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ELASTIC STABILITY ANALYSIS OF THIN-WALLED C- AND Z-SECTION BEAMS WITHOUT LATERAL RESTRAINTS

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Summary

The paper analyzes the elastic stability of steel thin-walled C- and Z-cross-section beams without lateral restraints. Firstly, mechanical properties of the beams material (steel) are determined by testing standard specimens in a laboratory. Based on the obtained data, the stability analysis of beams is carried out and critical forces are determined analytically by using the theory of thin-walled beams, numerically by using the finite element method (FEM), and experimentally by testing the C- and Z-cross-section beams in a laboratory. The analysis of critical forces and stability shows that the calculation according to the theory of thin-walled beams does not take the effect of local buckling into account, and that the resulting critical global forces do not correspond to the actual behaviour of the beam. The FEM gives the value of the critical force by taking the effect of the local buckling into consideration. The experimental test shows that the assumptions and simplifications, which have been introduced into the theory of thin-walled beams with open cross sections, significantly affect final results of the level of the critical force.

Key words: *elastic stability, critical force, thin-walled beam, C- and Z-section, finite element method*

1. Introduction

Constructions which are made of thin-walled open cross-section beams are increasingly used in engineering practice because of safety requirements and economic conditions. The behaviour of such beams under specified load is considerably more complex than the behavior of beams with solid cross sections. Thin-walled open cross-section beams tend to buckle locally because of the shapes of their cross sections. The basic theory of thin-walled members with open cross sections was developed by Vlasov [1]. He applied the term "sectorial coordinate" for the first time and presented the subject of mixed torsion in a most outstanding manner. The numerical procedures as well as necessary modifications were introduced later by Gjelsvik [2], Murray [3] and Ojalvo [4].

Several researchers have investigated the behaviour of thin-walled open cross-section beams [5-8]. According to the form of the open cross-section beams, the position of the shear centre and the plane load, special attention should be paid to the stability of the thin-walled open cross-section beams. Paczos [9] compared the experimental values of the critical load with those obtained with the help of the finite element method (FEM) and theoretical analyses. The experimental study of the thin-walled open cross-section beams subjected to pure bending is presented by Pastor [10]. There, the thin-walled open cross-section beams are exposed to the distortion of cross sections. In such beams, apart from the flexural form of the loss of stability, torsional and torsional-flexural losses of stability are possible, too. Lots of studies about this topic have been carried out [11-13].

The aim of this study is to determine whether the assumptions and simplifications which have been introduced in the theory of thin-walled open cross-section beams significantly affect the form of the loss of stability of the beam and the level of the critical force. Critical forces obtained by the theory of open-cross thin walled beams, by the FEM and experimentally will be compared, and the advantages and disadvantages of each method will be analyzed. It will give us an insight into the importance of the calculation of the stability of open-cross section thin walled beams.

2. Preview of elastic stability using the theory of thin-walled beams

It is assumed that a flat thin-walled beam with an arbitral cross section is loaded by centric compressive force F, Figure 1.

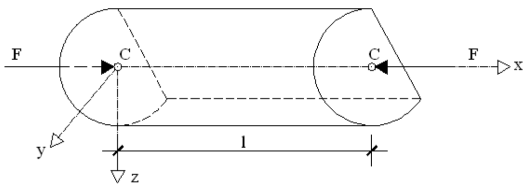


Fig. 1 Thin-walled open cross-section beam loaded by centric compressive force

The axes y, z are the main inertia axes of the cross section, while the axis x is a longitudinal axis of the beam. Point A is the shear centre, Figure 3. Displacement components of the shear centre A in the direction of the main axes of inertia y and z are marked ξ (x) and η (x), while φ (x) is a rotation angle of the cross section around the axis of the shear centre.

Equations of equilibrium for the beam after the loss of stability may be written in the following form [15, 16]:

$$E \cdot I_y \cdot \eta^{IV} + F \cdot \eta'' - y_A \cdot F \cdot \varphi'' = 0 \tag{1a}$$

$$E \cdot I_z \cdot \xi^{IV} + F \cdot \xi'' + z_A \cdot F \cdot \varphi'' = 0 \tag{1b}$$

$$E \cdot I_\omega \cdot \varphi^{IV} + (r^2 \cdot F - G \cdot I_t) \cdot \varphi'' + z_A \cdot F \cdot \xi'' - y_A \cdot F \cdot \eta'' = 0 \tag{1c}$$

I_y and I_z are the main axial moments of inertia, and I_t is the torsional moment of inertia (Saint-Venant torsional constant). I_ω is the main sectorial moment of inertia of the cross section. y_A and z_A are coordinates of the shear centre. E is elasticity modulus and G is shear modulus. r^2 is given in Eq. (8).

The general solution to the system of Eq. (1) contains 12 constants of integration. In order to determine these constants, it is necessary to specify 12 boundary conditions. For homogeneous boundary conditions, a system of linear homogeneous equations is obtained. The critical load is determined from the condition that the determinant of the system is equal to zero. The beam freely laid on two bearings with span l, loaded by the centric compressive force is considered, Figure 2.

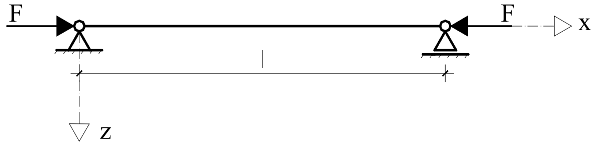


Fig. 2 The thin-walled beam freely laid on two bearings loaded with centric compressive force

For the freely laid beam, shown in Figure 2, the boundary conditions are:

$$\text{for} \quad x=0, \quad \xi=\eta=\varphi=0, \quad \xi''=\eta''=\varphi''=0$$

$$\text{for} \quad x=1, \quad \xi=\eta=\varphi=0, \quad \xi''=\eta''=\varphi''=0 \tag{2}$$

For the given boundary conditions, the solution to Eq. (1) can be assumed as follows:

$$\xi = A\cdot\sin\frac{n\cdot\pi}{l}\cdot x, \quad \eta = B\cdot\sin\frac{n\cdot\pi}{l}\cdot x, \quad \varphi = C\cdot\sin\frac{n\cdot\pi}{l}\cdot x \tag{3}$$

where A, B and C are unknown constants, and n is the whole positive number of sine half waves through the beam length. Once we put the expression (3) in Eq. (1) we get:

$$\begin{pmatrix} n^2\cdot\frac{\pi^2\cdot E\cdot I_z}{l^2} & 0 & -z_A\cdot F \\ 0 & n^2\cdot\frac{\pi^2\cdot E\cdot I_y}{l^2}-F & y_A\cdot F \\ -z_A\cdot F & y_A\cdot F & n^2\cdot\frac{\pi^2\cdot E\cdot I_\omega}{l^2}+G\cdot I_t-r^2\cdot F \end{pmatrix}\cdot\begin{Bmatrix} A \\ B \\ C \end{Bmatrix}=0 \tag{4}$$

To make the system of equations (4) have a solution different from a trivial one, the determinant of the system must be equal to zero:

$$\begin{vmatrix} F_z-F & 0 & -z_A\cdot F \\ 0 & F_y-F & y_A\cdot F \\ -z_A\cdot F & y_A\cdot F & r^2\cdot(F_\omega-F) \end{vmatrix}=0 \tag{5}$$

Euler critical buckling forces in the main planes (pure flexural buckling) F_z, F_y are shown in the expression (6):

$$F_z = n^2\cdot\frac{\pi^2\cdot E\cdot I_z}{l^2} \qquad F_y = n^2\cdot\frac{\pi^2\cdot E\cdot I_y}{l^2} \tag{6}$$

Critical force for the torsional loss of stability (pure torsional buckling) F_ω is shown in expression (7):

$$F_\omega = \frac{1}{r^2}\cdot\left(n^2\cdot\frac{\pi^2\cdot E\cdot I_\omega}{l^2}+G\cdot I_t\right) \tag{7}$$

In expressions (4), (5), (7) it applies:

$$r^2 = \frac{I_y+I_z}{A}+y_A^2+z_A^2 \tag{8}$$

By analysing the determinant (5) we get:

$$\begin{aligned} &\left(y_A^2+z_A^2-r^2\right)\cdot F^3+\left[\left(F_y+F_z+F_\omega\right)\cdot r^2-z_A^2\cdot F_y-y_A^2\cdot F_z\right]\cdot F^2- \\ &-r^2\cdot\left(F_y\cdot F_z+F_y\cdot F_\omega+F_z\cdot F_\omega\right)\cdot F+F_y\cdot F_z\cdot F_\omega\cdot r^2=0 \end{aligned} \tag{9}$$

Since the matrix of the system (4) is a symmetrical one, the cubic equation (9) has three real and positive roots: F_1, F_2, F_3. For each of the three roots, the elastic line consists of n sine half waves. Of practical importance is only the case when the elastic line consists of a single sine half wave, which, according to expressions (3), (6) and 7 for n=1, gives:

$$\xi = A \cdot \sin\frac{\pi}{l} \cdot x, \quad \eta = B \cdot \sin\frac{\pi}{l} \cdot x, \quad \varphi = C \cdot \sin\frac{\pi}{l} \cdot x \tag{10}$$

$$F_z = \frac{\pi^2 \cdot E \cdot I_z}{l^2}, \qquad F_y = \frac{\pi^2 \cdot E \cdot I_y}{l^2} \tag{11a}$$

$$F_\omega = \frac{1}{r^2} \cdot \left(\frac{\pi^2 \cdot E \cdot I_\omega}{l^2} + G \cdot I_t \right) \tag{11b}$$

In expression (9), figures F_z, F_y and F_ω which are given by expressions (11a) and (11b):

for: $F_y < F_z < F_\omega$ it will be $F_1 \le F_y \le F_2 \le F_z \le F_\omega \le F_3$;

for: $F_\omega < F_y < F_z$ it will be $F_1 \le F_\omega \le F_y \le F_2 \le F_z \le F_3$.

Relevant is the critical buckling force of a beam

$$F_{kr} = F_{\min} = F_1 \tag{12}$$

which suits both buckling and twisting of a beam and is always less than F_y, F_z, F_ω.

If the cross section has one axis of symmetry, for example if the major axis of inertia y is at the same time the axis of symmetry of the cross section, then z_A=0, which indicates that the system of equations (1) takes the following form:

$$E \cdot I_y \cdot \eta^{IV} + F \cdot \eta'' - y_A \cdot F \cdot \varphi'' = 0 \tag{13a}$$

$$E \cdot I_z \cdot \xi^{IV} + F \cdot \xi'' = 0 \tag{13b}$$

$$E \cdot I_\omega \cdot \varphi^{IV} + (r^2 \cdot F - G \cdot I_t) \cdot \varphi'' - y_A \cdot F \cdot \eta'' = 0 \tag{13c}$$

The first equation (13a) corresponds to in-plane buckling, and the other two equations (13b) and (13c) correspond to simultaneous twisting and out-plane buckling. The boundary conditions (2) of the system of equations (13) give a result determined by the expression (10).

From Eq. (5) for z_A=0 it is obtained:

$$F_z - F = 0 \tag{14a}$$

$$\begin{vmatrix} F_y - F & y_A \cdot F \\ y_A \cdot F & r^2 \cdot (F_\omega - F) \end{vmatrix} = 0 \tag{14b}$$

From Eq. (14a) the Euler critical force is determined [15]:

$$F = F_z = \frac{\pi^2 \cdot E \cdot I_z}{l^2} \tag{15}$$

By analysing the determinant (14), we get the equation for determining the other two critical forces:

$$(r^2 - y_A^2) \cdot F^2 - r^2 \cdot (F_y + F_\omega) \cdot F + F_y \cdot F_\omega \cdot r^2 = 0 \tag{16}$$

From where [15]:

$$F_{2,3} = \frac{r^2}{r^2 - y_A^2} \cdot \left[\frac{F_y + F_\omega}{2} \mp \sqrt{\left(\frac{F_y + F_\omega}{2} \right)^2 - F_y \cdot F_\omega \cdot \frac{r^2 - y_A^2}{r^2}} \right] \tag{17}$$

Since F_2<F_3, the relevant critical force is equal to the lower force between F_2 and F_z. If F_z<F_2, in-plane buckling of the beam appears, and if F_z>F_2, simultaneous twisting and lateral out-plane bending appear. In other ways of supporting the beam, regarding the analogy of the problems of stability of a flat beam with a solid cross section, the expressions for critical compressive forces can be summarized in the following form [15]:

$$F_z = \frac{\pi^2 \cdot E \cdot I_z}{(\mu \cdot l)^2} \qquad F_y = \frac{\pi^2 \cdot E \cdot I_y}{(\mu \cdot l)^2} \tag{18a}$$

$$F_\omega = \frac{1}{r^2} \cdot \left(\frac{\pi^2 \cdot E \cdot I_\omega}{(\nu \cdot l)^2} + G \cdot I_t \right) \tag{18b}$$

In Eq. (18) different coefficients of apparent length μ and ν are introduced, as the boundary conditions for deflection and rotation in the end sections of the beam do not have to be identical. For the beam which is at one end fixed and on the other end freely supported, the coefficient of apparent length of the beam by bending μ=0.7, and the ratio of the apparent length of the beams at the loss of stability by twisting ν = 0.7. For the beam which is freely supported at both ends μ= ν=1.0, for the beam which is fixed at both ends μ= ν=0.5, for the beam fixed at one end and free on the other one μ= ν=2.0.

3. Determination of critical force and form of loss of stability

The calculation and analysis of stability of the steel thin-walled C-section beam (C 100/50/15/2 and C 100/50/15/4), Fig. 3a, and Z-section beam (Z 100/50/15/2 and Z 100/50/15/4), Fig. 3b, will be carried out:

- experimentally, by testing the models in the laboratory [16],
- analytically, according to the theory of thin-walled open cross-section beams [15],
- numerically, by the FEM with the computer program SAP2000 v14 („buckling“ analysis) [17].

The thicknesses of the beam walls are t=2 mm and t=4 mm. These beams, with a span l=120 cm, are fixed at one end, and freely supported at the other one, and loaded by compressive force in the centre of gravity of the cross section, Figure 3c.

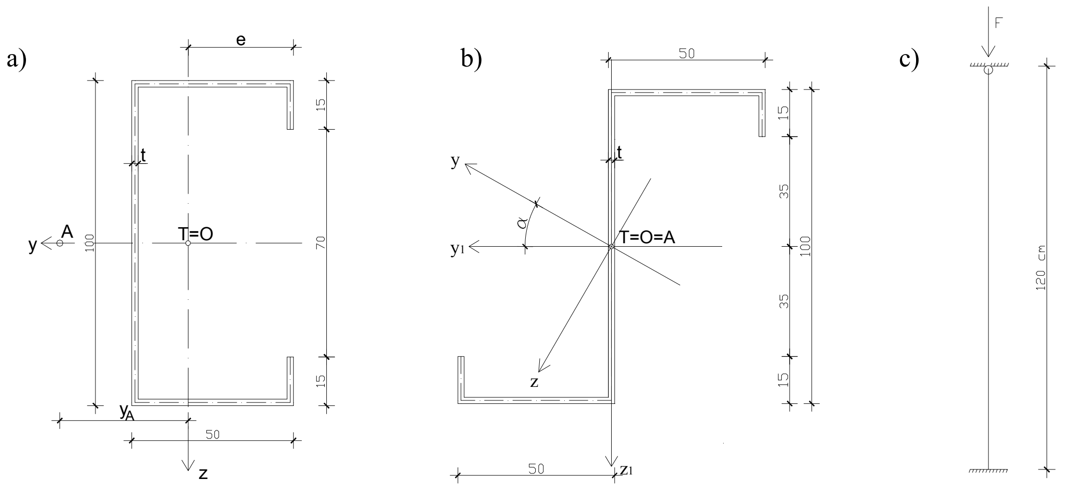


Fig. 3 Thin-walled open section beams: a) C 100/50/15/2 and C 100/50/15/4, b) Z 100/50/15/2 and Z 100/50/15/4, c) beam supports

3.1 Experimental determination of critical force

Mechanical properties of steel, from which the thin-walled C- and Z-cross-section beams were made, were determined in a laboratory by examining a series of three standard specimens with electroresistive tensiometers. The specimens were prepared for testing according to HRN EN ISO 6892-1:2010. The obtained values of the mechanical properties of steel are: tensile strength σ_M=402.34 MPa, liquid limit σ_T=321.16 MPa, modulus of elasticity E=183.76 GPa, Poisson's ratio ν=0.23 and shear modulus G=74.699 GPa.

The stability of thin-walled C- and Z-section beams is tested in a static press. For laboratory testing, solid steel plates were welded on these beams, 100/50/30 mm in dimension, through which compressive forces are acting. The steel plates and beams are made of the same material with the same mechanical properties.

During the laboratory testing, inductive tensiometers were placed on the thin-walled C-section and Z-section beam to measure the lateral displacement and electroresistive tensiometers were used to measure the longitudinal strain in the beams. The disposition of the measurement points on the C-section and Z-section beam is shown in Figure 4.

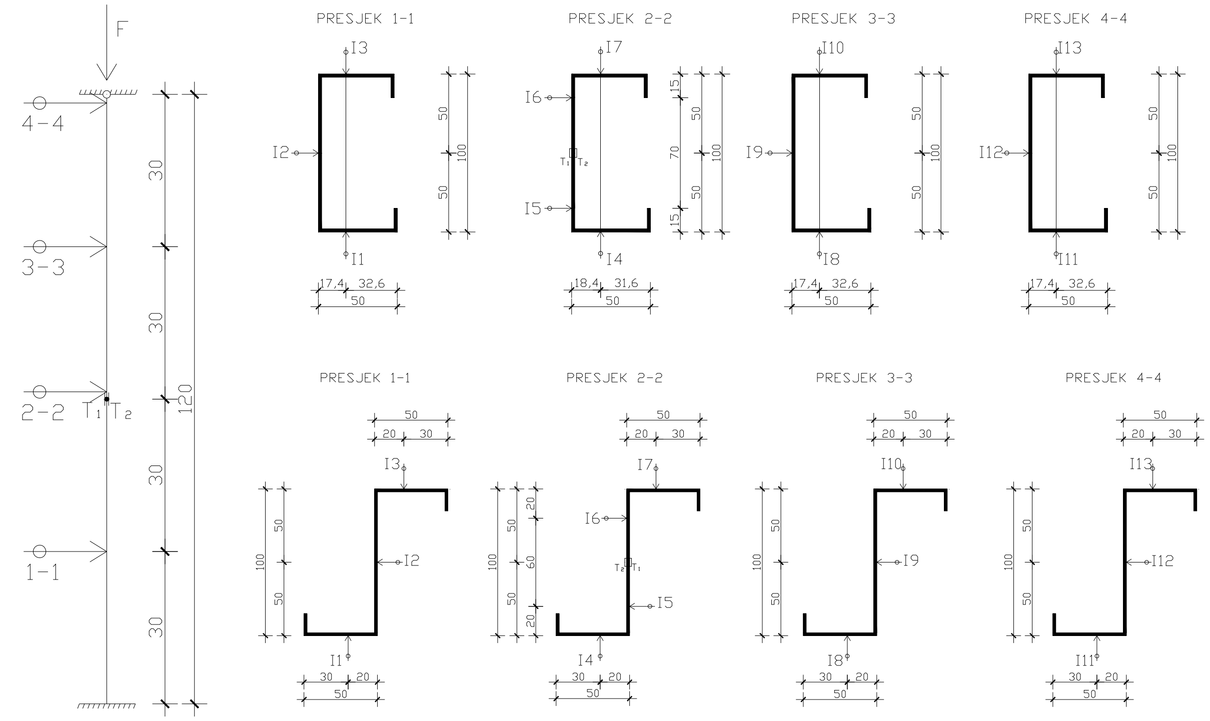
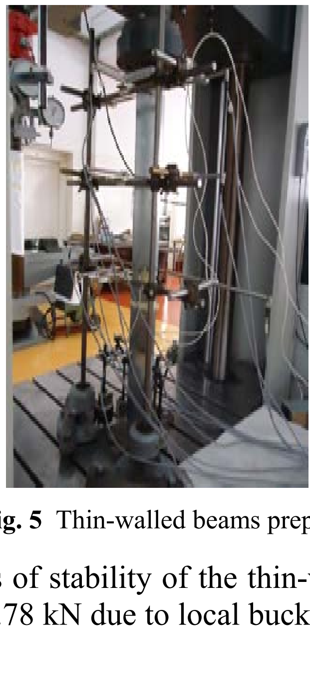


Fig. 4 Disposition of measurement points on the thin-walled C-section and Z-section beam

Fig. 5a shows a thin-walled C-section 100/50/15/2 beam which is prepared for testing, and Fig. 5b shows a thin-walled Z-section 100/50/15/2 beam prepared for testing. The beams were gradually loaded by the increasing force ΔF=50 kN up to the loss of stability.

a)



b)

Fig. 5 Thin-walled beams prepared for testing: a) C 100/50/15/2, b) Z 100/50/15/2

The loss of stability of the thin-walled C-section 100/50/15/2 beam appeared under the force F_{kr}=152.78 kN due to local buckling of the web, Fig. 6a. Fig. 6b shows the expansion of beam flanges, and the deformed thin-walled C-section 100/50/15/2 beam is shown in Fig. 6c. Once the maximum force is achieved, strain and force are in a nonlinear relationship. Strain growths and axial stiffness of the beam fall nonlineary. It is obvious that the global critical load will not be reached due to the local buckling.

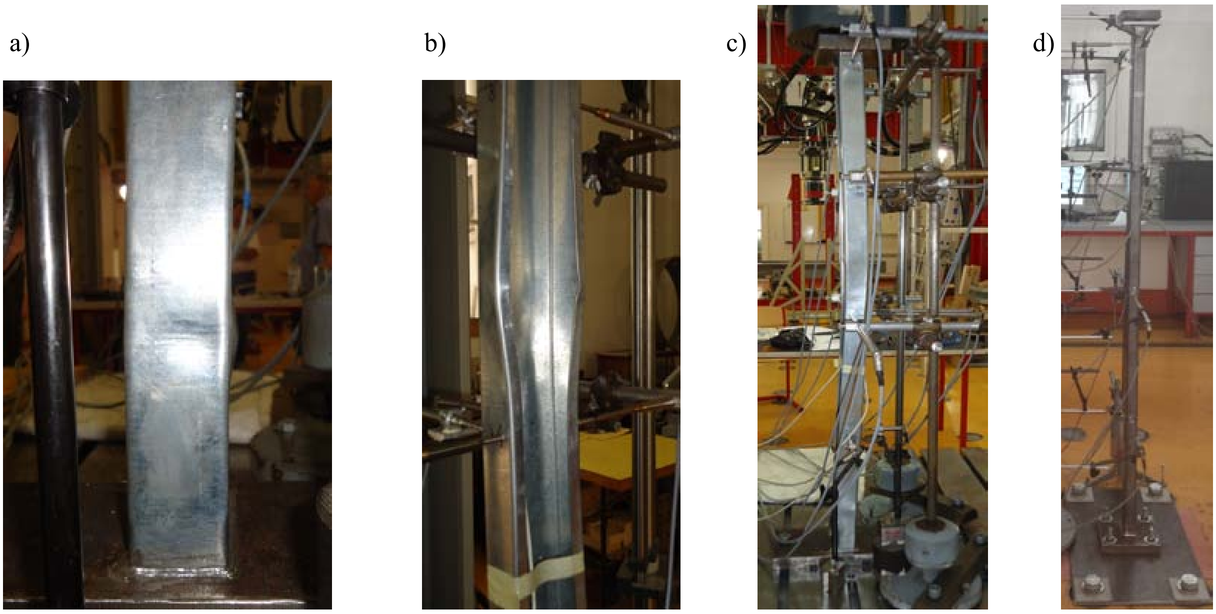


Fig. 6 Loss of stability of the thin-walled C-section beam: a) local buckling of the web of beam C 100/50/15/2, b) expansion of beam flanges C 100/50/15/2, c) deformed beam C 100/50/15/2, d) deformed beam C 100/50/15/4

The loss of stability of the thin-walled beam C 100/50/15/4 appeared under the influence of the global critical force F_{kr}=583.47 kN. Fig. 6d shows the deformed thin-walled beam C 100/50/15/4. Local bucklings of the webs in the beams do not appear due to higher wall thickness t=4 mm.

The loss of stability of the thin-walled beam Z 100/50/15/2 appeared under the influence of force F_{kr}=145.89 kN due to local buckling of the web, Fig. 7a. Fig. 7b shows the deformed thin-walled beam Z 100/50/15/2.

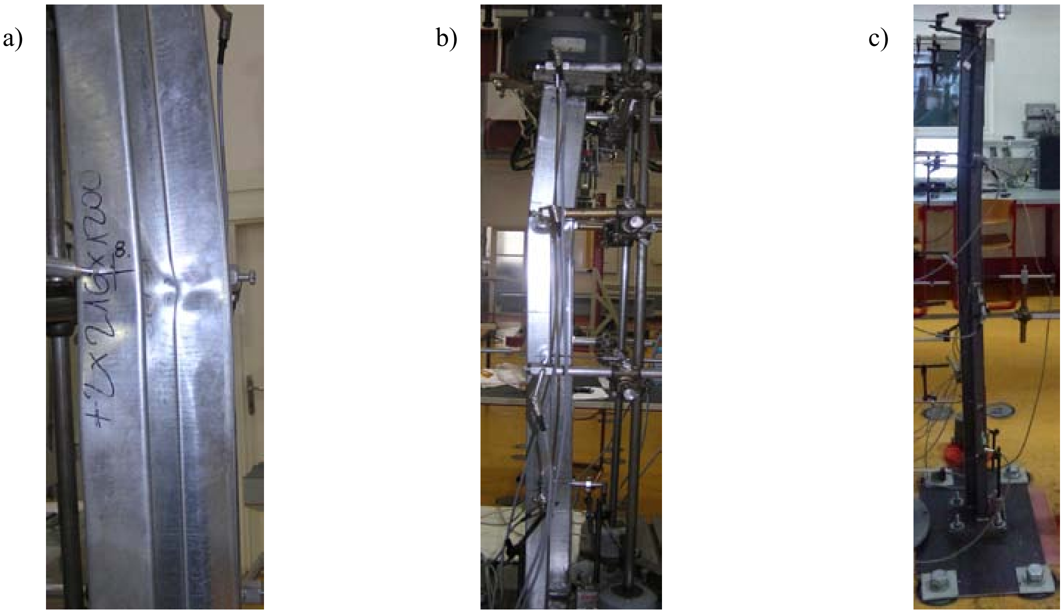


Fig. 7 Loss of stability of the thin-walled Z-section beam: a) local buckling of the web of beam Z 100/50/15/2, b) deformed beam Z 100/50/15/2, c) deformed beam Z 100/50/15/4

The loss of stability of the thin-walled beam Z 100/50/15/4 appeared under the influence of the global critical force F_{kr}=398.76 kN. Fig. 7c shows the deformed thin-walled beam Z 100/50/15/4.

3.2 Analytical determination of the critical force

The values of critical forces will be determined analytically, based on the theory of thin-walled open cross-section beams. In the structural analysis, it is often necessary to determine geometrical properties of a thin-walled beam with an open cross section [18].

3.2.1 C-section beam

For the thin walled C-section 100/50/15/2 beam with the wall thickness t=2 mm the cross-sectional area is A=444 mm^2, Fig. 3a. The coordinate of the centre of gravity regarding the centre line of the open vertical is e = 31.57 mm. The main moments of inertia of the cross-section are: I_y=717620 mm^4; I_z=156672.973 mm^4. The torsional moment of inertia of the cross-section is I_t=592 mm^4, and the main sectorial moment of inertia of the cross-section is I_ω=341640615.1 mm^6. r^2=3617.49 mm^2; y_A=40.60 mm; z_A=0. Using the expressions (18), we get the following values:

F_y=1844.53 kN ; F_z=460.21 kN ; F_ω=254.97 kN.

Critical force:

F_1=F_z=460.21 kN

By solving the expression (17), we get the other two values of the critical force:

F_2=238.79 kN; F_3=3618.2 kN.

Relevant critical force: F_{kr}=F_{min}=238.79 kN.

The loss of beam stability occurs under twisting and lateral out-plane buckling. The critical stress in the beam at the time of the loss of stability is

$$\sigma_{kr} = \frac{F_{kr}}{A} = 537.82 \text{ MPa} .$$

The critical stress under pure buckling in the plane of symmetry is:

$$\sigma_z = \frac{F_z}{A} = 1036.51 \text{ MPa} .$$

The critical stress is only 52% of the critical Euler stress.

For the thin walled C-section 100/50/15/4 beam with the wall thickness t=4 mm the cross-sectional area is A=856 mm^2, Fig. 3a. The coordinate of the centre of gravity regarding the centre line of the open vertical is e=32,52 mm. The main moments of inertia of the cross-section are: I_y=1323853.33 mm^4; I_z=275242.87 mm^4. The torsional moment of inertia of the cross-section is I_t=4565.33 mm^4, and the main sectorial moment of inertia of the cross-section is I_ω=569632394.7 mm^6. r^2=3337.29 mm^2; y_A=38.33 mm; z_A=0. Using the expressions (18), we get the following values:

F_y=3402.76 kN ; F_z=707.47 kN ; F_ω=540.91 kN.

Critical force:

F_1=F_z=707.47 kN

By solving the expression (17), we get the other two values of the critical force:

F_2=503.38 kN ; F_3=6541.99 kN.

Relevant critical force: F_{kr}=F_{min}=503.38 kN.

The loss of beam stability occurs under twisting and lateral out-plane buckling. The critical stress in the beam at the time of the loss of stability is

$$\sigma_{kr} = \frac{F_{kr}}{A} = 580.06 \text{ MPa} .$$

The critical stress under pure buckling in the plane of symmetry is:

$$\sigma_z = \frac{F_z}{A} = 826.48 \text{ MPa} .$$

The critical stress is 70% of the critical Euler stress.

3.2.2 Z-section beam

For the thin walled Z-section 100/50/15/2 beam with the wall thickness t=2 mm the cross-sectional area is A=444 mm^2. The position of the main axis of inertia is α=-28.29^o, Fig. 3b. The main moments of inertia of the cross-section are: $I_y=I_{max}$=901248 mm^4; $I_z=I_{min}$=92936 mm^4. The torsional moment of inertia is I_t=592 mm^4, and the main sectorial moment of inertia of the cross-section is I_ω=394405371.1 mm^6. r^2=2239.15 mm^2; $y_A=z_A$=0. Using the expressions (18), we get the following values:

F_y=2316.52 kN ; F_z=238.88 kN ; F_ω=650.24 kN.

Critical force:

$F_{kr}=F_{min}$=238.88 kN

The loss of the beam stability appears under pure twisting in the plane of minimum flexural stiffness (the plane x, y). The critical stress in the beam at the time of loss of stability is: $\sigma_{kr} = \frac{F_{kr}}{A} = 538.02 \text{ MPa}$.

For the thin walled Z-section 100/50/15/4 beam with the wall thickness t=4 mm the cross-sectional area is A=856 mm^2. The position of the main axis of inertia is α=-27.57^o, Fig. 3b. The main moments of inertia of the cross-section are: $I_y=I_{max}$=1638279.83 mm^4; $I_z=I_{min}$=164394.83 mm^4. The torsional moment of inertia is I_t=4565.38 mm^4, and the main sectorial moment of inertia of the cross-section is I_ω=735052717.8 mm^6. r^2=2105.93 mm^2; $y_A=z_A$=0. Using the expressions (18), we get the following values:

F_y=4210.94 kN ; F_z=422.55 kN ; F_ω=1059.01 kN.

Critical force is:

$F_{kr}=F_{min}$=422.55 kN.

The loss of the beam stability appears under pure twisting in the plane of minimum flexural stiffness (the plane x, y). Critical stress in the beam at the time of the loss of stability is: $\sigma_{kr} = \frac{F_{kr}}{A} = 493.63 \text{ MPa}$.

4. Numerical determination of the critical force

The calculation of the thin-walled C- and Z-section beams is done by using a finite element method in the computer program SAP2000 v14. Thin-walled beams are modelled by plate elements. The thin-walled C-section beam is divided into 1800 rectangular finite elements. The thin-walled Z-section beam is divided into 1440 rectangular finite elements. Dimensions of finite elements that make up the web are 12 x 12 mm, the flange is 11.5 x 12 mm and the stiffener is 13x12mm in size. The beams are modelled by using the steel plate 100/50/30 mm in dimension and the mechanical properties equal to the plate which was used in the experimental study.

For the C-section 100/50/15/2 beam, the critical factor of buckling obtained from the "buckling" analysis in SAP2000 v14 is 162.44 kN. The beam was loaded by unit force (in the centre of gravity of the cross section), which means that the critical force is F_{kr}=162.44 kN. On the deformed shape of the C-section beam, under the influence of the critical force, the occurrence of the largest local buckling in the lower half of the height of the beam can be seen, Fig. 8a.

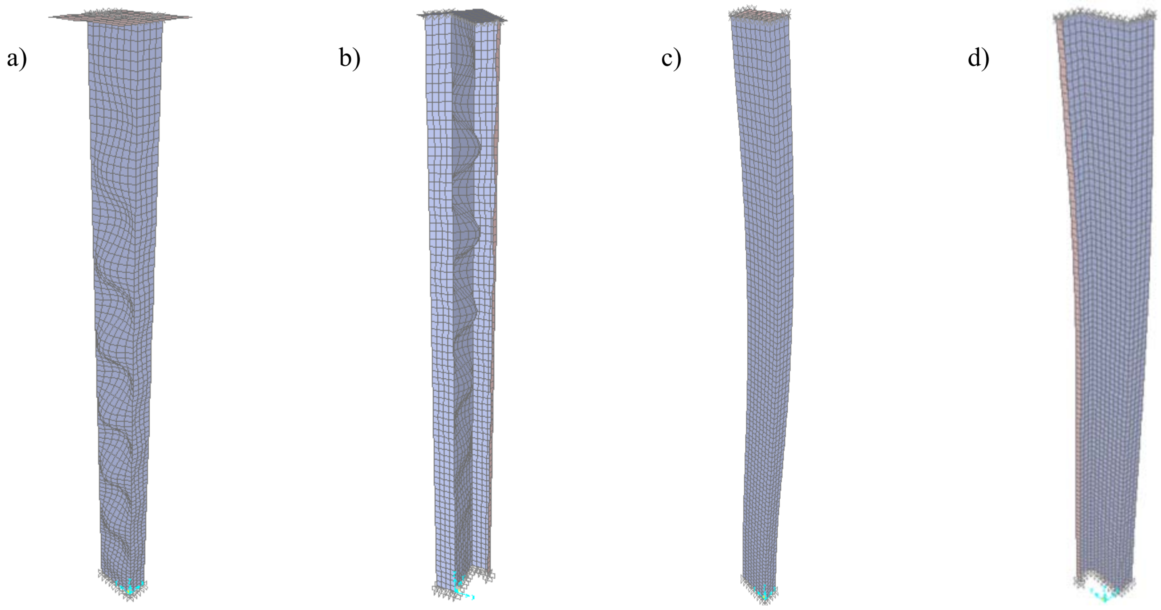


Fig. 8 Deformed shape of the thin-walled beam under the action of the critical force: a) C-section 100/50/15/2, b) Z-section 100/50/15/2, c) C-section 100/50/15/4, d) Z-section 100/50/15/4

For the Z-section 100/50/15/2 beam, the critical factor of buckling obtained from the "buckling" analysis in SAP2000 v14 is 160.53. The beam was loaded by unit force (in the centre of gravity of the cross section), which means that the critical force is F_{kr}=160.53 kN. Regarding the deformed shape of the Z-section beam under the action of the critical force, it can be seen that the largest local buckling appears in the upper half of the beam, Fig. 8b.

For the C-section 100/50/15/4 beam, the critical factor of buckling is 601.87. The beam was loaded by the unit force in the centre of gravity of the cross-section, which means that the global critical force is F_{kr}=601.87 kN. The shape of the buckling of the beam is shown in Fig. 8c.

For the Z-section 100/50/15/4 beam, the critical factor of buckling is 415.56, which means that the global critical force is F_{kr}=415.56 kN. The loss of beam stability appears by pure twisting in the plane of the minimum flexural stiffness. The shape of beam buckling is shown in Fig. 8d.

5. Comparison of the results of theoretical and experimental research

The values of critical forces F_{kr} for the thin-walled C- and Z-section beams which will be analyzed and compared are obtained analytically based on the theory of thin-walled open cross-section beams, numerically, with the FEM and experimentally, by testing. The results are shown in Table 1.

Table 1 Comparison of critical forces

F_{kr} [kN]	C-section beam		Z-section beam	
	100/50/15/2	100/50/15/4	100/50/15/2	100/50/15/4
F_{eks}	152.78	583.47	145.89	398.76
F_{teor}	238.79	503.38	238.88	422.51
F_{SAP}	162.44	601.87	160.53	415.56

The critical force determined by the theory of thin-walled beams has the same value for the C-section 100/50/15/2 beam and the Z-section 100/50/15/2 beam, but it is incorrect because it does not take local buckling into account, yet it gives a critical value of global forces. Values of that critical force F_{teor} are higher than those obtained by the FEM and experimental testing. The FEM, carried out according to the "Buckling" analysis in SAP, provides critical forces with the occurrence of local buckling. These values F_{SAP} deviate from the results of the experimental tests F_{eks}, which could be explained with imperfections of real elements (geometric imperfections, material inhomogeneity, residual stresses), as well as with model imperfections in the approximation of the mathematical and the numerical model.

With the thin-walled C-section 100/50/15/2 beam, local buckling appears in the lower half of the height of the beam, Fig. 8a, while with the thin-walled Z-section 100/50/15/2 beam it appears in the upper half of the height of the beam, Fig. 8b. The "Buckling" analysis gives us a range of possible local buckling which was confirmed by the experimental tests.

Thin-walled beams with a greater wall thickness are also observed, t=4 mm. With the C-section 100/50/15/4 beam and the Z-section 100/50/15/4 beam global stability loss appears without the occurrence of local buckling. For the Z-section 100/50/15/4 beam with an asymmetrical cross section, the critical forces F_{SAP} obtained by the FEM and F_{teor} obtained analytically by the theory of thin-walled beams are equal.

In the calculation of the beam stability of the symmetrical C-section 100/50/15/4 beam, it is necessary to take torsional stability into account. For the C-section 100/50/15/4 beam, the value of the global critical force F_{teor} obtained analytically is smaller than the global critical force F_{SAP} which was obtained by the FEM. The difference in the size of the critical force obtained analytically and numerically for beams with symmetrical C-sections occurs because of the stiffness of the plate through which the load is transferred.

6. Conclusion

For the thin-walled C-section 100/50/15/2 and Z-section 100/50/15/2 beams, after reaching the maximum force, strain and force are in a nonlinear relationship. Strain grows and axial stiffness of the beam falls nonlineary. Global critical force will not be achieved due to local buckling. For these beams, the finite element method gives the value of the critical force with the occurrence of local buckling, and the area of possible local buckling is determined, which coincides with the experimental tests. To the thin-walled open-cross section beams that are subject to the local loss of stability, we cannot apply the theory of thin-walled beams for calculating the critical force of the loss of stability, because that calculation gives a value of the critical global force without taking the local buckling of parts of a beam into account.

The relationship between the dimensions of the cross-section of thin-walled beams, the thickness of the wall and the height of the beam affect the way of cancellation of the beam, i.e. the loss of stability (local or global). By increasing the thickness of the beam wall there occurs the loss of beam stability reaching a global critical force, without having local buckling, for the C-section 100/50/15/4 and Z-section 100/50/15/4 beam.

To obtain the dependence of the cross-section dimensions and the height of the beam, which affect the way of the loss of the beam stability, further experimental tests and numerical calculations should be carried out.

For the thin-walled Z-section beam where the plane load goes through the shear centre, torsional stability should not be taken into account, and the critical forces obtained analytically and numerically are equal. For the C-section beam, the plane load does not go through the shear centre and here buckling and torsion appear simultaneously, so the rigidity of the plate, through which the load is transferred, has an influence on the value of the critical load.

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