ISSN 1330-3651 (Print), ISSN 1848-6339 (Online)
UDC/UDK 519.876.5:531.112/.113]:681.1

COMPARISON OF KINEMATIC ANALYSIS METHODS AT ONE DEMONSTRATION MECHANISM

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Subject review

The mechanisms are systems of moving parts that perform some function. One of their main advantages is converting the input forces and movements into a desired set of output forces and movements. The mechanism movement simulation is described in this paper. The solution includes calculation of velocities, accelerations and overall performance analysis. Numerical simulation of model mechanism was performed in the software package SAM. The velocities optimal for the drive element and the initial position of mechanism were given for initial conditions. All velocities and accelerations of mechanism points in this position were calculated. Results of computer simulation were compared with the results obtained by vector method. By comparison of the results it was found that the deviations were small, therefore it was concluded that the simulation was successfully performed.

Keywords: *acceleration, MathCad, mechanism, SAM, velocity*

Usporedba metoda kinematske analize kod jednog demonstracijskog mehanizma

Pregledni članak

Mehanizmi su sustavi pokretnih dijelova koji obavljaju neku funkciju. Jedna od njihovih glavnih prednosti je pretvaranje ulazne sile i pokreta u željeni set izlaznih sila i pokreta. U ovom radu simulira se kretanje mehanizma. Rješenje obuhvaća proračun brzina, ubrzanja i numeričku analizu mehanizma. Numerička simulacija modela mehanizma provedena je u softverskom paketu SAM. Za početne uvjete zadane su brzine pogonskog elementa i početni položaj mehanizma. Izračunate su sve brzine i ubrzanja točaka mehanizma u tom položaju. Rezultati računalne simulacije uspoređeni su s rezultatima dobivenim vektorskom metodom. Usporedbom rezultata utvrđeno je da su odstupanja mala te se zaključuje da je simulacija uspješno provedena.

Ključne riječi: *brzina, MathCad, mehanizam, SAM, ubrzanje*

1 Introduction

The mechanism is a device designed to transform the input force and movement into the desired set of output forces and movements. Generally, mechanisms consist of moving parts such as gears and gear chambers, belts, chain drives and articulated joints. Also, mechanisms consist of devices that cause friction such as brakes and clutches, structural components such as frames, fasteners, bearings, springs and seals as well as various specialized machinery parts, such as pins and keys.

To achieve motions that are accurately performed through the mechanism, members of the mechanism should be resistant to deformation. Therefore, the definition of the mechanism can be extended so that the mechanism implies a complex of rigid or solid bodies whose general purpose is conversion of one form of movement into a different form of movement.

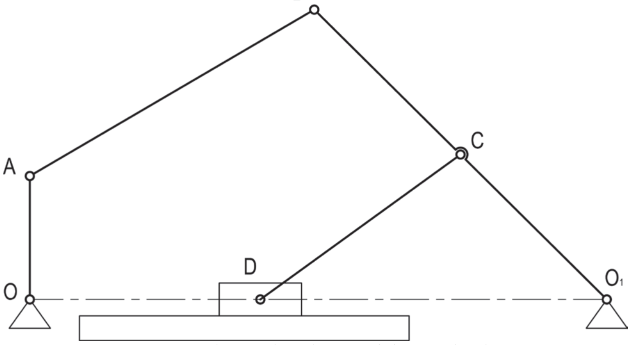

Figure 1 Kinematic scheme of the mechanism

The mechanism can be shown by detailed drawings, preliminary sketches, kinematic and structural scheme. Kinematic diagram or kinematic scheme shows only those elements of the mechanisms that have an impact on its movement [1, 2, 3, 7]. Kinematic scheme of specific four crank linkages with additional dyad is made in the scale necessary for noticing the movement. It is also the basic drawing for calculation of the mechanism's kinematics. In kinematic schemes mechanisms members are shown simplified. Fig. 1 shows the kinematic diagram of the mechanism treated in this paper.

For the purpose of structural analysis of the mechanism and choosing a calculation method, structural scheme of the mechanism is used. In this scheme mechanism members are shown regardless of dimensions which have no impact to mechanism kinematics (width, thickness). The structural scheme of the mechanism is shown in Fig. 2.

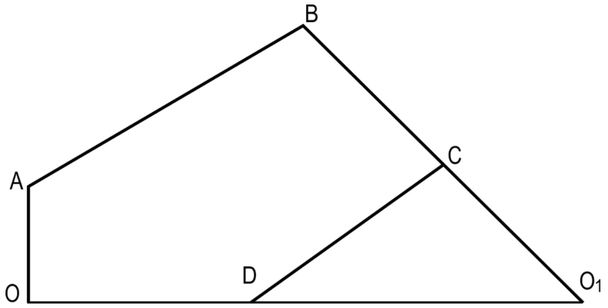

Figure 2 Structural scheme of the mechanism

2 Kinematic analysis

2.1 Model of demonstration mechanism

Although the mechanism presented in the paper is two dimensional, SolidWorks package is used to create a three-dimensional model of the mechanism with minimum thickness. Graphical solution is obtained adding constraints between model and the solution, and changing position of model, results in new graphical solution.

In this paper, the mechanism is in the initial position, which is shown in Fig. 3 and velocity and acceleration of points in this position are analysed. Engine that has two velocities is used to run the demonstration mechanism. Rotational speed of engine is thus measured in the first gear, its value is 55 rpm and according angular velocity of drive element – crank $\overline{\mathrm{OA}}$ is calculated.

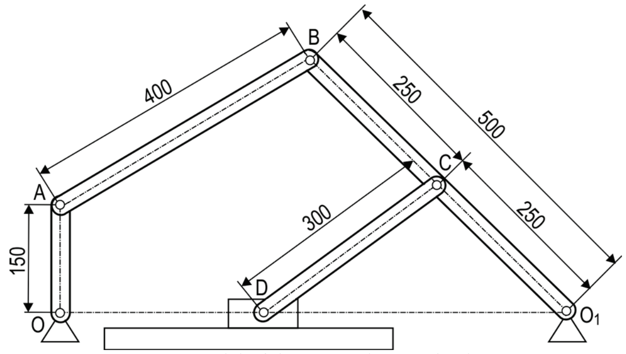

Figure 3 Model of demonstration mechanism [2]

2.2 Graphoanalytical solution in SolidWorks

The main role of simulation modeling can be prediction, optimization and process control [5, 6, 7].

Graphoanalytical method of calculating the velocity and acceleration used in this paper is based on a graphical presentation of vector equations which describe the relationship between velocity and acceleration of two points of a rigid body in planar movement [2, 3, 4, 8].

Using features of SolidWorks (SW) [9], line lengths in velocity and acceleration plan are related with formulas and constraints to the position plan. Added geometrical relationships allow, with moving the slider or a bar, for the mechanism to be arranged in a new position and also generate a new velocity and acceleration plan.

From the model of demonstration mechanism the position plan shown in Fig. 4 is obtained.

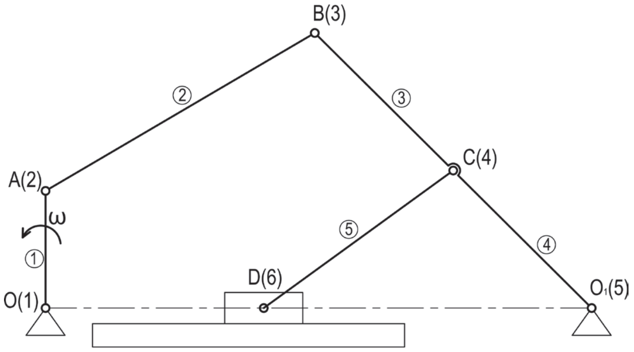

Figure 4 Position plan

The data used to calculate the velocity and acceleration are: $\overline{\mathrm{OA}} = 0{,}15$ m, $\overline{\mathrm{AB}} = 0{,}4$ m, $\overline{\mathrm{BO}_1} = 0{,}5$ m, $\overline{\mathrm{BC}} = \overline{\mathrm{CO}_1} = 0{,}25$ m, $\overline{\mathrm{CD}} = 0{,}3$ m, $\overline{\mathrm{OO}_1} = 0{,}7$ m, n = 55 rpm, $\omega = \dfrac{\pi\cdot n}{30} = 5{,}756$ rad/s.

First, the fixed point P_v = O' = O'$_1$, which is also called velocity pole, is selected (Fig. 5). In the velocity plan, it represents all points of the mechanism in which the velocity equals zero, and from which the velocity plan is further drawn.

The calculating velocity $\vec{v}_\mathrm{A}$ is following from vector Eq. (1):

$$\vec{v}_\mathrm{A} = \vec{v}_\mathrm{O} + \vec{v}_\mathrm{A/O}. \tag{1}$$

Since the initial velocity $\vec{v}_\mathrm{O} = \vec{0}$ it follows:

$$\vec{v}_\mathrm{A/O} = \omega\cdot\overline{\mathrm{OA}} = 5{,}756\cdot 0{,}15 = 0{,}8634\ \mathrm{m/s} = \vec{v}_\mathrm{A}. \tag{2}$$

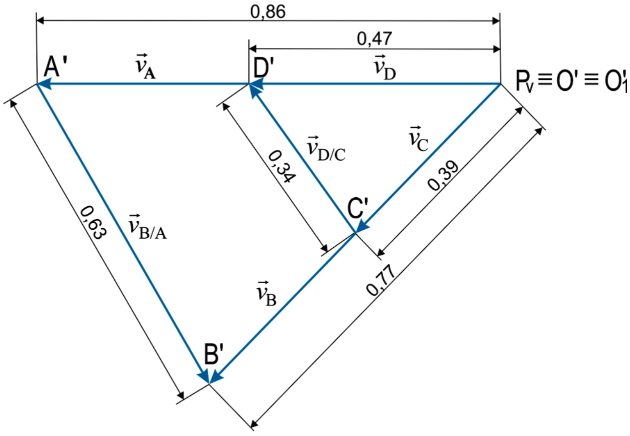

Figure 5 Velocity plan at 55 rpm, values in m/s

Velocity $\vec{v}_\mathrm{A}$ is drawn perpendicular to the length $\overline{\mathrm{OA}}$ (Fig. 4) from the velocity pole for the value which is calculated, and the point A' is obtained (Fig. 5). To determine the point B', two vector Eqs. (3) and (4) are used:

$$\vec{v}_\mathrm{B} = \vec{v}_\mathrm{A} + \vec{v}_\mathrm{B/A}, \tag{3}$$

$$\vec{v}_\mathrm{B} = \vec{v}_{\mathrm{O}_1} + \vec{v}_{\mathrm{B/O}_1}. \tag{4}$$

Line perpendicular to the length $\overline{\mathrm{AB}}$ (Fig. 4) starts from the point A'. To obtain the point B' (Fig. 5), it is necessary to draw another perpendicular line to the length $\overline{\mathrm{BO}_1}$ (Fig. 4) and this line starts at the velocity pole, since initial velocity is $\vec{v}_{\mathrm{O}_1} = \vec{0}$. The intersection of these two lines is the point B' (Fig. 3). To draw the point C' an Eq. (5) follows:

$$\vec{v}_\mathrm{C} = \vec{v}_{\mathrm{O}_1} + \vec{v}_{\mathrm{C/O}_1}. \tag{5}$$

In position plan, the point C is exactly midpoint of the length $\overline{\mathrm{BO}_1}$ (Fig. 3) and therefore, in the velocity plan, the point C' is on midpoint of the length $\overline{\mathrm{B'O'}_1}$ (Fig. 5). Point D' is obtained from Eq. (6):

$$\vec{v}_\mathrm{D} = \vec{v}_\mathrm{C} + \vec{v}_\mathrm{D/C}. \tag{6}$$

From the point C' a perpendicular line is drawn to the length $\overline{\mathrm{CD}}$ (Fig. 4). On the intersection of that line and the horizontal line that comes from the velocity pole (because the slider can move only in a horizontal direction) is the point D' (Fig. 5). All major velocities are measured from a fixed point.

As in the velocity plan, in the acceleration plan is also chosen the fixed point P_a = O" = O"$_1$ (Fig. 7) named acceleration pole representing each mechanism point having zero acceleration. Acceleration plan is drawn from acceleration pole and acceleration $\vec{a}_A$ is first obtained from vector Eq. (7).

$$\vec{a}_A=\vec{a}_O+\vec{a}^{\,n}_{A/O}+\vec{a}^{\,t}_{A/O}. \qquad (7)$$

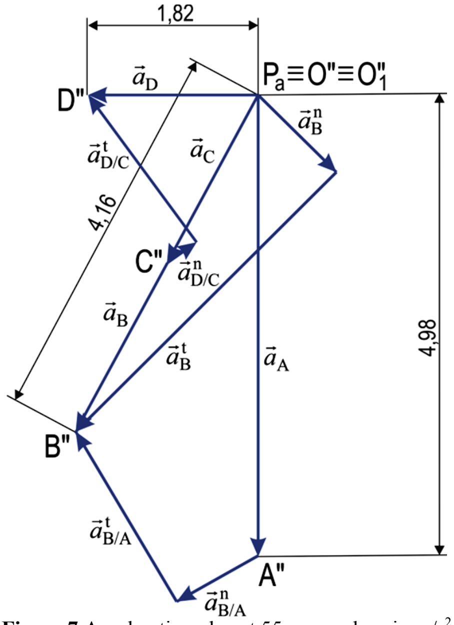

Figure 7 Acceleration plan at 55 rpm, values in m/s^2

Because of ω = const., acceleration $\vec{a}_O=\vec{0}$ and therefore the tangential acceleration of the point A around the point O is $\vec{a}^{\,t}_{A/O}=\vec{0}$. The normal component of the acceleration of the point A around O is calculated according to Eq. (8):

$$a^{n}_{A/O}=\omega^2\cdot\overline{OA}=5{,}756^2\cdot 0{,}15=4{,}97\ \mathrm{m/s^2} \qquad (8)$$

and is drawn from the acceleration pole parallel to the length $\overline{OA}$ (Fig. 4) for the value obtained from the vector Eq. (8). This gives the point A" (Fig. 7). From the point A", line parallel with length $\overline{AB}$ is drawn (Fig. 4) for the value from Eqs. (9) and (10):

$$\vec{a}_B=\vec{a}_A+\vec{a}^{\,n}_{B/A}+\vec{a}^{\,t}_{B/A}, \qquad (9)$$

$$a^{n}_{B/A}=\frac{v^2_{B/A}}{\overline{AB}}=\frac{0{,}634^2}{0{,}4}=1{,}01\ \mathrm{m/s^2}. \qquad (10)$$

Since the tangential acceleration of the point B around A cannot be calculated, from the point P_a = O" = O"$_1$ a line parallel with length $\overline{BO_1}$ is drawn (Fig. 4) for the value obtained from Eqs. (11) and (12):

$$\vec{a}_B=\vec{a}_{O_1}+\vec{a}^{\,n}_{B/O_1}+\vec{a}^{\,t}_{B/O_1}, \qquad (11)$$

$$a^{n}_{B/O_1}=\frac{v^2_{B/O_1}}{\overline{BO_1}}=\frac{0{,}77^2}{0{,}5}=1{,}19\ \mathrm{m/s^2}. \qquad (12)$$

After construction of the previous two lines, at their endpoints two lines perpendicular to them are drawn and at the intersection point B" is located (Fig. 7). At the midpoint of the length $\overline{BO_1}$ is located the point C" (Fig. 7). The calculation of the acceleration point D" is following. Vector Eq. (13) for the acceleration point D" states:

$$\vec{a}_D=\vec{a}_C+\vec{a}^{\,n}_{D/C}+\vec{a}^{\,t}_{D/C}. \qquad (13)$$

Normal component of the acceleration of the point D around the point C follows from Eq. (14):

$$a^{n}_{D/C}=\frac{v^2_{D/C}}{\overline{CD}}=\frac{0{,}338^2}{0{,}3}=0{,}38\ \mathrm{m/s^2}. \qquad (14)$$

It is drawn from the point C" and it is parallel with the length $\overline{CD}$ (Fig. 4). At the end of the normal component, a line perpendicular to that line is drawn and it represents the tangential acceleration. Its value is unknown.

Direction of the total acceleration of the point D is defined by the direction of the slider movement. As the slider can move only in a horizontal direction, its movement is called the linear movement and so the direction of the acceleration vector is also horizontal.

In the acceleration plan (Fig. 7) a horizontal line is drawn from the acceleration pole. The intersection of that horizontal line and the line of tangential component $\vec{a}^{\,t}_{D/C}$ is the point D" (Fig. 7). Accelerations of points of the mechanism are measured from the acceleration pole to the appropriate point in the acceleration plane.

3 Kinematic analysis in SAM

SAM is an interactive software package for the design, analysis and optimization of planar mechanisms, combines shaping, numerical analysis and post-processing of results, such as the development of animation and kinematic diagrams.

The mechanism can be generated using a wizard or can be composed from basic components including beams, sliders, gearboxes, belts, springs, dampers and friction elements.

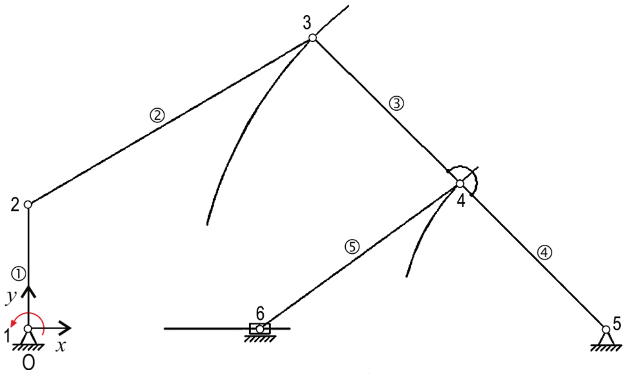

Figure 9 Points trajectory of the mechanism

After defining the geometry of the mechanism by entering the coordinates of the points and selecting the type of elements and supports, nodes trajectory of the mechanism are plotted by running animations (Fig. 9).

Node 1 is used as origin though not shown in SAM application thus not shown in Fig 9. Figs. 9 to 11 are snapshots from application and show models as built in SAM.

For this observation the essential nodes are 2, 3, 4 and 6 (points A, B, C and D in position plan, Fig. 4). It has been found in the animation that points 2 and 6 do not mutually overlap, which is important to build a mechanism and bars ① and ⑤ (Fig. 9) can be located in the same plane.

Location coordinates for points 3 and 4 in each interval are listed. Angular speed of the rocker $\overline{BO_1}$ can be calculated in 36 instants of time during 1 cycle, as per table of results (Tab. 1) there are 36 results of numerical analysis.

Table 1 Position of points 3 and 4

Nr	t	$x(3)$	$y(3)$	$x(4)$	$y(4)$
-	s	mm	mm	mm	mm
0	0,000	345,119	352,220	522,559	176,110
1	0,028	329,404	335,646	514,702	167,823
2	0,056	312,931	316,509	506,466	158,254
3	0,083	296,416	295,161	498,208	147,580
4	0,111	280,515	272,089	490,258	136,044
5	0,139	265,804	247,939	482,902	123,970
6	0,167	252,751	223,537	476,376	111,768
7	0,194	241,692	199,884	470,846	99,942
8	0,222	232,799	178,111	466,400	89,056
9	0,250	226,070	159,345	463,035	79,672
10	0,278	221,334	144,494	460,667	72,247
11	0,306	218,300	134,033	459,150	67,017
12	0,333	216,643	127,930	458,322	63,965
13	0,361	216,077	125,771	458,038	62,886
14	0,389	216,393	126,981	458,196	63,490
15	0,417	217,465	130,996	458,732	65,498
16	0,444	219,236	137,355	459,618	68,677
17	0,472	221,705	145,718	460,852	72,859
18	0,500	224,910	155,852	462,455	77,926
19	0,528	228,928	167,604	464,464	83,802
20	0,556	233,863	180,877	466,932	90,438
21	0,583	239,847	195,599	469,923	97,799
22	0,611	247,028	211,698	473,514	105,849
23	0,639	255,562	229,074	477,781	114,537
24	0,667	265,588	247,560	482,794	123,780
25	0,694	277,189	266,891	488,595	133,446
26	0,722	290,347	286,679	495,174	143,339
27	0,750	304,875	306,392	502,438	153,196
28	0,778	320,362	325,384	510,181	162,692
29	0,806	336,144	342,941	518,072	171,471
30	0,833	351,336	358,376	525,668	179,188
31	0,861	364,926	371,114	532,463	185,557
32	0,889	375,927	380,758	537,964	190,379
33	0,917	383,525	387,096	541,762	193,548
34	0,944	387,190	390,064	543,595	195,032
35	0,972	386,724	389,690	543,362	194,845
36	1,000	382,237	386,040	541,118	193,020

3.3 Determining deadlock of mechanism

Deadlock position of mechanism is when the mechanism reaches its end position. At that moment, the speed of a point element which cannot continue to move is equal to 0, i.e. it is stopped in its movement. At these points, the mechanism ends its movement or changes the direction of movement.

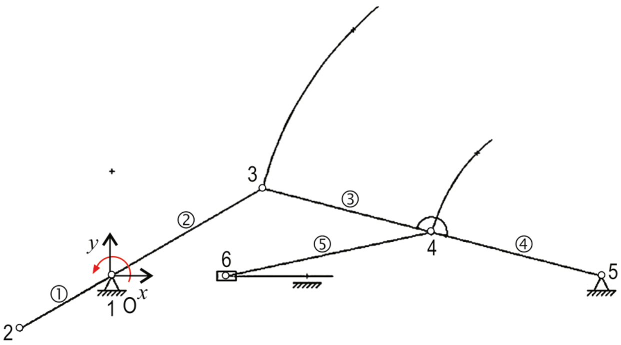

Figure 10 Lower deadlock position of mechanism

To determine the deadlock position of this mechanism (Fig. 10) numerical data were analysed (Tab. 1). At time t = 0,361 s points 3 and 4 get a minimum value of the x and y coordinates. That position of the mechanism is called the lower deadlock. The values are in millimetres, and the origin of the coordinate system is placed at the node 1.

Table 2 Lower deadlock position (mm)

	x-axis	y-axis
Point 3	216,077	125,771
Point 4	458,038	62,886

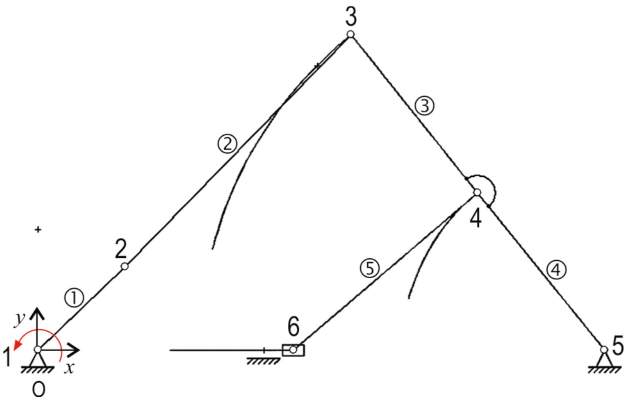

Figure 11 Upper deadlock position of the mechanism

Table 3 Upper deadlock position (mm)

	x-axis	y-axis
Point 3	387,190	390,064
Point 4	543,595	195,032

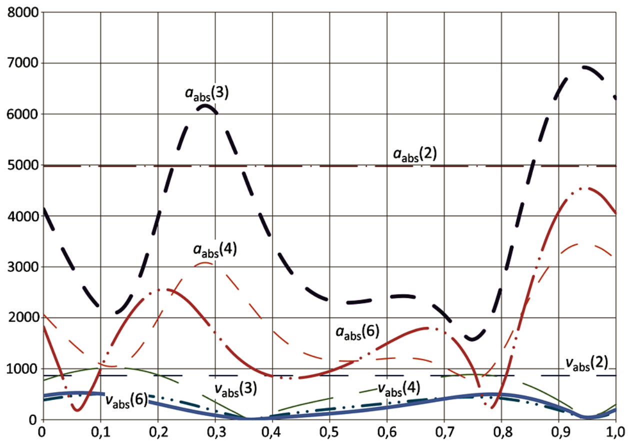

Figure 12 Velocites v_{abs} (mm/s) and accelerations a_{abs} (mm/s^2) graph

The same procedure was repeated for the upper position (Fig. 11). Tab. 1 shows that at time t = 0,944 s points 3 and 4 get maximum values of the x and y coordinates. That position of the mechanism is called the upper deadlock position.

It is possible, the same as in deadlock, to export the value of velocity and acceleration for the entire mechanism (Tab. 4). Tab. 4 shows that the velocity and acceleration of point 2 i.e. point A is constant as expected, because that point is on the crank which has a constant angular speed. Most intensive velocity and acceleration of the mechanism are achieved at point 3 i.e. point B.

Table 4 Absolute values of mechanism node velocity and acceleration (55 rpm)

Nr	t	$v_{abs}(2)$	$a_{abs}(2)$	$v_{abs}(3)$	$a_{abs}(3)$	$v_{abs}(4)$	$a_{abs}(4)$	$v_{abs}(6)$	$a_{abs}(6)$
-	s	mm/s	mm/s^2	mm/s	mm/s^2	mm/s	mm/s^2	mm/s	mm/s^2
0	0,000	863,938	4975,925	771,151	4133,409	385,576	2066,704	470,058	1824,152
1	0,028	863,938	4975,925	869,635	3473,661	434,818	1736,830	509,396	1005,848
2	0,056	863,938	4975,925	944,559	2880,010	472,279	1440,005	525,982	192,154
3	0,083	863,938	4975,925	994,748	2391,261	497,374	1195,631	520,451	580,503
4	0,111	863,938	4975,925	1018,376	2102,282	509,188	1051,141	494,447	1275,818
5	0,139	863,938	4975,925	1012,719	2191,430	506,360	1095,715	450,632	1856,566
6	0,167	863,938	4975,925	974,281	2770,500	487,141	1385,250	392,724	2284,007
7	0,194	863,938	4975,925	899,675	3732,060	449,837	1866,030	325,516	2520,347
8	0,222	863,938	4975,925	787,782	4831,146	393,891	2415,573	254,727	2539,419
9	0,250	863,938	4975,925	642,982	5743,127	321,491	2871,564	186,398	2348,567
10	0,278	863,938	4975,925	477,264	6157,362	238,632	3078,681	125,646	2008,573
11	0,306	863,938	4975,925	307,845	5962,354	153,922	2981,177	75,246	1621,263
12	0,333	863,938	4975,925	150,488	5317,944	75,244	2658,972	35,126	1280,784
13	0,361	863,938	4975,925	13,954	4507,012	6,977	2253,506	3,203	1034,770
14	0,389	863,938	4975,925	100,456	3751,841	50,228	1875,920	23,276	886,925
15	0,417	863,938	4975,925	195,972	3155,872	97,986	1577,936	46,827	820,757
16	0,444	863,938	4975,925	277,363	2737,470	138,681	1368,735	69,456	817,677
17	0,472	863,938	4975,925	349,230	2476,137	174,615	1238,068	92,708	863,776
18	0,500	863,938	4975,925	415,363	2340,134	207,681	1170,067	117,815	950,118
19	0,528	863,938	4975,925	478,614	2297,312	239,307	1148,656	145,809	1070,697
20	0,556	863,938	4975,925	540,930	2316,996	270,465	1158,498	177,563	1219,714
21	0,583	863,938	4975,925	603,358	2368,112	301,679	1184,056	213,755	1388,357
22	0,611	863,938	4975,925	665,959	2415,990	332,979	1207,995	254,733	1560,850
23	0,639	863,938	4975,925	727,607	2419,965	363,804	1209,983	300,257	1709,855
24	0,667	863,938	4975,925	785,712	2334,812	392,856	1167,406	349,114	1792,195
25	0,694	863,938	4975,925	835,951	2123,397	417,976	1061,698	398,652	1747,650
26	0,722	863,938	4975,925	872,198	1801,786	436,099	900,893	444,374	1505,504
27	0,750	863,938	4975,925	886,900	1573,822	443,450	786,911	479,875	1003,625
28	0,778	863,938	4975,925	872,145	1905,633	436,073	952,817	497,495	219,504
29	0,806	863,938	4975,925	821,395	2864,219	410,697	1432,110	489,872	799,195
30	0,833	863,938	4975,925	731,458	4082,552	365,729	2041,276	452,068	1927,206
31	0,861	863,938	4975,925	603,880	5257,594	301,940	2628,797	383,386	2992,987
32	0,889	863,938	4975,925	445,020	6183,584	222,510	3091,792	287,866	3836,985
33	0,917	863,938	4975,925	264,702	6745,206	132,351	3372,603	173,202	4361,628
34	0,944	863,938	4975,925	74,089	6916,761	37,045	3458,381	48,728	4545,283
35	0,972	863,938	4975,925	116,342	6743,034	58,171	3371,517	76,469	4422,466
36	1,000	863,938	4975,925	298,081	6308,200	149,040	3154,100	194,679	4052,825

It is expected that the velocity in the position of the deadlock is 0 m/s. However, due to a finite number of intervals, which is quite small, it is evident that the velocity does not reach 0 m/s but is near exact deadlock position. For accurate determination it is necessary to increase the number of intervals or use a numerical method such as the method of the parabola.

4 Analytical vector solution in MathCad

To calculate the velocity and acceleration in MathCad, software for solving and analysing engineering calculations, it is necessary to write the vector equation of velocity and acceleration for each point. From each vector equation it is possible to create three scalar equations which can be solved in matrix form. Such system can be quickly solved by using the command block Given-Find. In this calculation engineering units are used which accelerates calculation process with automated unit recalculation [6]. Mathcad syntax though similar, differs from usual mathematical notation. Vectors, matrices, line lengths and angle measures are defined as variables.

For obtaining solutions numerical Given-Find block algorithm is used. Velocities and acceleration magnitudes are calculated using absolute value function (|), determinant function can be used for square matrices only.

To achieve clear and accurate results, calculation angles are measured from the horizontal line (positive direction of the x axis). From the position plan, for the current position of the mechanism (Fig. 4), the following angles are: $\varphi_0\angle(\overline{\mathrm{OA}}, x)$, $\varphi_1\angle(\overline{\mathrm{AB}}, x)$, $\varphi_2\angle(\overline{\mathrm{BO}_1}, x)$ and $\varphi_3\angle(\overline{\mathrm{CD}}, x)$.

From the data used to calculate the velocity and acceleration at 55 rpm following variables are defined: $\overline{\mathrm{OA}}$ = 0,15 m, $\overline{\mathrm{AB}}$ = 0,4 m, $\overline{\mathrm{BO}_1}$ = 0,5 m, $\overline{\mathrm{BC}} = \overline{\mathrm{CO}_1}$ = 0,25 m, $\overline{\mathrm{CD}}$ = 0,3 m, $\omega_{\mathrm{OA}} = \dfrac{\pi \cdot n}{30}$ = 5,756 rad/s, φ_0 = 90°, φ_1 = 30,37°, φ_2 = 135,22° and φ_3 = 215,95°.

From vector Eq. (15) for determining the velocity $\vec{v}_{\mathrm{A}}$

$$\vec{v}_{\mathrm{A}} = \vec{v}_{\mathrm{O}} + \vec{v}_{\mathrm{A/O}} \tag{15}$$

an expression in MathCad is created:

$$[\vec{v}_{\mathrm{A}}] = \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{OA}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{OA} \cdot \cos\varphi_0 \\ \mathrm{OA} \cdot \sin\varphi_0 \\ 0 \end{pmatrix} = \begin{pmatrix} -0{,}863 \\ 0 \\ 0 \end{pmatrix} \mathrm{m/s} \tag{16}$$

as well as result (17) of the velocity magnitude $\vec{v}_{\mathrm{A}}$

$$|\vec{v}_{\mathrm{A}}| = 0{,}863\ \mathrm{m/s}. \tag{17}$$

To determine the velocity $\vec{v}_{\mathrm{B}}$, vector Eqs. (18) and (19) are used:

$$\vec{v}_{\mathrm{B}} = \vec{v}_{\mathrm{A}} + \vec{v}_{\mathrm{B/A}}, \tag{18}$$
$$\vec{v}_{\mathrm{B}} = \vec{v}_{\mathrm{O}_1} + \vec{v}_{\mathrm{B/O}_1}. \tag{19}$$

From those equations follow scalar Eqs. (20) to (25) below:

$$\left. \begin{array}{l} v_{\mathrm{B}x} = 1\ \mathrm{m/s},\ v_{\mathrm{B}y} = 1\ \mathrm{m/s}, \\ \omega_{\mathrm{AB}} = 1\ \mathrm{rad/s},\ \omega_{\mathrm{BCO}} = 1\ \mathrm{rad/s}, \end{array} \right\} \text{initial values} \tag{20}$$

given

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{OA}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{OA} \cdot \cos\varphi_0 \\ \mathrm{OA} \cdot \sin\varphi_0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{AB}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{AB} \cdot \cos\varphi_1 \\ \mathrm{AB} \cdot \sin\varphi_1 \\ 0 \end{pmatrix} = \begin{pmatrix} v_{\mathrm{B}x} \\ v_{\mathrm{B}y} \\ 0 \end{pmatrix} \tag{21}$$

$$\begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{BCO}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{BO}_1 \cdot \cos\varphi_2 \\ \mathrm{BO}_1 \cdot \sin\varphi_2 \\ 0 \end{pmatrix} = \begin{pmatrix} v_{\mathrm{B}x} \\ v_{\mathrm{B}y} \\ 0 \end{pmatrix} \tag{22}$$

$$\begin{pmatrix} v_{\mathrm{B}x} \\ v_{\mathrm{B}y} \\ \omega_{\mathrm{AB}} \\ \omega_{\mathrm{BCO}} \end{pmatrix} = \mathrm{Find}\ (v_{\mathrm{B}x}, v_{\mathrm{B}y}, \omega_{\mathrm{AB}}, \omega_{\mathrm{BCO}}) \tag{23}$$

$$\left. \begin{array}{l} v_{\mathrm{B}x} = -0{,}543\ \mathrm{m/s},\ v_{\mathrm{B}y} = -0{,}547\ \mathrm{m/s}, \\ \omega_{\mathrm{AB}} = -1{,}585\ \mathrm{rad/s},\ \omega_{\mathrm{BCO}} = 1{,}541\ \mathrm{rad/s}, \end{array} \right\} \text{results} \tag{24}$$

$$v_{\mathrm{B}} = \begin{pmatrix} v_{\mathrm{B}x} \\ v_{\mathrm{B}y} \\ 0 \end{pmatrix} = \begin{pmatrix} -0{,}543 \\ -0{,}547 \\ 0 \end{pmatrix} \mathrm{m/s} \tag{25}$$

as well as the result of velocity $\vec{v}_{\mathrm{B}}$

$$|\vec{v}_{\mathrm{B}}| = \sqrt{v_{\mathrm{B}x}^2 + v_{\mathrm{B}y}^2} = 0{,}771\ \mathrm{m/s}. \tag{26}$$

Vector equation for the velocity $\vec{v}_{\mathrm{C}}$ is:

$$\vec{v}_{\mathrm{C}} = \vec{v}_{\mathrm{O}_1} + \vec{v}_{\mathrm{C/O}_1}, \tag{27}$$

from which is obtained Eq. (28):

$$[\vec{v}_{\mathrm{C}}] = \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{BCO}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{CO}_1 \cdot \cos\varphi_2 \\ \mathrm{CO}_1 \cdot \sin\varphi_2 \\ 0 \end{pmatrix} = \begin{pmatrix} -0{,}271 \\ -0{,}274 \\ 0 \end{pmatrix} \mathrm{m/s} \tag{28}$$

and result (29) of velocity $\vec{v}_{\mathrm{C}}$:

$$|\vec{v}_{\mathrm{C}}| = \sqrt{v_{\mathrm{C}x}^2 + v_{\mathrm{C}y}^2} = 0{,}385\ \mathrm{m/s}. \tag{29}$$

Finally, it is necessary to determine velocity $\vec{v}_{\mathrm{D}}$ from Eq. (30):

$$\vec{v}_{\mathrm{D}} = \vec{v}_{\mathrm{C}} + \vec{v}_{\mathrm{D/C}}, \tag{30}$$

from which follow four scalar Eqs. (31) to (34)
Initial values: $v_{\mathrm{D}x} = 1\ \mathrm{m/s}, \omega_{\mathrm{CD}} = 1\ \mathrm{rad/s},$ (31)

given

$$[\vec{v}_{\mathrm{C}}] + \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{CD}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{CD} \cdot \cos\varphi_3 \\ \mathrm{CD} \cdot \sin\varphi_3 \\ 0 \end{pmatrix} = \begin{pmatrix} v_{\mathrm{D}x} \\ 0 \\ 0 \end{pmatrix}, \tag{32}$$

$$\begin{pmatrix} v_{\mathrm{D}x} \\ \omega_{\mathrm{CD}} \end{pmatrix} = \mathrm{Find}\ (v_{\mathrm{D}x}, \omega_{\mathrm{CD}}), \tag{33}$$

$$v_{\mathrm{D}} = \begin{pmatrix} v_{\mathrm{D}x} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -0{,}47 \\ 0 \\ 0 \end{pmatrix} \mathrm{m/s}, \tag{34}$$

and result of velocity $\vec{v}_{\mathrm{D}}$:

$$|\vec{v}_{\mathrm{D}}| = 0{,}47\ \mathrm{m/s}. \tag{35}$$

To determine the acceleration $\vec{a}_{\mathrm{A}}$, vector Eqs. (36) is used (36):

$$\vec{a}_{\mathrm{A}} = \vec{a}_{\mathrm{O}} + \vec{a}_{\mathrm{A/O}}^{\mathrm{n}} + \vec{a}_{\mathrm{A/O}}^{\mathrm{t}}. \tag{36}$$

from which is obtained the expression (37):

$$[\vec{a}_{\mathrm{A}}] = \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{OA}} \end{pmatrix} \cdot \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{OA}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{OA} \cdot \cos\varphi_0 \\ \mathrm{OA} \cdot \sin\varphi_0 \\ 0 \end{pmatrix} \right] = \begin{pmatrix} 0 \\ -4{,}97 \\ 0 \end{pmatrix} \mathrm{m/s^2} \tag{37}$$

and result of velocity $\vec{a}_{\mathrm{A}}$ (38):

$$|\vec{a}_{\mathrm{A}}| = 4{,}97\ \mathrm{m/s^2}. \tag{38}$$

As well as velocity $\vec{v}_{\mathrm{B}}$, acceleration $\vec{a}_{\mathrm{B}}$ requires two vector Eqs. (39) and (40):

$$\vec{a}_{\mathrm{B}} = \vec{a}_{\mathrm{A}} + \vec{a}^{\,\mathrm{n}}_{\mathrm{B/A}} + \vec{a}^{\,\mathrm{t}}_{\mathrm{B/A}}, \tag{39}$$

$$\vec{a}_{\mathrm{B}} = \vec{a}_{\mathrm{O_1}} + \vec{a}^{\,\mathrm{n}}_{\mathrm{B/O_1}} + \vec{a}^{\,\mathrm{t}}_{\mathrm{B/O_1}}, \tag{40}$$

from which follow scalar equations:

$$\left.\begin{array}{l} a_{\mathrm{B}x} = 1\ \mathrm{m/s^2},\ a_{\mathrm{B}y} = 1\ \mathrm{m/s^2}, \\ \alpha_{\mathrm{AB}} = 1\ \mathrm{rad/s^2},\ \alpha_{\mathrm{BCO}} = 1\ \mathrm{rad/s^2}, \end{array}\right\}\ \text{initial values} \tag{41}$$

given

$$\begin{gathered}
[\vec{a}_{\mathrm{A}}] + \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{AB}} \end{pmatrix} \cdot \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{AB}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{AB}\cdot\cos\varphi_1 \\ \mathrm{AB}\cdot\sin\varphi_1 \\ 0 \end{pmatrix} \right] + \\
+ \begin{pmatrix} 0 \\ 0 \\ \alpha_{\mathrm{AB}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{AB}\cdot\cos\varphi_1 \\ \mathrm{AB}\cdot\sin\varphi_1 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{\mathrm{B}x} \\ a_{\mathrm{B}y} \\ 0 \end{pmatrix},
\end{gathered} \tag{42}$$

$$\begin{gathered}
\begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{BCO}} \end{pmatrix} \cdot \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{BCO}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{BO_1}\cdot\cos\varphi_2 \\ \mathrm{BO_1}\cdot\sin\varphi_2 \\ 0 \end{pmatrix} \right] + \\
+ \begin{pmatrix} 0 \\ 0 \\ \alpha_{\mathrm{BCO}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{BO_1}\cdot\cos\varphi_2 \\ \mathrm{BO_1}\cdot\sin\varphi_2 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{\mathrm{B}x} \\ a_{\mathrm{B}y} \\ 0 \end{pmatrix},
\end{gathered} \tag{43}$$

$$\begin{pmatrix} a_{\mathrm{B}x} \\ a_{\mathrm{B}y} \\ \alpha_{\mathrm{AB}} \\ \alpha_{\mathrm{BCO}} \end{pmatrix} = \mathrm{Find}\,(a_{\mathrm{B}x},\ a_{\mathrm{B}y},\ \alpha_{\mathrm{AB}},\ \alpha_{\mathrm{BCO}}) \tag{44}$$

$$a_{\mathrm{B}} = \begin{pmatrix} a_{\mathrm{B}x} \\ a_{\mathrm{B}y} \\ 0 \end{pmatrix} = \begin{pmatrix} -1{,}942 \\ -3{,}643 \\ 0 \end{pmatrix}\ \mathrm{m/s^2} \tag{45}$$

and result of acceleration $\vec{a}_{\mathrm{B}}$:

$$|\vec{a}_{\mathrm{B}}| = \sqrt{a^2_{\mathrm{B}x} + a^2_{\mathrm{B}y}} = 4{,}129\ \mathrm{m/s^2}. \tag{46}$$

Vector Eq. (47) for acceleration $\vec{a}_{\mathrm{C}}$ is:

$$\vec{a}_{\mathrm{C}} = \vec{a}_{\mathrm{O_1}} + \vec{a}^{\,\mathrm{n}}_{\mathrm{C/O_1}} + \vec{a}^{\,\mathrm{t}}_{\mathrm{C/O_1}}. \tag{47}$$

from which are obtained the following expressions:

$$\text{Initial values: } a_{\mathrm{C}x} = 1\ \mathrm{m/s^2},\ a_{\mathrm{C}y} = 1\ \mathrm{m/s^2}, \tag{48}$$

given

$$\begin{gathered}
[\vec{a}_{\mathrm{C}}] = \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{BCO}} \end{pmatrix} \cdot \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{BCO}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{CO_1}\cdot\cos\varphi_2 \\ \mathrm{CO_1}\cdot\sin\varphi_2 \\ 0 \end{pmatrix} \right] + \\
+ \begin{pmatrix} 0 \\ 0 \\ \alpha_{\mathrm{BCO}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{CO_1}\cdot\cos\varphi_2 \\ \mathrm{CO_1}\cdot\sin\varphi_2 \\ 0 \end{pmatrix} = \begin{pmatrix} a_{\mathrm{C}x} \\ a_{\mathrm{C}y} \\ 0 \end{pmatrix},
\end{gathered} \tag{49}$$

$$\begin{pmatrix} a_{\mathrm{C}x} \\ a_{\mathrm{C}y} \end{pmatrix} = \mathrm{Find}\,(a_{\mathrm{C}x}, a_{\mathrm{C}y}), \tag{50}$$

$$a_{\mathrm{C}} = \begin{pmatrix} a_{\mathrm{C}x} \\ a_{\mathrm{C}y} \\ 0 \end{pmatrix} = \begin{pmatrix} -0{,}971 \\ -1{,}822 \\ 0 \end{pmatrix}\ \mathrm{m/s^2} \tag{51}$$

and result of acceleration $\vec{a}_{\mathrm{C}}$:

$$|\vec{a}_{\mathrm{C}}| = \sqrt{a^2_{\mathrm{C}x} + a^2_{\mathrm{C}y}} = 2{,}064\ \mathrm{m/s^2}. \tag{52}$$

Eq. (53) for acceleration $\vec{a}_{\mathrm{D}}$ is:

$$\vec{a}_{\mathrm{D}} = \vec{a}_{\mathrm{C}} + \vec{a}^{\,\mathrm{n}}_{\mathrm{D/C}} + \vec{a}^{\,\mathrm{t}}_{\mathrm{D/C}} \tag{53}$$

from which is obtained the expression with initial values and three scalar equations as follows:

$$a_{\mathrm{D}} = 1\ \mathrm{m/s^2},\ \alpha_{\mathrm{CD}} = 1\ \mathrm{rad/s^2}, \tag{54}$$

given

$$\begin{gathered}
[\vec{a}_{\mathrm{C}}] + \begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{CD}} \end{pmatrix} \cdot \left[\begin{pmatrix} 0 \\ 0 \\ \omega_{\mathrm{CD}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{CD}\cdot\cos\varphi_3 \\ \mathrm{CD}\cdot\sin\varphi_3 \\ 0 \end{pmatrix} \right] + \\
+ \begin{pmatrix} 0 \\ 0 \\ \alpha_{\mathrm{CD}} \end{pmatrix} \cdot \begin{pmatrix} \mathrm{CD}\cdot\cos\varphi_3 \\ \mathrm{CD}\cdot\sin\varphi_3 \\ 0 \end{pmatrix} = \begin{pmatrix} \vec{a}_{\mathrm{D}} \\ 0 \\ 0 \end{pmatrix},
\end{gathered} \tag{55}$$

$$\begin{pmatrix} a_{\mathrm{D}} \\ \alpha_{\mathrm{CD}} \end{pmatrix} = \mathrm{Find}\,(a_{\mathrm{D}},\ \alpha_{\mathrm{CD}}), \tag{56}$$

and result of acceleration $\vec{a}_{\mathrm{D}}$:

$$|\vec{a}_{\mathrm{D}}| = 1{,}822\ \mathrm{m/s^2}. \tag{57}$$

5 Comparison of results

For the final analysis of kinematic values, results of kinematic analysis are entered in Tab. 5 and compared. It is evident that the results almost do not differ. It can be concluded that the analysis conducted through all three software packages is successful.

Expected variations of the results obtained in the SAM occur in the comparison of acceleration due to a defect of the numerical algorithm. The assumption is that they can be avoided by increasing the number of intervals or more accurate definition of mechanism geometry.

Table 5 Comparison of results

Value	Unit	Graphical SW	SAM solution	MathCad solution
v_A	m/s	0,8634	0,863	0,863
v_B		0,771	0,771	0,771
v_C		0,385	0,386	0,385
v_D		0,470	0,470	0,470
a_A	m/s^2	4,970	4,976	4,970
a_B		4,128	4,133	4,129
a_C		2,064	2,066	2,064
a_D		1,822	1,824	1,822

6 Conclusion

Performing a kinematic analysis using software tools, according to the results, it is possible to reach the following conclusions:

- Kinematics analysis in SAM 6.1 is performed very quickly, with satisfactory accuracy, knowledge of vector algebra to determine kinematic values is not required. Disadvantage is that in complex mechanisms deviations in acceleration results are possible. Experience has shown that it is not possible to simulate all types of kinematics without additional simplification of the model.
- Graphical analytical solution obtained using SolidWorks enables fast solution recalculation for different mechanism positions and using formulas for different initial values of velocity and acceleration. Disadvantage is the necessity of creating vector equations and possessing the knowledge of traditional methods of kinematic analysis. To obtain dynamic solution it is necessary to set constraints between kinematic scheme and solution graph, which is time consuming.
- Analytical solution obtained using MathCad implies knowledge of vector algebra and software syntax but provides accurate results and easy generation of solution for different mechanism position and initial values of velocity and acceleration.
- Vector solution can be expanded with position functions of each node dependent on position of drive element, to eliminate reading of bar angle and automate the rest of kinematic solution.
- Given that the teaching of kinematics if performed in the form of lectures accompanied with recitation class, to use software packages in recitation it is necessary to work with smaller groups in the lab and accordingly adjust the curriculum. It is also necessary for students to previously master required IT skills. The stated reasons are the cause of avoiding the use of software while teaching base subjects, but they can be used as part of the final and diploma thesis, because the application of software tools provides plenty of topics for papers in the basic theoretical courses.

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