. (6) gives Eigen values, the square roots of which are the natural frequencies, Eigen vectors of amplitudes corresponding to the natural frequencies.

In this analysis we have made use of subspace method in ANSYS. Modal analysis has been performed after creating the chassis finite element model and meshing in

free-free state and with no constraints. The results have been calculated for the first 20 frequency modes and show that road and excitations are the most important problems for truck chassis. Also the global and local vibrations in different frequencies are illustrated.

The mode shapes of the truck chassis at certain natural frequencies are critical issues to determine optimized mounting point of the components like engine, suspension, transmission and more. Therefore it is important to include the dynamic effect in designing the chassis.

5   Dynamic analysis

For the purpose of vehicle simulation several models with different degrees of freedom are available in the literature. Models with large number of degrees of freedoms generally result in increased computing time while the accuracy of the results only slightly increases. In this paper, in order to ride and handling analysis of a truck during double lane change manouver, a nonlinear 8 degree-of-freedom (dof) vehicle model in [26] that includes both lateral and longitudinal dynamics is utilized. The degrees of freedom associated with this model are the longitudinal velocity, the lateral velocity, the yaw rate, the roll rate, and four wheel rotational speeds. The equations of motion for the vehicle model read [26]:

Longitudinal motion:

$$m(\dot{u} - rv) = \sum F_x. \tag{7}$$

Lateral motion:

$$m(\dot{v} + ru) + m_s h_s \ddot{\Phi} = \sum F_y. \tag{8}$$

Yaw motion:

$$I_{zz}\dot{r} - I_{xz}\dot{\Phi} = \sum M_z. \tag{9}$$

Roll motion:

$$I_{xx}\ddot{\Phi} - I_{xz}\dot{r} = \sum M_x, \tag{10}$$

where m and m_s are the total mass and the rolling mass respectively, I_{zz} and I_{xx} are mass moment of inertia about z-axis and x-axis, I_{xz} is the product of inertia with respect to x and z axes. h_s is the height of sprung mass CG to roll axis. The terms ΣF_x and ΣF_y are the total external forces along the x and y directions. ΣM_x and ΣM_z are the resultants of the moments acting around the roll and yaw axes of the vehicle-fixed coordinate system.

6   Result and discussion

Finite element analysis for truck chassis stress and strain distribution through component load is performed in ABAQUS. Then, the modelled ladder chassis in an earlier section is used for the vibration analysis (natural frequencies and mode shapes) in ANSYS. The chassis and component used in the analysis is given in Tab. 1.

6.1 Chassis optimal weight result

In order to optimally design the chassis, by consideration of its support condition at joint point with suspension systems and component loads on it, the chassis is divided to three sections as illustrated in Fig. 4.
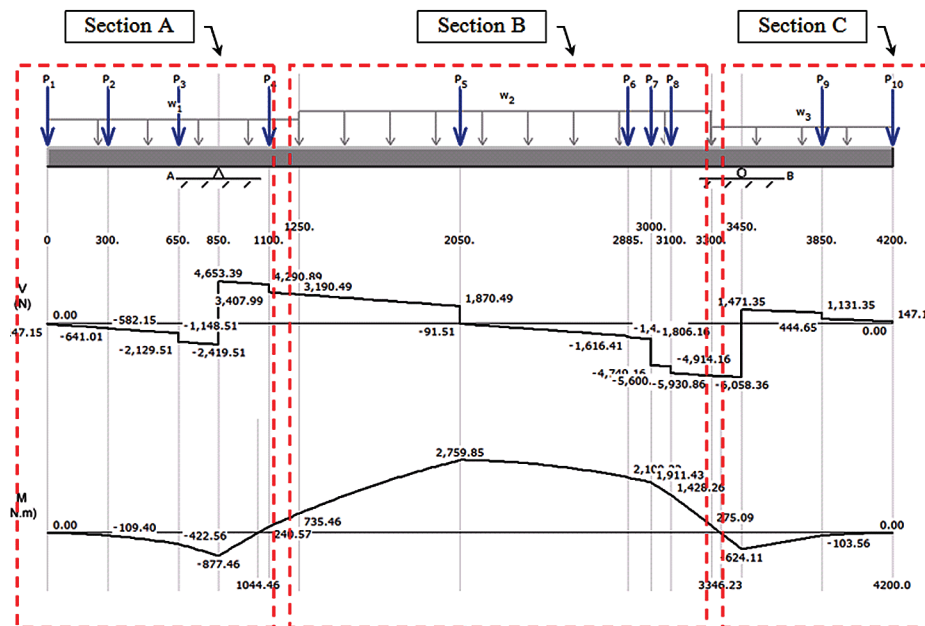

Figure 4 Bending moment and shear force diagrams

According to maximum moment and shear forces for each section the minimum section module is calculated and the results given in Tab. 2.

In order to reach optimal chassis, both maximum stress and strain theories should be satisfied for any sections of A, B, C simultaneously. Design criteria for

each section are calculated and the results are presented in Tab. 3.

Section	Position / mm	Max. shear / N	Max. moment / N·m
A#	0 ÷ 1250	7072,90 at X= 850	−877,46 at X= 850
B#	1250 ÷ 3300	5930,86 at X=3100	2759,85 at X=2050
C#	3300 ÷ 4200	7529,71 at X=3450	−624,11 at X=3450

Table 3 Design criteria for each section

Section	Max stress theory	Max strain theory
A#	$S_A \geq 5{,}27\,\text{cm}^3$	$I_A \geq 269{,}8\,\text{cm}^4$
B#	$S_B \geq 16{,}56\,\text{cm}^3$	$I_B \geq 334{,}01\,\text{cm}^4$
C#	$S_C \geq 3{,}75\,\text{cm}^3$	$I_C \geq 300\,\text{cm}^4$

Note to the design criteria presented in Tab. 1, open web studs (U-shape) and closed rectangular blank profiles are selected from [19], which results in chassis minimum weight. Results, as below, for each case are compared in Tab. 4.

Table 4 Chassis weight and safety factor for profiles

Section	Profile	Profile num. / mm	Weight kg/m	Safety factor
A#	U-Shape Rectangular	120/60*6	13,375 16,25	1,92 2,03
B#	U-Shape Rectangular	133/60*6,5	23,575 33,01	2,11 1,85
C#	U-Shape Rectangular	133/60*6,5	10,35 11,7	2,54 2,32
U-shape chassis weight			94,6 kg	
Rectangular chassis weight			121,91 kg	

Results in Tab. 3 show that the dynamic safety factor for each profile is approximately the same, whereas the U-shaped profiles result in less weight. Moreover, safety factor results show that section A and B are critical sections, because of engine mounting in front part of chassis (section A). Also, section B which has more length than other parts, has maximum deflection.

6.2 Static and dynamic analysis

Stress analysis is used to determine the stress and strain distribution when the chassis structure receives the loads. Figs. 5 ÷ 8 compare the stress and strain distribution in the chassis for static and dynamic loads.
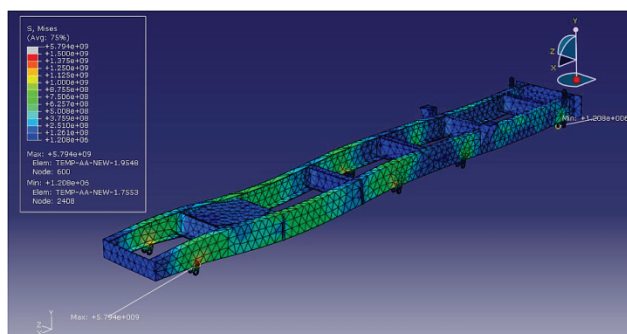

Figure 5 Stress distribution (static load)

Figs. 5 and 6 clearly show that the component mounting place has higher stress concentration. Moreover, maximum stress in both cases is in section A, where engine and transmission system which have more weights, are mounted. In other words, maximum stress occurs just under the loading point. Also, it is obvious that stress distribution in dynamic condition is higher than static loads. Nonetheless, the stress level in the frames is below the yield or ultimate strength of the chassis material. Also, vertical deflection of chassis is observed to be maximum (4 mm) at the front and middle of the chassis.
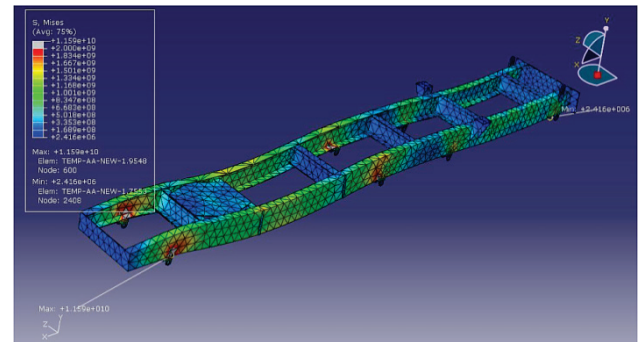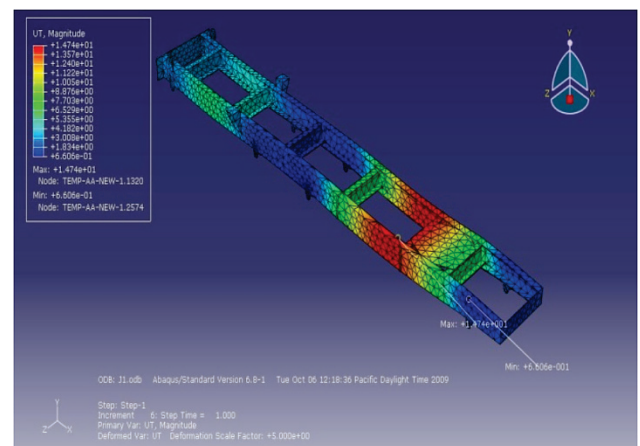

Figure 6 Stress distribution (dynamic load)

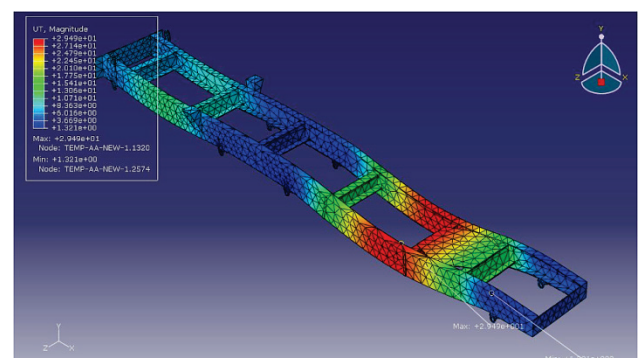

Figure 7 Strain distribution (static load)

Figure 8 Strain distribution (dynamic load)

6.3 Vibrational analysis

Modal analysis provides results which indicate the natural vibration characteristics of a chassis. It involves determination of the natural frequencies and modes of vibration. Modal analysis has been performed after creating the chassis finite element model and meshing in free-free state and with no constraints in ANSYS. Since chassis has no constraints, the first 6 frequency modes are almost vanished. These modes are related to the chassis displacement and rotation in x, y and z directions. First 16

analysed modes for optimized and un-optimized chassis and their mode shapes are compared in Tab. 5.

Table 5 Chassis natural frequencies (Hz)

Mode shape	Non-optimized (Optimized)	Mode shape	Non-optimized (Optimized)
Torsion	9,52 (8,23)	Torsion	92,478 (85,7)
Torsion	21,19 (19,57)	Bending	113,75 (108,15)
Bending	33,517 (27,20)	Torsion & Bending	127,13 (125,21)
Torsion	34,68 (29,70)	Local Bending	130,84 (128,95)
Torsion	45,78 (41,00)	Local Bending	138,01 (132,76)
Torsion & Bending	**53,861** (46,56)	Torsion & Bending	153,80 (143,89)
Bending	68,91 (65,35)	Bending	163,14 (159,38)
Bending	81,30 (71,73)	Torsion & Bending	168,21 (163,56)

The major areas of concern in the truck chassis were found to be coincidence of a structural resonance at 50 ÷ 55 Hz [27], and experienced the torsional and bending mode. Sixth natural frequency of un-optimized chassis which has integrated bending and torsional mode (53,861 Hz) is in critical range. Whenever, optimized chassis natural frequencies are placed outside the critical range.

Figure 9 First mode shape (torsional)

Figure 10 Second mode shape (torsional)

Figs. 9 ÷ 12 depict first (torsion), second (bending), 6th (bending and torsional) and 12th (local bending) modes respectively. The other mode shapes are presented in appendix.
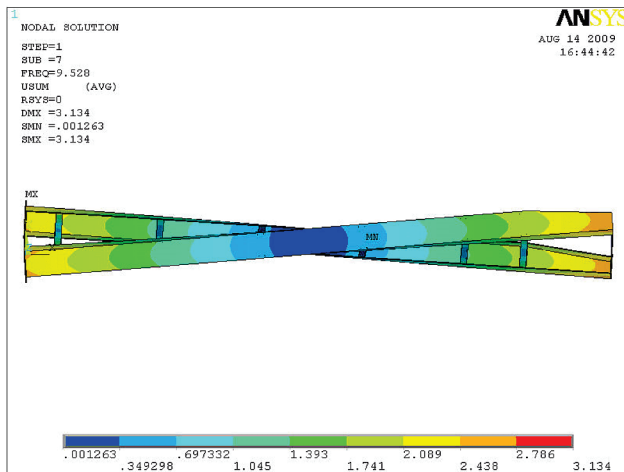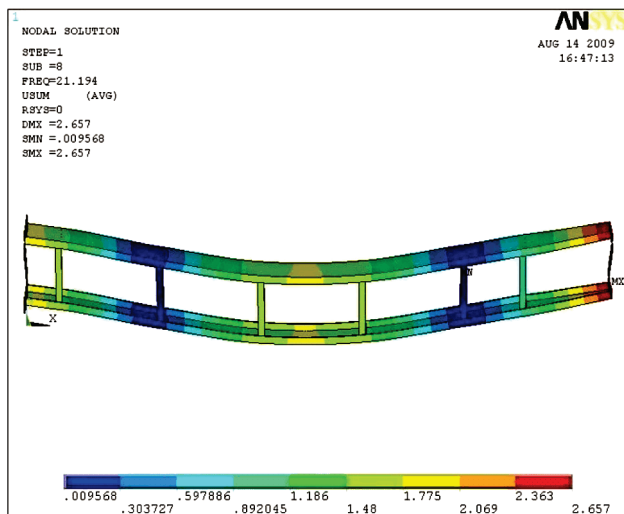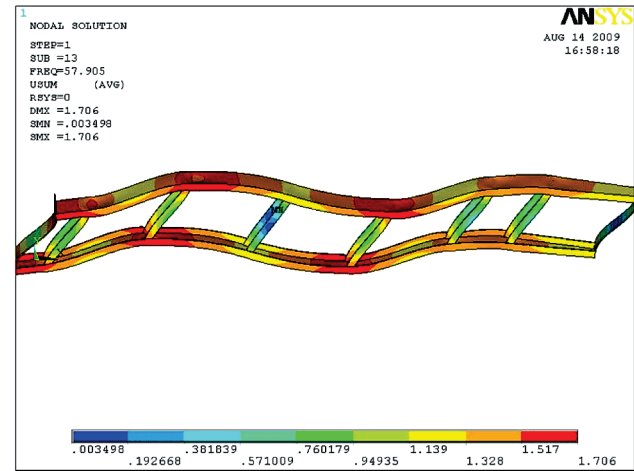

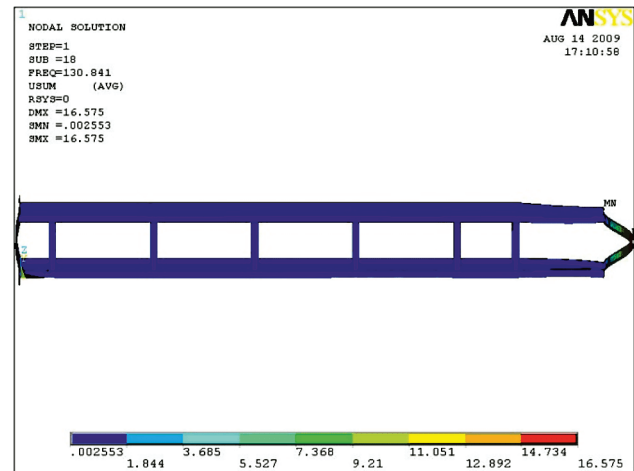**Figure 11** Sixth mode shape (combined)

Figure 12 12th mode shape (local bending)

In low frequency noises (50 ÷ 200 Hz), engine and road excitations are major sources. Whereas, in high frequency noises (200 ÷ 4000 Hz), with note that engine power limited to produce more noises, tire and aerodynamic noises are considerable. Therefore, the main excitations are at low speeds, while at high speeds the excitations to the chassis are much less. In order to avoid resonance phenomena, the natural frequency of the truck chassis should not coincide with the excitation source frequencies, which are obtained by chassis optimization. Also, this improves vehicle ride and handling conditions.

The local bending vibration occurs at high frequencies (130 ÷ 138 Hz) at the top hat cross member. Road excitation is the main disturbance to the truck chassis when the truck travels along the road. In practice, the road excitation has typical values varying from 0 to 100 Hz. At high speed cruising, the excitation is about 3000 rpm or 50 Hz [27, 28].

Distribution of mass and stiffness of the vehicle has a significant impact on its vibration behaviour. The equipment installed on the chassis of truck increases the chassis mass which leads to the decrease of natural frequencies. Regarding the previous discussions about the diesel engine speed, we can say that natural frequencies

are in critical range. Hence with decreasing the chassis length which increases the chassis stiffness, we increase the natural frequencies to place them in the appropriate range. In addition, with changing the gasoline tank situation and performing similar changes, we can prevent coinciding the simulation force frequencies and natural frequencies. Otherwise resonance phenomenon occurs and if these two frequencies coincide, this phenomenon destroys the chassis.

Chassis harmonic analysis is conducted by applying a cyclic load (harmonic) with amplitude 1 N at the end of the chassis as shown in Fig. 13.
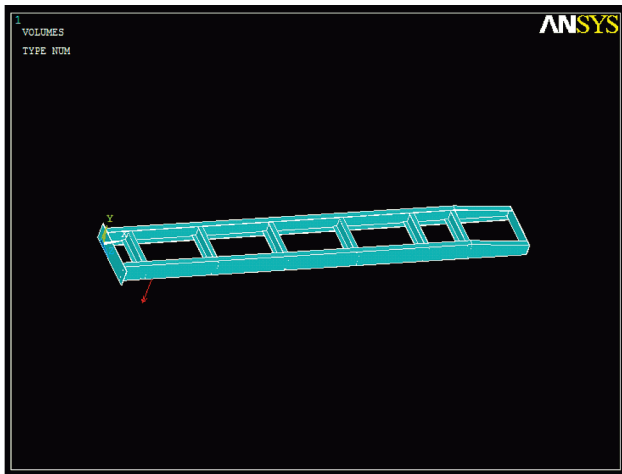

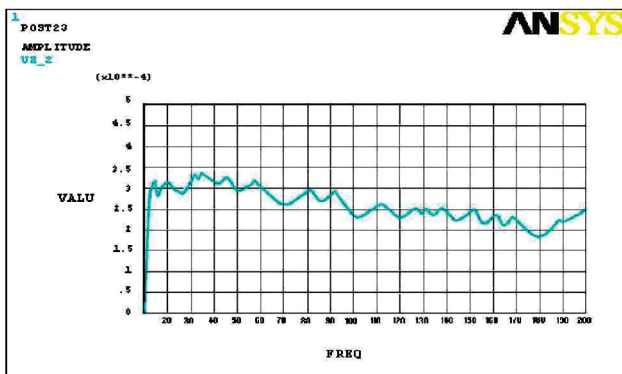**Figure 13** Harmonic load with 1 N magnitude applied

Figure 14 Chassis harmonic excitation by 1 N load

With the analysis of harmonic analysis, the frequency behaviour of the load on the truck chassis is illustrated in Fig. 14.

As shown in Fig. 14 the optimized chassis oscillations are observed near the natural frequencies of chassis which are limited.

6.4 Dynamic analysis

In order to investigate the effects of chassis weight and its distribution on vehicle performance, vehicle lateral dynamic is simulated by using Simulink/MATLAB. The simulation scenario consists of the double lane change maneuver. The vehicle velocity is kept constant during the maneuver, the vehicle is able to brake and accelerate in order to keep the velocity 54 km/h simulation results for lateral acceleration. Side slip, yaw rate and roll angle for open (optimized) and closed (non-optimized) profiles are compared in Figs. 14 ÷ 17, respectively.
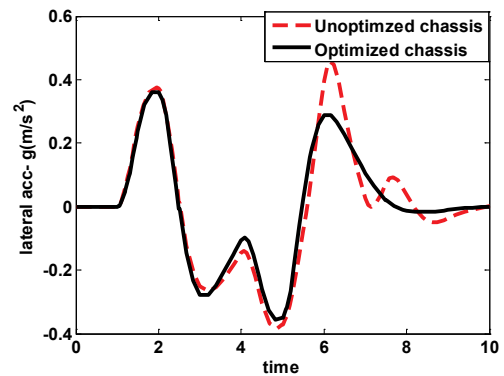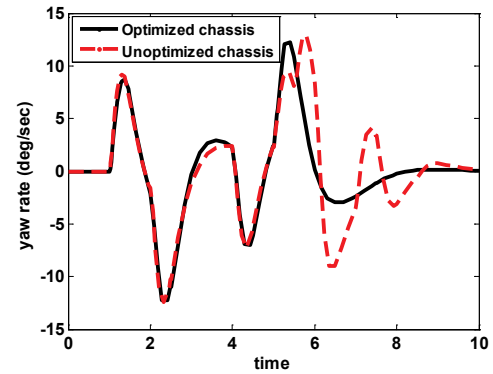

Figure 15 Lateral acceleration

Figure 16 Yaw rate

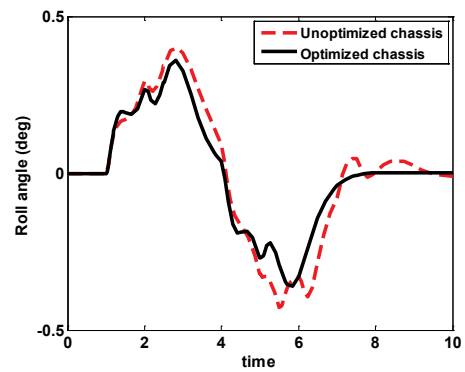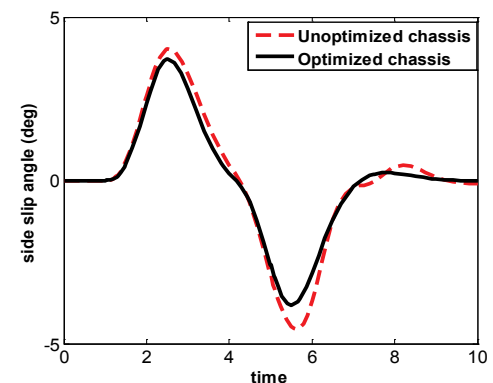**Figure 17** Roll angle

Figure 18 Side slip angle

The results indicate that the optimized chassis (open profile) has improved the performance of the vehicle compared with that of the non-optimized (closed profile) one. Non-optimized chassis is not converged to steady-state values at a finite time and remains severely oscillatory so that the dynamic behaviour of the vehicle is quite unsafe.

For practical instance, in order to improve FH16 trailer ladder chassis performance, adding double chassis known as auxiliary chassis, using open U-shaped section instead of closed rectangular profiles in manufacturing process, optimal mounting location of the engine and transmission system, strengthening the chassis by adding stiffener and also changing the connecting bars are some usual tricks to enhance the rigidity and adjusting vehicle mass and stiffness, which can affect the chassis vibrational and dynamic behaviour of vehicle effectively. Using these methods, we can prevent resonance phenomenon and unusual chassis vibration. Hence they will improve ride and durability of vehicle and chassis respectively in dynamic loads conditions.

7 Conclusion

In order to conduct stress and vibration analysis of the existing truck chassis, finite element method is used to assess the static and dynamic structural behaviours of truck chassis. Cross-section of chassis reduced the overall weight of chassis by 21 %, which is optimized according to loading; after modifications the finite element analysis was carried out. The stress/strain distribution and natural frequencies were calculated along the chassis. Maximum stress and strain levels are found in front section of chassis, where engine and transmission are installed, and this section should be modified by U-shape stiffeners. In order to improve static and dynamic characteristic of chassis to endure equipment loads, cross section and mass distribution of the chassis were optimized. The simulation results indicate that the chassis weight optimization and cross-section have considerable effect on ride comfort, handling, stability and prevention of vehicle rollover through quick speed maneuvers. Also, this optimization placed chassis natural frequencies out of the critical range, which can improve vehicle ride and handling condition considerably. The major areas of concern in the truck chassis were found to be coincidence of a structural and engine ideal resonance at 50 ÷ 55 Hz, at which combined torsional and bending modes occurred. In the optimized chassis, the structural natural frequency can be moved out of this critical range.

8 References

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Appendix:
Mode shapes