

# Vehicle Routing Optimization Using Multiple Local Search Improvements

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Original scientific paper

Combinatorial optimization problems on graphs arise in many practical applications. One of the most studied practical combinatorial optimization problem is the Vehicle Routing Problem (VRP). When coupled with modern in-car navigation and fleet management software, real world applications of VRP optimization result in significant cost savings. In this paper novel multiple improvements pivoting rule for Capacitated VRP (CVRP) is proposed. Its application significantly reduces computational time needed for CVRP optimization. A novel pivoting rule is implemented as part of the search step selection mechanism in the Iterated Local Search algorithm. Augmented iterated local search algorithm is tested on 4 large scale real-world problems in Croatia with up to 7,065 customers and 236 vehicles, and on standard CVRP benchmark sets. Real-world problem data was obtained from a large Croatian logistics company. Comparison of well known first and best pivoting rules with proposed novel multiple improvements pivoting rule regarding travel distance, number of search moves and computational time is given. Achieved computational speed-ups are up to 29 times compared to the first improvement pivoting rule and 9 times compared to the best improvement pivoting rule, without any substantial degradation in quality of the obtained solution.

**Key words:** VRP, CVRP, iterated local search, multiple improvements

**Optimizacija usmjeravanja vozila primjenom višestrukih poboljšanja u lokalnom pretraživanju.** Kombinatoričke optimizacije na grafu pojavljuju se u mnogim aplikacijama u praksi. Jedan od najviše proučavanih kombinatoričkih optimizacijskih problema je problem usmjeravanja vozila. Ukoliko se optimizacija usmjeravanja vozila poveže sa suvremenim u vozila ugrađenim sustavima navigacije i nadgledanja voznog parka moguće je postići značajne uštede u troškovima dostave. U ovom radu je predložen novi mehanizam odabira smjera lokalnog pretraživanja zasnovan na višestrukim poboljšanjima za rješavanje kapacitivnog problema usmjeravanja vozila. Predloženi novi mehanizam je implementiran kao dio mehanizma odabira smjera lokalnog pretraživanja u algoritmu iterativnog lokalnog pretraživanja. Prošireni algoritam iterativnog lokalnog pretraživanja je provjeren na 4 vrlo velika optimizacijska problema sa stvarnim podacima iz Hrvatske (skup od 7.065 kupaca i 236 dostavnih vozila) i na standardnim testnim skupovima. Stvarni testni podaci dobiveni su od jedne velike hrvatske logističke tvrtke. U radu je napravljena usporedba između mehanizama odabira smjera lokalnog pretraživanja zasnovanih na prvom i najboljem poboljšanju te predloženog mehanizma poboljšanja lokalne pretrage. Usporedba je napravljena prema prijednom putu, broju pomaka lokalnog pretraživanja i vremenu izračuna. Dobiveni rezultati pokazuju ubrzanje u vremenu izračuna za 29 puta u usporedbi sa prvim smjerom poboljšanja lokalne pretrage te 9 puta u usporedbi sa najboljim smjerom poboljšanja lokalne pretrage bez značajnijih degradacija u kvaliteti dobivenog rješenja.

**Ključne riječi:** VRP, CVRP, iterativno lokalno pretraživanje, višestruka poboljšanja

## 1 INTRODUCTION

Graphs are common way of presentation when some kind of network or connections among elements in a system have to be described mathematically. Very different mathematical or practical problems can then be solved using the same graph related algorithms. Range of problems that can be solved covers problems like drilling a

given set of holes in a printed circuit board, route planning in computer or road networks, etc.

Regarding route planning in road networks a very common problem is the Traveling Salesman Problem (TSP). The goal is to find the tour between  $n$  cities with minimal travel cost. The salesman has to visit each city exactly once and then return to its home (starting) city. The TSP is solved by searching for a Hamiltonian tour of minimal

length on a graph. The choice of the best tour becomes a combinatorial optimization problem.

Another practical problem is the delivery of goods to shops around a city or region. This problem can be seen as  $m$ -TSP where route optimization for each of the  $m$  available delivery vehicles is done separately. If a depot is added, the  $m$ -TSP problem becomes a Vehicle Routing Problem (VRP). Both mentioned problems (TSP and VRP) including their extensions, like Capacitated Vehicle Routing Problem (CVRP) where capacity of vehicle is an additional constraint, are  $NP$ -hard problems. Such problems can be solved by an exact algorithm only for cases with relatively small number of locations. Because the number of solutions grows with factorial complexity, an exhaustive search algorithm is impractical for real world TSP and VRP solving. The methods that are often used in practice for those  $NP$ -hard combinatorial problems on graphs are heuristic methods which are used to speed up the process of finding a satisfactory good solution.

The novel multiple improvements pivoting rule for solving CVRP is proposed in this paper. Proposed rule is implemented as part of the search step selection mechanism in the Iterated Local Search (ILS) algorithm. The used local search operators and ILS algorithm are presented in detail. Comparison of new pivoting rule with first and best pivoting rule is conducted. The testbed for comparison consist of standard Christofides CVRP, Taillard CVRP problem benchmarks [1, 2], and four large scale real-world problems. The real-world benchmark set was generated from real distribution demand data in order to test proposed search step selection mechanism in conditions identical to its most probable application area. Obtained results show significant speed up regarding computation time and reduction of needed algorithm iterations.

The rest of the paper is organized as follows. In Section 2, mathematical definition of the CVRP problem is given. In Section 3, state of the art approaches are described. In Section 4, novel search step selection mechanism is proposed. In Section 5, implementation of the proposed search step selection as part of the ILS algorithm is described. Section 6 contains description of experimental data and obtained results discussion. Last Section 7 ends the paper with conclusion.

## 2 CAPACITATED VEHICLE ROUTING PROBLEM

VRP was introduced by Dantzig and Ramser [3] and is one of the most significant problems in distribution management. CVRP is considered to be a classical version of the VRP. Often, when VRP is mentioned, capacitated variant is assumed. Additional constraint in CVRP is that every customer has a certain demand, and delivery vehicles have limited capacity in serving those demands.

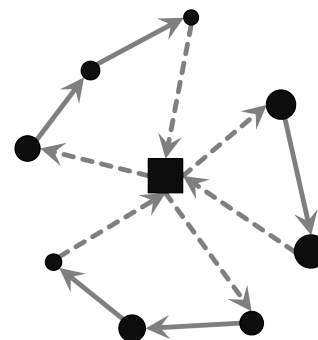


Fig. 1. Capacitated vehicle routing problem

More formally speaking, let  $G = (V, E)$  (Fig. 1) be a connected digraph where  $V = \{0, \dots, n\}$  is set of nodes, and  $E = \{(i, j) : i, j \in V, i < j\}$  is set of edges with non-negative weights that are connecting nodes. Digraph is a directed graph which has no loops and no multiple oriented edges (with the same starting and ending nodes). Every node  $i \in V \setminus \{0\}$  represents a customer with non-negative demand  $q_i$ , and node 0 represents a depot. Node 0 presents an exception in the digraph used for CVRP presentations and denotes starting and ending point of every delivery route. Therefore it has multiple connections to its neighboring nodes. Every edge has a cost  $c_{ij}$ . In Fig. 1 solved CVRP is presented. Rectangle denotes a depot, circles represent customers with radius proportional to customer demand, dashed lines are oriented edges which have connection with depot and solid lines represent edges between customers. Homogeneous fleet of  $m$  vehicles with equal capacities  $Q$  is located at depot (in Fig. 1  $m = 3$ ).

The objective in CVRP is to minimize total distance, such that every customer is served only once by one vehicle, routes must start and end in depot, and total demand of customers in route does not exceeds vehicle capacity  $Q$ . Distance Constrained VRP (DCVRP) is variant in which overall route distance  $D_i$  should not exceed upper limit  $L$ . The implemented ILS algorithm presented in this paper is applied only to the problems without the overall route distance constraint.

As mentioned before, the CVRP is a  $NP$ -hard problem [4], so exact algorithms are suitable only for solving smaller sized problems with up to 50 customers in reasonable computing time [5]. Problems with larger number of customers are most commonly solved with metaheuristic algorithms. Most metaheuristics consists of some construction and improvement heuristics (i.e. local search) [6]. Various metaheuristics for vehicle routing problem can be found in Bräysy and Gendreau survey [7].

### 3 STATE OF THE ART APPROACHES IN SOLVING CVRP

Local search algorithms are often applied to hard combinatorial problems. They iteratively generate candidate solutions and evaluate them. Local search algorithms are essential elements for vast majority metaheuristic algorithms for VRP. Some of metaheuristics algorithms for solving VRP that use local search are simulated annealing, tabu search and ILS. In many cases, the performance of the evolutionary algorithms for combinatorial problems can be significantly improved by adding a local search phase after applying mutation and recombination or by incorporating a local search process into the recombination operator. Local search algorithms start at some location of the search space and move from one solution to a neighboring solution [8]. This approach is often more appropriate for real-world applications, because available time to find a solution is often limited. Although exact methods can guarantee that eventually optimal solution will be found or prove that a solution does not exist.

Over the years researchers have shown that (meta)heuristic algorithms can produce high quality solutions in reasonable time. Today, virtually every state of the art algorithm for VRP uses some sort of local search mechanism [9]. Often high performance algorithms use randomized choices in generating or selecting candidate solutions [8].

#### 3.1 Local search operators

As stated previously, local search is based on moves from one solution to a neighbor one. To perform those moves, neighboring solutions need to be evaluated. Neighboring moves are evaluated with neighborhood operators. For detailed description of operators see [10]. Most common operators for CVRP are relocate, exchange, 2-opt

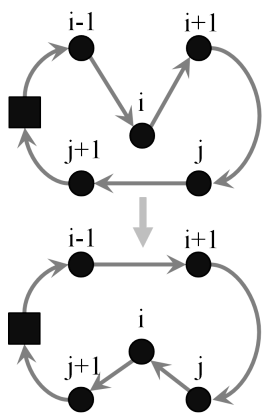


Fig. 2. Relocate move

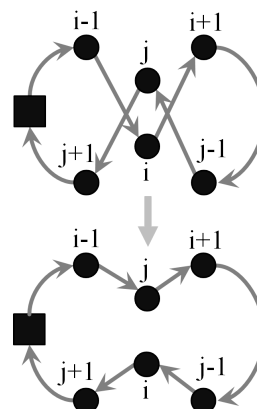


Fig. 3. Exchange move

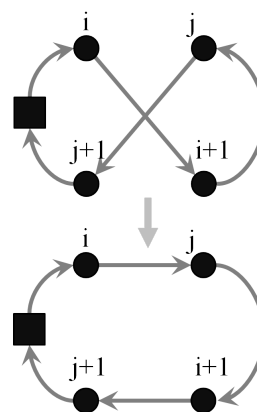


Fig. 4. 2-opt move

and 2-opt\*. In CVRP local search operators are divided into two groups, first group that changes position of one or more customers in a single route and second group of operators that are relocating or exchanging one or more customers between two routes. Local search operators are routes modifiers which improve objective function when possible improvements are found. Every operator works on some pattern and modifies one route or more of them. Objective function for our example of operators is route length. The pattern of route modification for each operator is presented in Figs. 2- 4. In mentioned figures rectangle presents depot, black circles customers and lines delivery routes.

Figure 2 illustrates relocate operator for one route, where customer  $i$  is relocated to the new position between customers  $j$  and  $j + 1$ . Figure 3 illustrates exchange operator for one route, where customers  $i$  and  $j$  are exchanged. Figure 4 illustrates 2-opt operator for a single route. The edges  $(i, i + 1)$  and  $(j, j + 1)$  are replaced by edges  $(i, j)$  and  $(i + 1, j + 1)$ , causing reversal in the direction of edges

between  $i + 1$  and  $j$ .

### 3.2 Search step selection mechanism

Important part of local search is how to select new improving step. The search step selection mechanism decides which improving moves will be performed. Moves are determining the new solution neighborhood and further course of the local search.

The search step selection mechanism is also called acceptance strategy. Commonly used pivoting rules in this selection mechanism are best improvement and first improvement. Best improvement selects in each search step the one that achieves maximal reduction of total distance. Best improvement is also called greedy hill-climbing or discrete gradient descent. First improvement tries to avoid the time complexity of evaluating all possible neighbors by performing first improving step found during neighborhood search [8].

Other pivoting rules that are used also are  $d$ -best improvement [11], random improvement and least improvement [8]. The  $d$ -best improvement terminates the search when  $d$  improving neighbor solutions are found. The best solution from this set is then taken as the next solution. Random improvement selects the next step by choosing one of the improving move randomly. Least improving strategy chooses the improvement that reduces total distance of the solution by a smallest amount. Those strategies are rarely used and are not examined in scope of this paper.

## 4 NOVEL SEARCH STEP SELECTION MECHANISMS

First improvement search step can be calculated more efficiently because smaller subset of the neighborhood is evaluated. But, there is one negative effect. Evaluated improvements are smaller or equal to the best possible improvement, therefore more search steps have to be calculated. More detailed analysis is given in Section 6. Best improvement evaluates whole neighborhood, but only one, the most improving step, is performed while all other less improving steps are discarded in that iteration.

It is quite possible that one or more of those less improving steps will be best improvement later as local search iterates. It is beneficial not just to make one best step, but also less improving ones as well. Since every improvement changes neighborhood, making more than one improving move at one iteration can lead to an infeasible solution. For example, vehicle capacity  $Q$  is not enough to accept more than one new customer in a particular route. Thus, it is necessary to check that new solution will be feasible after performing all improving moves.

Example of this idea is illustrated in Fig. 5 showing relocate moves between two routes. Let say that only three

improving moves can be made, best improving move between routes  $v_1$  and  $v_2$  from solution  $s$  to solution  $s'$ . Next move is the one between routes  $v_3$  and  $v_4$  that generates new solution  $s''$ . Last one is moving customer from route  $v_5$  to  $v_6$ , giving a solution  $s'''$ . Using best improvement pivoting rule three iterations of relocate operator would be needed to iterate from solution  $s$  to solution  $s'''$ . It is obvious that all moves can be performed in just one iteration since all those moves are independent from each other and there is no need to evaluate the neighborhood three times.

Multiple improvements can also be applied for single route operators as long as they do not interfere with each other. Virtually all standard operators can be modified to benefit from new described strategy. Proposed multiple improvement search step strategy is used for operators that change two routes. In our case the best improvement strategy is used for single route operators, since CVRP routes are often built from small number of customers, and thus, performance gain is relatively small.

In order to speed up a local search process the novel multiple improvement pivoting rule is proposed, which uses the fact that in one algorithm iteration several independent moves can be done. This mechanism can be applied whenever moves done by this pivoting rule do not make solution infeasible.

## 5 ILS ALGORITHM

To test performance of multiple improvement strategy, simple ILS algorithm is selected (Algorithm 1). It is good example of algorithm that uses local search because of its simplicity and it does not need excessive parameters tuning like simulated annealing or memetic algorithms. Same local search speed up approach can be applied for improving other commonly used metaheuristics algorithms for VRP mentioned in Section 3.

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### Algorithm 1 ILS

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```

init := Sweep()
s := LocalSearch(init)
best := s
while not Terminate() do
  s' := Escape(s)
  s'' := LocalSearch(s')
  if  $f(s'') < f(best)$  then
    best := s''
  end if
  s := s''
end while

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Initial solution is obtained by Sweep algorithm proposed by Gillett and Miller [12]. Local optimum is then

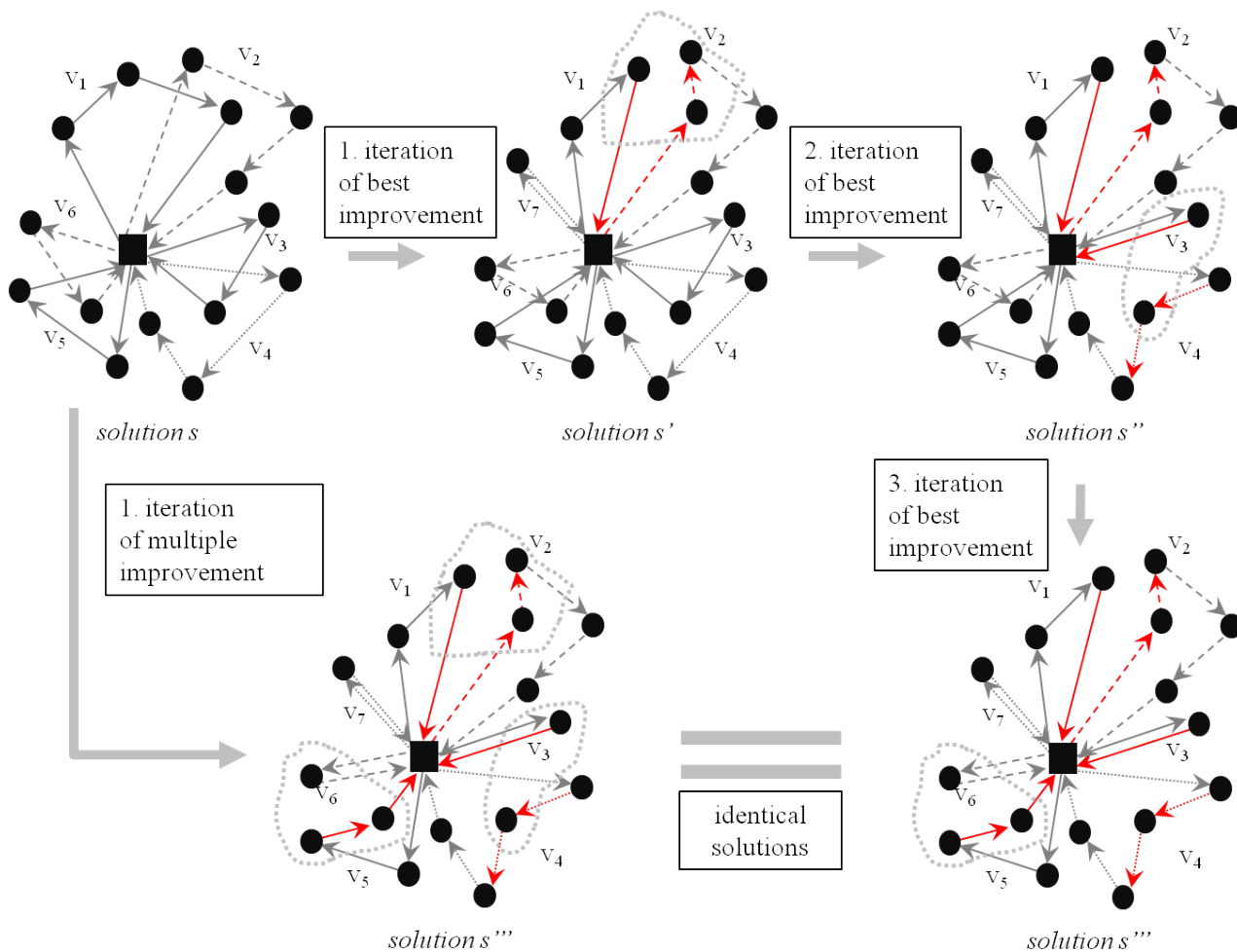


Fig. 5. Multiple relocate moves

obtained by applying *LocalSearch* procedure. It consists of relocate and 2-opt operators that are applied on a single route. Relocate, exchange and 2-opt\* operators are changing customers from two routes. When local search gets stuck into local optima *Escape* procedure tries to escape from it by modifying current solution. *Escape* procedure uses idea from Large Neighborhood Search (LNS) [13], and Ruin and Recreate framework proposed by Schrimpf et al. [14]. First, removal of  $r$  customers from solution is done randomly, and then removed customers are inserted with Regret insertion heuristics [15]. Local search is applied on modified solution  $s'$  until local optimum is reached. New solution  $s''$  is accepted if its distance is smaller than distance of currently best solution. Algorithm iterates until termination condition is met. Usually maximal number of iterations is used as termination criteria.

## 6 EXPERIMENTAL RESULTS

Proposed algorithm was implemented in C++ and compiled with MS Visual C++ 10.0 compiler. All tests were performed on a PC with Intel i5 – 2410M 2.3 GHz (with turbo boost up to 2.9 GHz) processor running Microsoft Windows 7 64-bit operating system. Termination criteria for ILS algorithm was set to 100,000 iterations.

Tests were conducted on 4 real-world problems in Croatia and on standard Christofides et al. [1], and Rochat and Taillard [2, 16] CVRP benchmarks. Christofides et al. problems with distance constraints were not tested since DCVRP is not considered in this research. The 4 real-world problems were used for comparison regarding solution quality and computational time. Used CVRP benchmarks were used for comparison of solution quality. Known computational time for CVRP benchmarks is related to the case where algorithms are run until best possible solution has been found and in this paper applications with

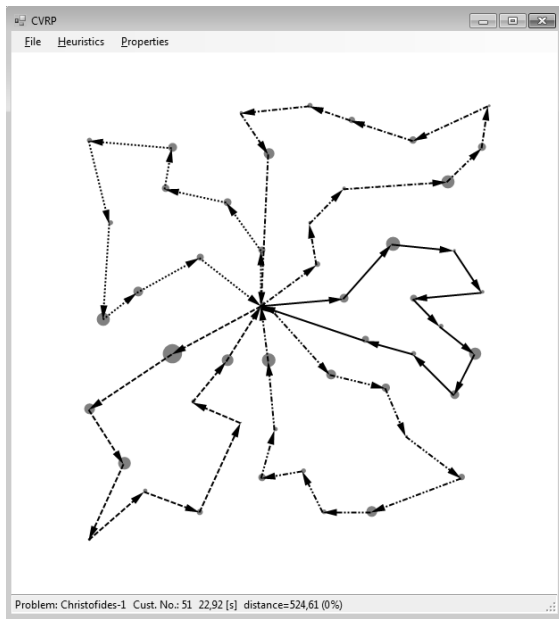


Fig. 6. GUI of developed application presenting CVRP problem (Christofides 1) solved optimally

limited available computational time are considered.

Figure 6 shows graphical user interface developed for easier testing of the algorithm. Selected pivoting rules for implemented ILS algorithm comparison were first improvement, best improvement and proposed multiple improvement in real-world benchmarks. In standard benchmarks only proposed multiple improvement was used and compared to best known benchmarks results.

### 6.1 Real-world benchmarks

As mentioned, performance of the proposed pivoting rule is tested on four real-world very large instances in Croatia (Fig. 7). This four benchmark problems were generated from data obtained from a large Croatian logistics company. Originally, problem was defined as location-allocation problem to obtain locations of four distribution centers. Total of 16, 515 customers were clustered and proposed depot locations were found [17]. Number of customers, depots names and number of vehicles per cluster can be found in Table 1. For this benchmark the demand weights for all customers were set to 1.0 and vehicles capacities were set to 30.0.

Described four benchmarks were used to compare proposed multiple improvement pivoting rule with well known first and best improvement because very large CVRP instances are needed. Comparison of first, best and multiple improvement pivoting rules can be found in Table 2. Total number of improving moves for local search is

Table 1. Croatia benchmark problems

Region	Depot	No of cust.	No of veh.
SW	Rijeka	3, 805	127
SE	Split	3, 154	106
NE	Đakovo	2, 492	84
Central	Zagreb	7, 065	236

given, from initial solution until local search is stuck in local optima. First and best improvement pivoting rules have the same number of iterations as the number of moves. For multiple improvement pivoting rule the number of iterations is considerably smaller than number of moves. The average moves/iterations ratio is about 16 moves per iteration and is the main reason for substantial speed-up of multiple improvement pivoting rule.

Distance wise, it is hard to distinguish best approach among compared three pivoting rules. Best improvement has smallest total distance for two problems. First and multiple improvements had smallest total distance in one benchmark, respectively. Since first improvement has smallest distance in largest problem, it has smallest average distance for all four problems. It is interesting to notice that local search with first improvement pivoting rule makes approximately 2.5 times more moves than best and multiple improvement. Since changing customers inside or among routes takes some amount of CPU time, first improvement is slower than best improvement approximately 3 times.

Major difference in performance comparison arises in execution time. As mentioned earlier, best improvement is 3 times faster than first improvement. Proposed pivoting rule, multiple improvement, outperforms them both, and



Fig. 7. CRO benchmark

Table 2. Pivoting rules comparison

Problem	Size	First improvement			Best improvement			Multiple improvement			
		Dist.	Moves	Time [s]	Dist.	Moves	Time [s]	Dist.	Moves	Iterations	Time [s]
SW	3806	186.92	6331	93.60	186.79	3049	50.97	186.99	3108	169	7.30
SE	3155	184.92	7313	197.12	185.06	3103	55.07	184.67	3268	271	11.41
NE	2493	114.64	4467	25.37	114.33	2148	20.94	114.86	2131	196	3.92
Central	7065	288.63	23791	2998.89	295.10	7534	985.52	293.55	8158	384	91.47
avg		193.78	10476	828.75	195.32	3959	278.13	195.02	4167	255	28.52

this by a quite significant margin. It is 9.75 times faster than best improvement, and 29.05 time faster than first improvement. This difference emerges from the fact that although number of moves is roughly the same as for best improvement more than one move is made per iteration, so local search gets stuck in local optimum substantially faster. For smaller sized problems speed-up is less pronounced, as a smaller number of moves per particular local search iteration are performed.

Currently, routes are implemented in *std::vector* container, and it could be wise for future work to test some faster route representations, i.e. linked list. For problem located in central Croatia (contains 7,065 customers and 236 vehicles) one complete neighborhood evaluation and performing move itself for best improvement is executed in 0.13 seconds, on average, which is very fast. Also, substantial speed-ups without significant loss in solution quality can be achieved with neighborhoods reduction using neighbors lists.

### 6.2 Standard CVRP benchmarks

CVRP benchmarks of Christofides et al. [1] are divided into random problems (1 – 10) and clustered problems (11 – 14). As mentioned earlier, we are using benchmarks that are not distance constraint (problems 1 – 5 and 11, 12). Termination criteria for ILS algorithm was set to 100,000 iterations. Best known results (distances) are taken from Nagata and Bräysy [18]. Table 3 shows comparison of best known results and obtained results for ILS algorithm with proposed multiple improvements local search selection mechanism.

Table 3. Christofides et al. benchmark problems

Problem	Best distance	ILS distance	$\Delta$ [%]	Time [s]
1(50)	524.61	524.61	0.00%	21.72
2(75)	835.26	835.32	0.01%	45.47
3(100)	826.14	826.14	0.00%	136.70
4(150)	1028.42	1034.97	0.63%	355.73
5(199)	1291.29	1336.02	3.35%	373.93
11(120)	1042.11	1042.11	0.00%	183.21
12(100)	819.56	819.56	0.00%	129.59
avg			0.57%	178.05

ILS algorithm found 4 out of 7 best known solutions. Problem 5 with distance 3.35% above best known solution shows poor performance of used escape mechanism. Biggest issue is in random removal of 10% customers from the local optima solution. Random removal is not good enough to overcome local optima and future work should include other removal techniques [15] or another more powerful escape procedure. With just random removal strategy this algorithm can be considered as a brute-force algorithm and thus enormous number of iterations (100,000) is needed to achieve average deviation of 0.57% above best known solution. With such large number of iterations in mind, algorithm can be considered as rather fast with slightly less than 3 minutes of average CPU time.

Another standard CVRP benchmark set contains 13 instances introduced by Rochat and Taillard [2]. All parameters of ILS algorithm are the same as for previous benchmark set. Results are given in Table 4. Already mentioned poor performance of escape mechanism has even greater impact on this benchmark set. Just 2 instances are solved equally to best known solution, and average deviation from best known solutions is 0.80%. Again largest *T385* instance stands out with 4.98%, while *T150c* instance is slightly above 1% margin. For remaining 11 instances distance is under 1% above best known solutions.

Future work should include further speed ups of lo-

Table 4. Taillard benchmark problems

Problem	Best distance	ILS distance	$\Delta$ [%]	Time [s]
<i>T75a</i>	1618.36	1618.36	0.00%	70.72
<i>T75b</i>	1344.62	1356.58	0.88%	58.62
<i>T75c</i>	1291.01	1291.01	0.00%	70.61
<i>T75d</i>	1365.42	1365.91	0.04%	64.47
<i>T100a</i>	2041.34	2047.90	0.32%	109.72
<i>T100b</i>	1939.90	1941.08	0.06%	132.86
<i>T100c</i>	1406.20	1415.28	0.64%	162.42
<i>T100d</i>	1580.46	1596.31	0.99%	152.05
<i>T150a</i>	3055.23	3068.67	0.44%	320.58
<i>T150b</i>	2727.89	2732.52	0.17%	384.68
<i>T150c</i>	2341.84	2365.89	1.02%	277.85
<i>T150d</i>	2645.39	2669.39	0.90%	432.14
<i>T385</i>	24431.44	25712.70	4.98%	3205.73
avg			0.80%	418.65

cal search procedure utilizing heterogeneous computing, by exploiting graphical processing unit [19].

## 7 CONCLUSIONS

In this paper novel local search step selection mechanism is proposed. The proposed search step mechanism uses the fact that in one algorithm iteration, several independent improvement moves can be done. This mechanism can be applied whenever capacity constraints for CVRP are fulfilled. In this way optimization procedure requires less iteration of local search procedure. Although number of search moves remains the same, significant speedup is achieved since search moves are less time demanding than algorithm iterations.

The proposed pivoting rule reduces, in order of magnitude, running time needed for performing local search while solution quality remains approximately the same. This novelty can be used in many approaches in solving CVRP because most of the state of the art algorithm use some kind of local search. As size of the problem grows, the contributions of archived speed-ups become more relevant as is shown on large real-world instances.

Multiple improvement pivoting rule is most suitable for very large CVRP problems, and it was tested on real-world instances with up to 7,065 customers and 236 vehicles. Experimental results confirm that substantial speedups can be achieved, up to 29 times compared to first improvement for largest tested instance without noticeable degradation in solution quality.

For vast majority of standard CVRP benchmarks ILS algorithm has obtained high-quality solutions. Although algorithm can not be considered as slow regarding CPU time, with better focused escape strategy algorithm would need less iterations, and as a consequence, less CPU time.

## ACKNOWLEDGMENT

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