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DETERMINATION OF ACTUAL DISCHARGE OF HIGH-PRESSURE LOW-DISCHARGE AXIAL PISTON PUMPS

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Summary

High-pressure low-discharge axial piston pumps are commonly used for the dosage of fluids of various physical properties and can be found in numerous technical systems. The discharge characteristics of these pumps should be very precise and reliable. Experiments performed with a high-pressure low-discharge axial piston pump show that in the operating range of the discharge pressure and stroke length, the actual discharge of the pump deviates from the theoretical one by up to 35 %. The operation of non-return valves during the suction and discharge phases are analysed and the analytical expression that corrects this deviation is derived. The proposed expression predicts the measured discharge with a relative error of less than 3%.

Key words: *axial piston pump, actual discharge, volumetric efficiency*

1. Introduction

High-pressure low-discharge axial piston pumps are commonly used for the dosage of fluids of various physical properties and can be found in many systems, e.g. in hydraulic drive systems, in systems for sea water desalinization, in the preparation of feed water, in wastewater processing systems, etc. [4]. Usually, they have only one piston in a cylinder equipped with suction and discharge non-return valves. The discharge of these pumps is variable and controlled by changing the piston stroke length. Because of their specific application, the accuracy of their discharge is very important. Thus, the discharge characteristics of these pumps should be very precise and reliable.

The actual discharge of an axial piston pump is usually defined as the difference between the theoretical pump discharge and the total leakage. Volumetric efficiency is defined as the ratio between the actual and the theoretical pump discharge. It depends on the properties of fluid and pump characteristics. Experiments [5,6] performed with a high-pressure low-discharge axial piston pump show that at high discharge pressure and short stroke length, the volumetric efficiency falls to 0.65.

2. Aim of the research

The goal of this paper is to propose an analytical expression that describes pump discharge at various discharge pressures and stroke lengths. In the paper, the acceptable deviation of the calculated discharge from the measured discharge is ±3 %. This issue is already addressed in [6], but in a statistical manner using the Fourier analysis. This paper extends the model from [6] by observing experimental indicator diagrams and analysing sources of volumetric losses. It is assumed that these losses are caused by liquid compression as well as by the inertia and the leakage of the valves. They will be described by observing the non-return valve and the expression derived will be checked throughout the measuring range. What follows is a brief description of the experiment and obtained experimental results. Theoretical analysis of losses and derivation of the expression for predicting the actual pump discharge will be compared to experimental indicator diagrams. Finally, the accuracy of the proposed expression will be verified at the described set of measurements.

3. Description of experiment

Basic technical data of the considered pump are given in Table 1. The pump discharge varies with a change in the stroke length at a constant pump drive shaft rotational speed. Measurements were carried out [5] by a hydraulic measuring system shown schematically in Fig. 1 where p_1 is the pressure measured in the cylinder, p_{atm} is the atmospheric pressure at the intake side, and p_2 at the discharge side. The system for data acquisition and processing (Fig. 2) consists of pressure transducers (P1 and P2), an acceleration transducer, an amplifier, a PC and a tape recorder for simultaneous signal recording. Total measurement uncertainty of the used measuring chain is estimated to be less than 1 % [2]. The crankshaft angle is measured indirectly from the piston acceleration and the kinematics of the reciprocating mechanism.

Table 1 Properties of the analysed axial piston pump

Category of pump	axial piston
Type	single acting, variable discharge
Number of pistons	1
Piston diameter	22 mm
Nominal discharge	30 cm3/s (at maximum stroke length)
Maximum stroke length, H	40 mm
Operating pressure	10 MPa (at piston stroke length of H)
Maximum pressure	30 MPa (at piston stroke length of 10% H)
Discharge control	continuous, by changing the piston stroke length
Rotational speed	2.02 strokes/s
Valve type	non-return

Table 2 Components of the data acquisition and processing system

	Type	Manufacturer	Measuring uncertainty
Pressure transducer	Model 2201	Teledyne Taber	$<\pm 0.2$ %
Acceleration transducer	B 12/200	Hottinger-Baldwin Messtechnik	$<\pm 0.15$ %
Amplifier	Alpha 3000	Hottinger-Baldwin Messtechnik	$<\pm 0.15$ %
Tape recorder	V-Store	Racal Records	$<\pm 0.3$ %

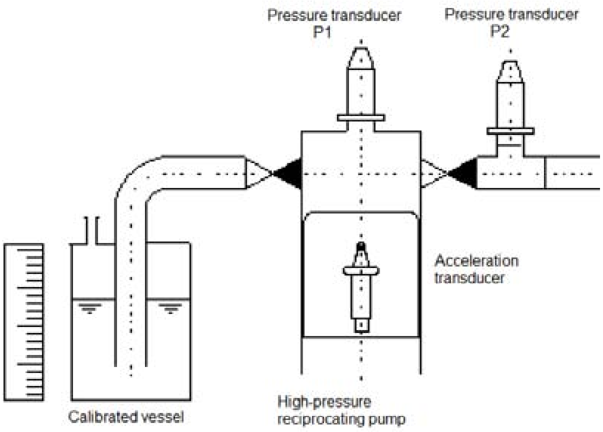


Fig. 1 Scheme of the hydraulic system

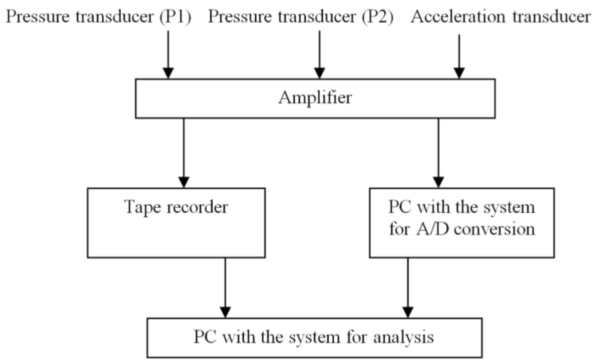


Fig. 2 Data acquisition and processing system

In total, 40 groups of measurements were carried out and data were recorded in measuring sheets. Each measurement is represented by an HxPy indicator diagram (e.g. H6P150 denotes the measurement for a stroke length of 0.6 *H* and a discharge pressure of about 150 bars). Fig. 3 gives an example of an experimental pressure indicator diagram for 0.4 *H* and pressures of 25 and 250 bar. It describes the pressure in the cylinder as a function of crankshaft angle: BDC denotes the bottom dead centre; in A, the suction valve closes, in B, the discharge valve opens; TDC denotes the top dead centre; in C, the discharge valve closes, and in D, the suction valve opens. [1] The position of the TDC remains fixed, while the position of the BDC changes with a change in the stroke length.

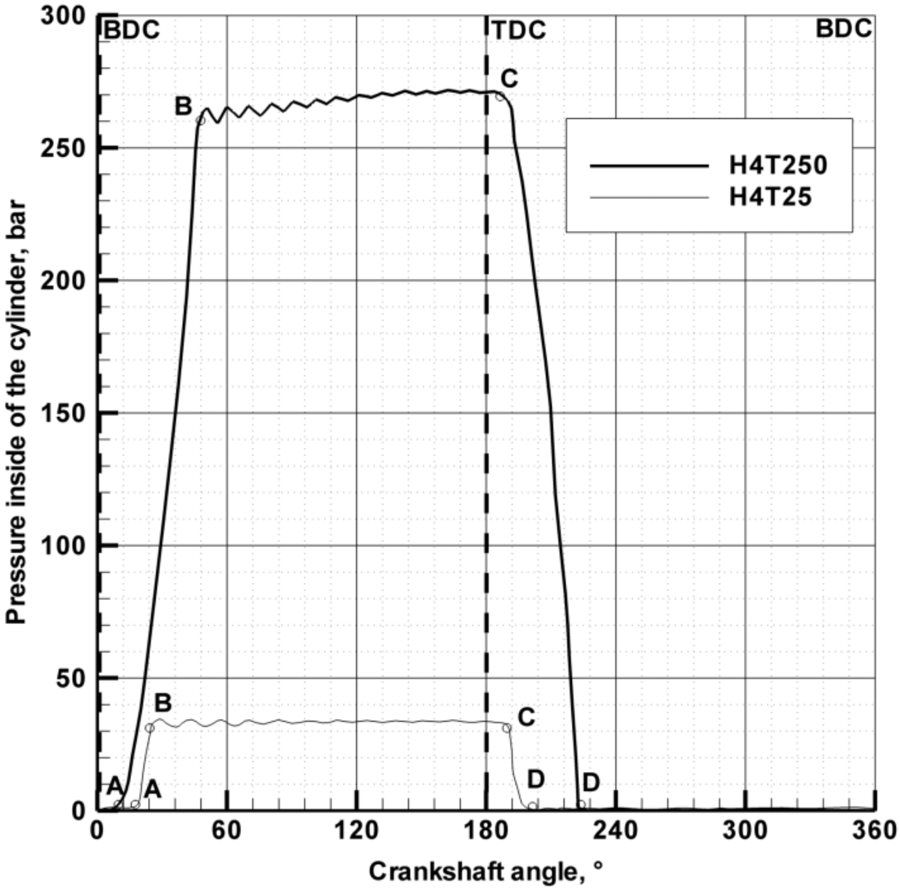


Fig. 3 Experimental diagram of H2P50 and H2P250

4. Mathematical model of actual discharge

Theoretically, the discharge phase starts at the bottom dead centre (BDC), and ends at the top dead centre (TDC), while the intake phase starts at TDC, and ends at BDC (Fig. 4). Obviously, the discharge phase cannot begin before the pressure in the cylinder rises above the discharge pressure, i.e. before point B. Also, the intake phase cannot begin before the pressure in the cylinder drops below the atmospheric pressure, i.e. before point D. [3]

Based on Fig. 3, the actual pump cycle clearly differs from the theoretical one because delays in the start of the phases can be observed. These delays can be divided into two parts. The first part of the delay depends on the time needed for the ball inside the valve to close the flow through the valve. Because of that, the suction valve will close the flow not at BDC, but at some point A which the piston will reach with some time delay $t_{BDC\text{-}A}$ and stroke offset ΔH_A. Analogously, the flow through the discharge valve will not be closed at TDC, but at some point C, with a time delay $t_{TDC\text{-}C}$ and stroke offset ΔH_C. The second part of the delay depends on the time needed for the pressure inside the cylinder to rise above the pressure needed for the opening of the discharge valve. Similar is valid for the intake phase. This means that the discharge phase will not start at point A, but at some point B with additional time delay $t_{A\text{-}B}$ and stroke offset ΔH_B, and the intake phase will start at some point D with a time delay $t_{C\text{-}D}$ and stroke offset ΔH_D.

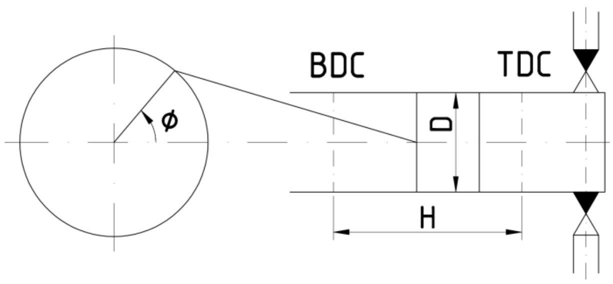


Fig. 4 Reciprocating mechanism

Because of imperfections in sealing, it can be expected that leakage will occur whenever the difference between pressures is present. The experiment showed that the leakage between the cylinder stuffing box and the piston is negligible, so it will not be taken into account. Leakage through the valve, due to a very small orifice, can be considered laminar, and the following linear expression is assumed:

$$m_L = \rho K_V \Delta p \Delta t \tag{1}$$

Where m_L is the leakage through the valve, kg, ρ is the density of fluid at higher pressure, kg/m^3, K_V is the leakage coefficient, m^3/Pas, Δp is the pressure difference, Pa, Δt is the time elapsed, s. Based on that, it is possible to derive an expression that analytically determines the mass of the fluid discharged during one cycle of axial piston pump. This will be done by observing the non-return discharge valve (Fig. 5). In Table 3, one pump cycle is divided into six phases.

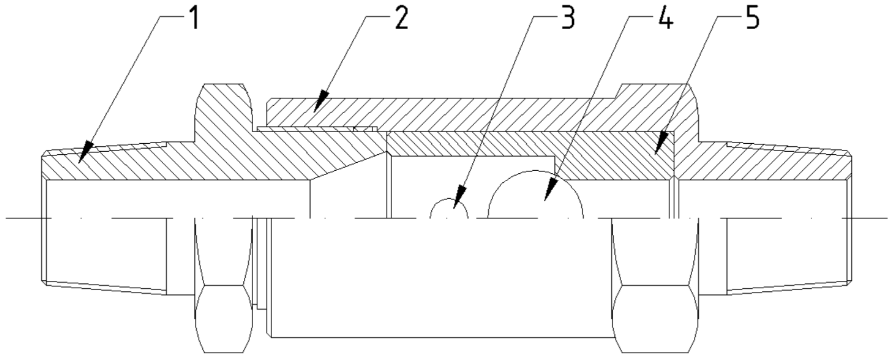


Fig. 5 Cross section of non-return valve: the valve consists of two outer parts (1) and (2), with a sleeve enclosed by them (5); a ball (4) moves between the valve sleeve (5) and a pin (3)

Phase BDC – A is the phase of the suction valve closing. The discharge valve is closed, but leakage is present because of the pressure difference (fluid returns into the cylinder through the discharge valve). The mass of the fluid returned through the discharge valve according to (1) is expressed as:

$$m_{BDC-A} = -K_V \rho (p_2 - p_{atm}) \Delta t_{BDC-A} \tag{2}$$

Phase A – B is the phase of compression of the fluid. Both valves are closed, but leakage is also present through the discharge valve because of the pressure difference that is not constant, so the mass of the fluid is expressed as:

$$m_{A-B} = -K_V \rho \int_{t_A}^{t_B} [p_2 - p_1] dt = -K_V \rho (p_2 - p_{atm}) \Delta t_{AB} + K_V \rho \int_{t_A}^{t_B} [p_1 - p_{atm}] dt \tag{3}$$

It has to be noted that during this phase, leakage through both valves is noted which will result in a shift of point B (point at which the pressure is p_2). Since the exact position of B is unknown, B can be defined in the way that the leakage through the suction valve is already taken into account.

Phase B – TDC is the discharge phase. The discharge valve is open, but leakage through the suction valve is present. Because of that and the delay from phase BDC – B, the mass of the discharged fluid is given by the following expression:

$$m_{B-TDC} = \frac{d^2 \pi}{4} \rho \left(H - \Delta H_A - \Delta H_B \right) - K_V \rho (p_2 - p_{atm}) \Delta t_{B-TDC} \tag{4}$$

Phase TDC-C is the phase of the discharge valve closing. The discharge valve is still open, so the fluid returns into the cylinder, and leakage through the suction valve is present. Thus, the mass of the returned fluid is given as:

$$m_{TDC-C} = -\frac{d^2 \pi}{4} \rho \Delta H_C - K_v \rho (p_2 - p_{atm}) \Delta t_{TDC-C} \tag{5}$$

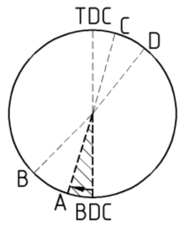
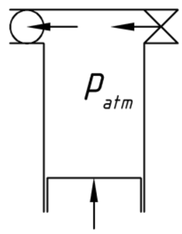
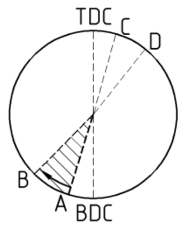
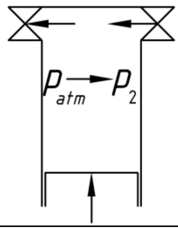
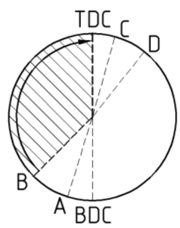
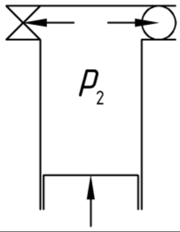
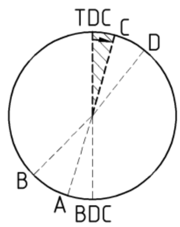
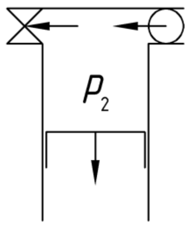
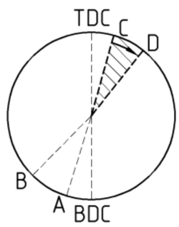
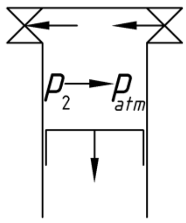
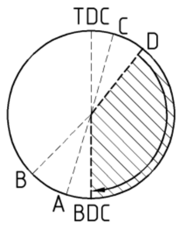
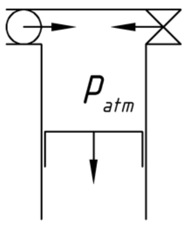
In the C – D phase, the discharge valve is closed and the pressure drops from p_2 to p_{atm}, but leakage is noted at both valves. Therefore, the total mass of the returned fluid in this phase is as follows:

$$m_{C-D} = -K_V \rho \int_{t_C}^{t_D} [p_2 - p_1] dt = -K_V \rho \left(p_2 - p_{atm} \right) \Delta t_{CD} + K_V \rho \int_{t_C}^{t_D} [p_1 - p_{atm}] dt \tag{6}$$

Phase D – BDC is the intake phase. The discharge valve is closed, but leakage is present because of the pressure difference and, as in (2), the mass of the returned fluid equals:

$$m_{D-BDC} = -K_V\left(p_2 - p_{atm}\right)\rho\Delta t_{D-BDC} \tag{7}$$

Table 3 Phases during one pump cycle: the angle at which the crankshaft rotates is marked bold and shaded, "x" denotes the closed valve, "o" denotes the open valve, direction of flow is marked with arrows, the pressure in the cylinder, p_1, is also indicated and it varies between the atmospheric pressure p_{atm} and the discharge pressure p_2

Phase	Crankshaft position	Flow schematic
BDC-A		
A-B		
B-TDC		
TDC-C		
C-D		
D-BDC		

Total mass of the fluid discharged during one pump cycle is obtained by summing up equations (2) – (7):

$$m = \frac{d^2\pi}{4}\rho\left[H - \left(\underbrace{\Delta H_A + \Delta H_B + \Delta H_C}_{\Delta H_X}\right)\right] - K_V\rho\left(p_2 - p_{atm}\right)\Delta t_{cycle} + \\ + K_V\rho\int_{t_A}^{t_B}\left[p_1 - p_{atm}\right]dt + K_V\rho\int_{t_C}^{t_D}\left[p_1 - p_{atm}\right]dt \tag{8}$$

Since $\Delta t_{AB} = t_B - t_A$ and $\Delta t_{CD} = t_D - t_C$ are much smaller than Δt_{cycle}, the last two members of the equation can be neglected and the total mass of the fluid discharged during one working cycle is:

$$m = \frac{d^2\pi}{4}\rho\left(H - \Delta H_X\right) - K_V\rho\left(p_2 - p_{atm}\right)\Delta t_{cycle} \tag{9}$$

Four members of that equation denote the following:

- m, actual discharge,
- $\rho H d^2\pi / 4$, theoretical discharge (compressibility of the fluid is already taken into account since ρ is defined at the discharge pressure),
- $\rho\Delta H_X d^2\pi / 4$, reduced discharge because of the delay in the closing of valves,
- $K_V\rho\left(p_2 - p_{atm}\right)\Delta t_{cycle}$, reduced discharge because of leakage.

5. Results and discussion

It can be seen from Fig. 3 that ΔH_A and ΔH_C are slightly smaller for higher discharge pressure (valves close faster), while ΔH_B is greater. Since the density of the fluid in (9) is variable, the difference in ΔH_B is much less pronounced, so it can be assumed that ΔH_X is constant. Since the leakage is expressed in (1), the leakage coefficient K_V can also be considered constant. This allows us to use the least square method to statistically analyse data obtained through the experiment using the following criterion:

$$K = \sum_{i=1}^{40}\left(\frac{m_{meas,i} - m_{calc,i}}{m_{meas,i}}\right)^2 = \min \tag{10}$$

Where m_{meas} represents the measured and m_{calc} the calculated (9) mass discharged during one pump cycle. When (10) is differentiated by ΔH_X and K_V, the following equations are valid:

$$\frac{\partial K}{\partial \Delta H_X} = 0 \tag{11}$$

$$\frac{\partial K}{\partial K_V} = 0 \tag{12}$$

By solving these equations, parameters describing the actual discharge are obtained: $\Delta H_X = 0.755\ \text{mm}$, and $K_V = 6.048\cdot 10^{-14}\ \text{m}^3/\text{Pas}$. The actual flow for different stroke lengths and pressures was calculated by using these parameters and equation (9).

Table 4 contains data on the mass of fluid discharged during one pump cycle. The results are plotted versus the theoretical and the experimental results and shown in Fig. 6. As can be observed from Table 4 and also from Fig. 6, the theoretical and the measured values differ substantially, especially at shorter stroke lengths and higher pressures, while the calculated results match quite well with the measured values.

Table 4 Mass (g) of fluid discharged during one pump cycle: the first (upper) number represents the measured value, the second the calculated value, and the third the theoretical value for different combinations of the stroke length and the discharge pressure

	H2	H4	H6	H8	H10
T25		5.3866 5.4020 5.7665	8.1657 8.0924 8.4600	10.5948 10.5611 10.9259	12.9691 12.8882 13.2401
T50	2.5334 2.4778 2.9212	5.2768 5.3170 5.7665	8.0696 8.0334 8.4600	10.5399 10.4988 10.9259	12.8483 12.8120 13.2401
T75		5.1670 5.2642 5.7665	8.0189 7.9682 8.4600	10.4576 10.4483 10.9259	12.6260 12.7595 13.2401
T100	2.3413 2.2970 2.9212	5.0627 5.1864 5.7665	7.8761 7.9018 8.4600	10.3794 10.3765 10.9259	12.6260 12.7287 13.2401
T125					12.5024 12.6636 13.2401
T150	2.1766 2.2037 2.9212	4.9598 5.0683 5.7665	7.6716 7.7927 8.4600	10.1831 10.2576 10.9259	12.4201 12.6157 13.2401
T200	2.0764 2.0308 2.9212	4.8445 4.9495 5.7665	7.5536 7.6173 8.4600	9.9978 10.1369 10.9259	
T250	1.8994 1.8794 2.9212	4.7004 4.7561 5.7665			

Equation (9) can be transformed into:

$$\underbrace{H-\frac{4m}{\rho d^2\pi}}_{y}=\Delta H_X+K_V\underbrace{\frac{4\left(p_2-p_{atm}\right)}{d^2\pi}\Delta t_{cycle}}_{x} \tag{13}$$

This represents a linear equation with variables y and x, while ΔH_X represents the y-intercept, and K_V the line gradient. Each side of the equation represents the difference between the theoretical and the actual discharge. The calculated difference in discharge is plotted in Fig. 7 along with the measured differences represented by points. This figure shows the absolute difference, while the least square method was used for data describing the relative difference, so it can be observed that this line is closer to points with lower discharge, i.e. H2 and H4. This is because the criterion K was defined as the sum of relative differences in the discharge. If the criterion was defined as the sum of absolute differences, then ΔH_X and K_V

would be slightly changed. The authors decided to minimize relative differences because the deviation from the theoretical discharge is much more pronounced at shorter stroke lengths, by up to 35 %, while at the maximum stroke length the differences are less than 7 % (Table 4).

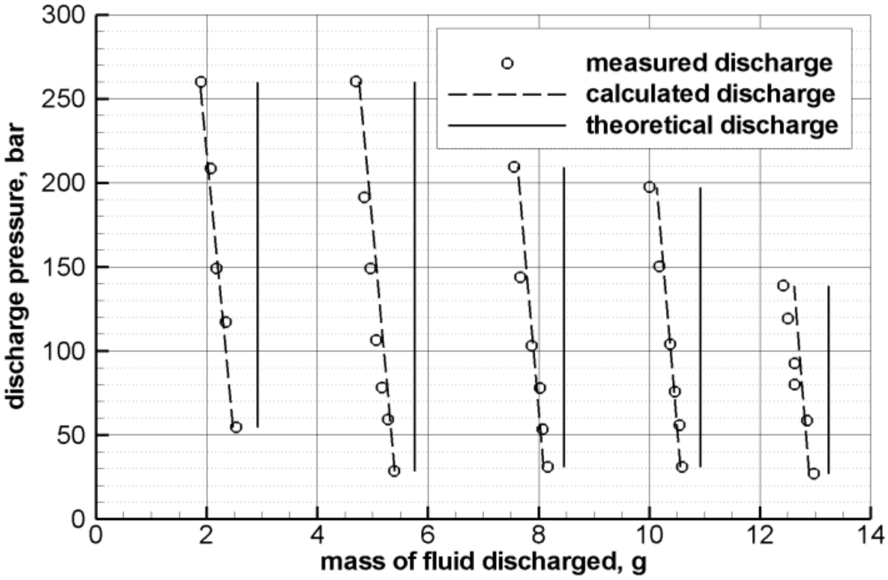


Fig. 6 Measured, calculated and theoretical discharge in g during one pump cycle at different pressures and stroke lengths: the theoretical discharge is represented by solid line, the calculated by dashed line, and the discharge measured in certain points by circles

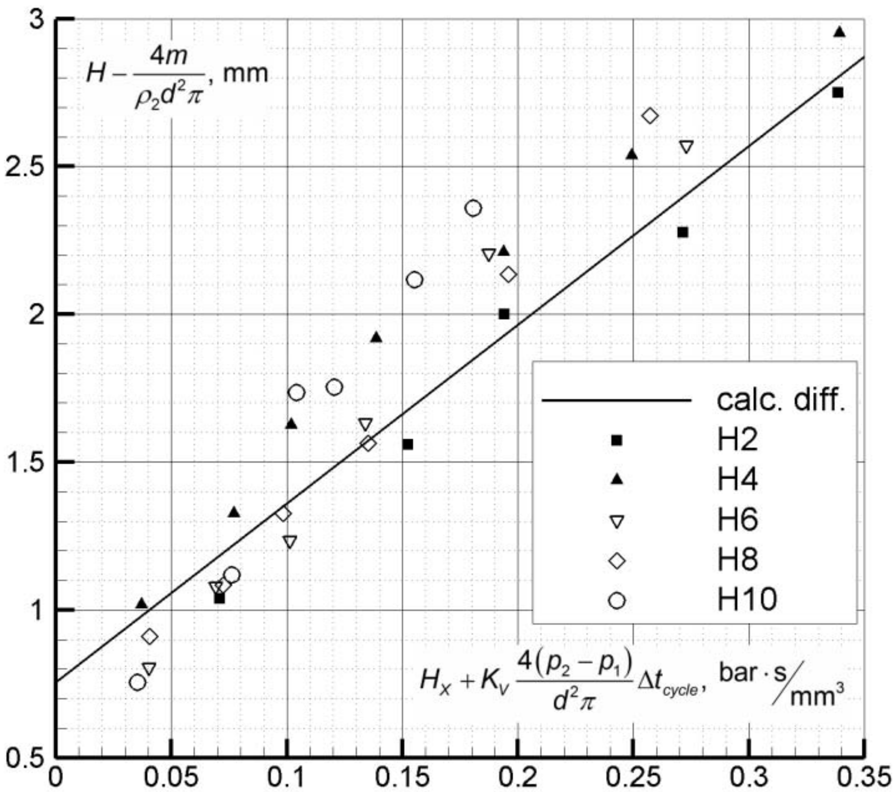


Fig. 7 Measured (points) and calculated (line) difference with respect to the theoretical discharge

6. Conclusion

An analytical expression determining the actual pump discharge has been derived. It predicts the actual discharge with the difference between the calculated and the actual discharge of less than 3 %. The expression represents a viable tool for determining discharge throughout the measuring range. This means the goal of this paper is achieved since the accuracy in pump discharge calculation is within acceptable limits. The expression is derived by observing the non-return valve operation and by using experimental results. Parameters describing the non-return valve operation, ΔH_X and K_V, are functions of non-return valves. Thus, the expression is only valid for similar pumps using the same valves. Further improvements could be made in determining parameters ΔH_X and K_V for any valve, and with that, the expression could be used for any axial piston pump.

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