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INFLUENCE OF THE EARTH'S TOPOGRAPHIC MASSES ON VERTICAL DEFLECTION

Milan Rezo, Danko Markovinović, Marko Šljivarić

Original scientific paper
Modern scientific research is largely focused on interdisciplinary aspect of individual technical disciplines in research and scientific approach in discovering new insights. In the field of geo-sciences, there is a wide range of mutual methods of measuring physical parameters needed for the determination of geologic and structural composition of the underground area, the density of surface layers, the Earth's gravity field, i.e. local gravity changes. This paper explains the influences of topography of the Earth masses in the determination of vertical deflections needed for the transformation of the heights by means of trigonometric levelling methods in the real gravity field acceleration.

Keywords: *relief correction and residual terrain modelling, topographic isostatic reduction, topographic reduction, vertical deflection*

Utjecaj Zemljinih topografskih masa na otklone vertikala

Izvorni znanstveni članak
Suvremena znanstvena istraživanja u velikoj su mjeri usmjerena k interdisciplinarnosti pojednih tehničkih grana u istraživačkom i znanstvenom pristupu iznalaženja novi spoznaja. U području geo-znanosti postoji široki spektar zajedničkih metoda mjerenja različitih fizikalnih parametara radi određivanja geološko-strukturnog sastava podzemlja, gustoće površinskih slojeva, gravitacijskog polja Zemlje odnosno lokalnih promjena sile teže. U ovom radu se objašnjavaju utjecaji topografije Zemljinih masa pri određivanju otklona vertikala nephodnih za prijenos visina metodom trigonometrijskog nivelmana u realnom polju ubrzanja sile teže.

Ključne riječi: *korekcija reljefa i rezidualno modeliranje reljefa, otklon vertikale, topografsko-izostatska redukcija, topografska redukcija*

1 Introduction

The accuracy of measurements of vertical deflection on the physical Earth's surface is not conditioned only by the precision of measuring instruments and measuring methods, but to a great extent by the fact that the geologic and structural composition of the underground area is not known, first of all related to the density of the Earth's crust and the depth of isostatic compensation that directly affect the amount of gravity acceleration value. The changes of gravity acceleration value should be known when transforming the heights from the land to the islands using the method of trigonometric levelling. Since the land area is connected with the sea area through the mountain chains (massifs), each measurement of distances and zenith angles requires the information about physical parameters – vertical deflection.

2 Methods of determining the vertical deflection

Depending on the method of determining vertical deflection, we distinguish the following methods: astrogeodetic [1], topo-isostatic, astrogeodetic and gravimetric, and gravimetric methods [2]. In this paper, the astrogeodetic, and topo-isostatic methods are described.

2.1 Astrogeodetic method

The astronomical measurements are used to determine the astronomical longitude and latitude of some point on the Earth's surface. The angle between the vertical and the plane of the prime meridian determines the position of this point (astronomical coordinates Φ and Λ), Fig. 1.

The normal line to an ellipsoid is defined by means of geodetic coordinates φ and λ, Fig. 1.

The differences between astronomical and geodetic coordinates define the vertical deflection υ. On the Earth's surface there is a point T, and we imagine the auxiliary celestial sphere around it, Fig. 2. If SKQN presents the horizon of the point T, and Z is the impact point of the normal line and the auxiliary celestial sphere (geodetic zenith), ZT is the normal line, and Z* is the impact point of the vertical line and the auxiliary celestial sphere (astronomical zenith).

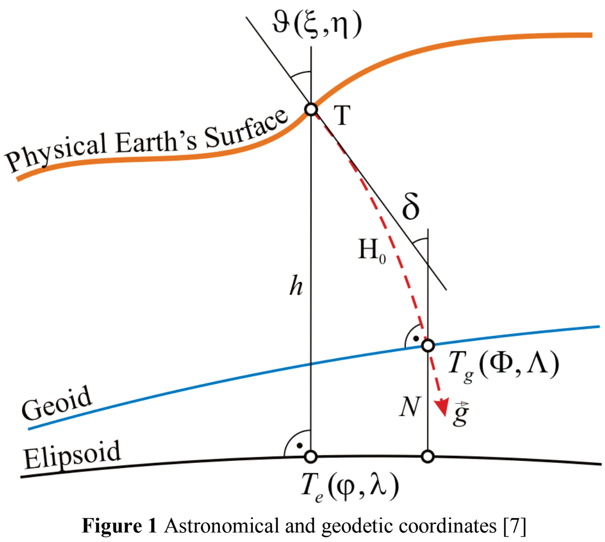

Figure 1 Astronomical and geodetic coordinates [7]

The point P_N presents the point of the celestial pole, ZP is the geodetic meridian of the point T, and Z^*P_N is the astronomical meridian. The size of the deflection shall then be $ZZ^*=\Theta$ in the direction of geodetic azimuth α, where $PZ=90° - \varphi$, and $PZ^*=90° - \Phi$. Important presumption is that the physical surface of the Earth coincides in the station point T with the reference ellipsoid surface or it intersects it in this point. By projecting the point Z* on to the geodetic meridian ZP into the point C, we separate the size of the deflection Θ into two components, northwards (meridian) ξ and eastwards (the prime vertical) η.

Since the angle value of the vertical deflection υ is very small, only a few seconds, the spherical triangle PZ*C can be identified with the plane triangle where it can be written as follows applying the plane trigonometry rules [1, 2]:

$$\xi = \Theta \cdot \cos\upsilon; \ \eta = \Theta \cdot \sin\upsilon \tag{1}$$

i.e.

$$\Theta = \sqrt{\xi^2 + \eta^2}; \quad \tan\Theta = \eta / \xi . \tag{2}$$

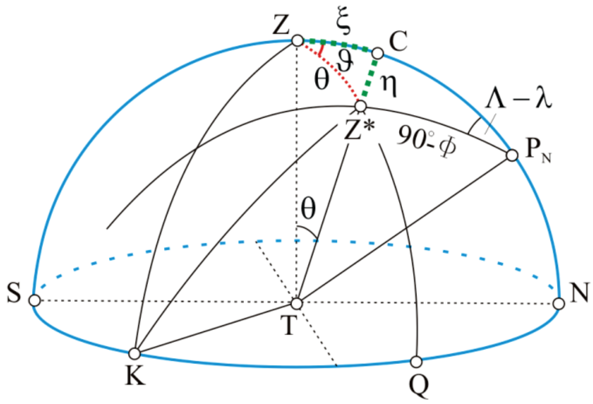

Figure 2 Geometric presentation of vertical deflection [1]

Due to the small component of the vertical deflection in the direction of the prime vertical η, it can be written that the final value of the component of the vertical deflection in the direction of the meridian plane ξ is equal to:

$$\xi = (90^\circ - \varphi) - (90^\circ - \Phi) = \Phi - \varphi . \tag{3}$$

The meridian component of the vertical deflection is equal to the difference between the geodetic and astronomical latitude. It is positive for $\Phi > \varphi$ from the spherical triangle PZ*C. The expression for the component of the vertical deflection in the direction of the first vertical line is as follows [3]:

$$\eta = (\Lambda - \lambda) \cdot \cos\varphi, \tag{4}$$

where $\Lambda - \lambda$ is the difference between the astronomical and geodetic longitude and it is positive if $\Lambda > \lambda$. The component η can be calculated also by means of the expression:

$$\eta = (A - \alpha) \cdot \mathrm{ctan}\varphi, \tag{5}$$

where $A - \alpha$ is the difference between the astronomical and geodetic azimuth. The previous expression presents the control in computing the component of the vertical deflection in the direction of the prime vertical, if the astronomical longitude and azimuth have been determined by means of astronomical measurements. On the basis of the Eqs. (4) and (5), the abbreviated form of the equation can be derived that is called Laplace equation [3]:

$$A - \alpha = (\Lambda - \lambda) \cdot \sin\varphi . \tag{6}$$

If we want to project the vertical deflection in the direction of a certain azimuth α, the following expression will be used:

$$\varepsilon = \Theta = \xi \cdot \cos\alpha + \eta \cdot \sin\alpha . \tag{7}$$

The astro-geodetic method of determining the values of the components of the vertical deflection is applied only on land, unlike the gravimetric procedure that is applied also at sea. The astro-geodetic determination of the vertical deflection is used even today because it makes it possible to notice local variations either of vertical deflection, or of geoid undulations [1].

2.2 Topographic and topo-isostatic method

Vertical deflections can be computed using previously mentioned formulas by reducing the astronomical observations to geodetic. This calculation is only approximate and temporary since we have not taken the inequality of mass distribution into account, i.e. the calculation has been made with the presumption that the density of the Earth's upper layers is the same everywhere, which is not the case in reality.

The deflections calculated with regard to the absolute size on the basis of visible masses are always larger than the vertical deflections obtained from the differences of astronomical and geodetic coordinates, and we interpret this phenomenon by means of the isostasy theory [4].

Isostasy is the term used in geology, and it refers to the gravitational balance between the Earth's lithosphere and asthenosphere. Since we see that the direction of the vertical line is much less deflected on certain parts of the Earth than expected, it indicates the shortage of the masses in the underground area. It means that the shortage of masses in the deeper areas corresponds to the topographically visible rising masses (mountains). The explanation is based on the fact that on average lighter Earth's crust and its upper mantle (lithosphere) swim on the plastic liquid substrate of thicker asthenosphere. The topographic components of the vertical deflection are in linear approximation calculated according to [5]:

$$\begin{Bmatrix}\xi_{\mathrm{tg}}\\ \eta_{\mathrm{tg}}\end{Bmatrix} = -\frac{G\rho}{g_{\mathrm{n}}}\int_{\pi} \int_{z=h_{\mathrm{ref}}}^{h} \frac{z-h}{r^3}\,\mathrm{d}z \begin{Bmatrix}\cos\alpha\\ \sin\alpha\end{Bmatrix} d\pi, \tag{8}$$

where G is gravitational constant, ρ is the density of the surface masses, g_{n} is normal gravity acceleration, h is the height of the observed point P on the physical surface of the Earth, h_{ref} is the height of the reference "polished" surface on which the topo-isostatic vertical deflections are calculated (frequently asked value is zero), r is the radius of visible masses divided into proper geometric bodies i.e. prisms.

The topographic vertical deflections are always larger than the astro-geodetic deflections related to the absolute size, which is attributed to the isostasy theory.

In order to correct the obtained data about the calculated topographic deflections, we must take the mass attraction down to the depth of the isostasy T into consideration.

By adding the isostatic correction to the topographic deflections, we obtain the topo-isostatic vertical deflections [4]:

$$\xi_{\mathrm{Ti}} = \xi_{\mathrm{tg}} + \xi_{\mathrm{i}}. \tag{9}$$

The topo-isostatic vertical deflections are determined out of topographic deflections whereby the hypothesis about the mass compensation (isostasy) is introduced.

3 Digital Elevation Model

Digital Elevation Model – DEM) presents a series of points on the Earth's surface the coordinates of which are suitable for computational processing [6]. Digital Elevation Model is used as the name for digital topographic and bathymetric data related to the Earth's surface, without vegetation and built objects properly distributed along the coordinate axes. The similar term is Digital Terrain Model – DTM that includes vegetation, built objects and break lines for the purpose of better terrain approximation.

It is of great significance for the calculation of physical parameters, i.e. in this case of vertical deflection, to know the topographic data of high resolution. DEM is only a model of the Earth's surface that is error-prone. Unfortunately, each error in DEM affects the accuracy of calculating the values of vertical deflections.

For the purpose of modelling the gravity field at the territory of Croatia, DMR4×5 has been used, that was made for the continental part of former Yugoslavia by means of digitizing the contour lines on topographic maps at the scale of 1:25000 needed by the former Federal Agency for Radio diffusion in 1983.

That DEM was made in the resolution of 4"×5" (a arc sec by 5 s) (~120×110 m), in such a way that the highest terrain elevation was attached to each 4"×5" window. The continental areas outside of the territory of the former state were supplemented with the heights of ETOPO5 DEM, and the territory of the Adriatic Sea was covered with bathymetric data 2,5'×2,5' (~4,6×3,3 km)[8].

The model DMR4×5 was defined by the position of nodal points of the raster 4"×5" on Bessel ellipsoid, and it had to be redefined in relation to the official GRS80 ellipsoid, Tab. 1. In the first line of the given model, the area of operation was defined as related to the geodetic latitude φ and longitude λ, and by means of the step $\Delta\varphi$ and $\Delta\lambda$.

Table 1 Example of specifying the digital elevation model in the raster 4"×5" (~120×110 m) [7]

41,00027777		47,99916666		111,99583332		0,99444444		0,00111111	0,00138889
542	542	542	542	542	542	542	542	542	542
542	522	522	522	522	522	522	522	522	522
522	522	522	522	522	522	522	522	522	522
522	522	522	522	522	522	522	522	522	522

Along with the above mentioned models, there were also the reference digital models in the raster 10'×15' and 20'×25' that were used all together with the constant density of 2670 kg/m^3 in modified TC computer programme for the calculation of various influences of topographic masses on physical parameters [5].

4 The influence of topography on vertical deflections

Topographic masses affect the physical parameters of the Earth's gravity acceleration referring to the indentedness of topography and the anomaly of topographic mass density. The influence of topography on vertical deflections can be expressed as follows [9]:

- direct topographic effect of all masses above the sea level,
- topographic-isostatic reduction with Airy isostatic model,
- relief correction – the effect of irregular actual relief as related to the Bouguer plate, and
- residual correction (Residual Terrain Modelling - RTM) effect of topographic masses as related to the reference surface of mean heights.

4.1 Calculation of topographic reduction

Various topographic reductions are calculated by means of the modified programme TC [9] where the sector method, spline interpolation within the zone close to calculation point, and the approximation that includes the curvature of the Earth surface are applied.

Various topographic effects used to be calculated in the past in such a way that the area around the point was divided into a series of concentric rings, and each ring into sectors, as for example Hayford, i.e. Hammer zone [4]. Using the calculated mean heights of the sector, the contribution of single sectors to the total effect of topographic masses was calculated with relatively simple equations in the cylindrical system.

When calculating by means of TC programme with the data specified by the digital elevation model, i.e. the network of mean values of the network of accurate values in raster points, it is common practice to keep such quasi square division of the value fields into sectors, and the calculation is made in the Cartesian coordinate system. Detailed information is needed only in the vicinity of the calculation point, and for the purpose of faster calculation, larger medium sectors are sufficient to be used for larger distances.

Since the area of interest is covered by two digital models with the increasing sector values where more rough model is obtained by means of creating mean fine model for the purpose of achieving homogeneous data, the creation of sectors is characterized by the radii r_i, where i = 1, 2. So called fine digital elevation model is used up to the distance r_1, and rough model up to the distance r_2, Fig. 3.

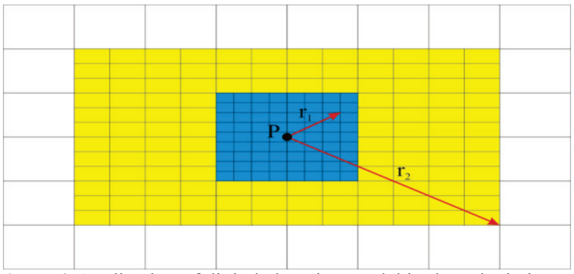

Figure 3 Application of digital elevation model in the calculation of various topographic effects depending on the distance *r* [7]

The value of the double raster value of a rough raster grid is usually chosen for r_1 [5]. By means of sector method, the size of the sectors as related to the distance r from the calculation point is chosen. Practice has shown, that the classical *sr* sector method where s is the size of a sector as related to the distance, yields good results for almost all types of topographic effects. For quasi Hayford sector method, the relation s/r is equal to 1,5. From the selected s/r relation, the distance r_i is determined. The relative error curves of the approximation of accurate formulas in the calculation of gravity acceleration, deflection of vertical components and potentials can be found, along with the $h_{st} = 0$, in [10].

The values of deflection of vertical components can be expressed as the linear functionals $L(T)$ of disturbing potential T defined by means of the difference between the actual W and normal gravity field potential U [5]:

$$\xi = -\frac{1}{rg_{\mathrm{n}}}\frac{\partial T}{\partial \varphi}; \quad \eta = -\frac{1}{rg_{\mathrm{n}}\cos\varphi}\frac{\partial T}{\partial \lambda}, \tag{10}$$

where φ and λ are geodetic latitude and longitude, and g_{n} is normal gravity acceleration.

Special attention has been given to the influence of close topography (up to 1 km) on the calculation point. The effect can be very large for the vertical deflections. The station surroundings are therefore additionally densified by means of interpolation. In spite of that, there is a divergence between the station height and the interpolated model. The numerical experiments dictate the movement of the station to the height of the model in the better case of deflection of vertical components and height anomalies (geoid undulation) calculation. For gravity acceleration, on the contrary, topographic and RTM effects are directly correlated with the model height, and the model height is therefore modified in such a way that it passes better through the station height [10].

The curvature of the Earth is taken into consideration in TC programme by means of super elevation Δz. The prism at the distance r is parallel with the local vertical line at the calculation point and it is lowered below the tangent plane through the calculation point for $\Delta z = r^2/2R_{\mathrm{Earth}}$. In this way, the prisms remain parallel. This approximation is valid up to the distance of a few thousand km [5].

Along with the above mentioned digital models, the following values have been chosen for the input parameters in the programme TC (expressed in units and with the quantity marks used in the programme): $r_{\mathrm{ho}\ 0} = 2670\ \mathrm{kg/m^3}$ (average crust density), $r_{\mathrm{ho}\ 1} = 2870\ \mathrm{kg/m^3}$ (average density at the top of the mantle), $r_{\mathrm{ho\ iso}} = -200\ \mathrm{kg/m^3}$ (average density jump at the border of crust/mantle for which Airy isostatic models have been used that are well adjusted to the observed area)[11], $d_{\mathrm{pt\ iso}} = 30$ km, (the depth of isostatic compensation), $r_{2\ \mathrm{exac}} = 20^2$ (the ratio or shift that determines the usage of accurate formulas), $i_{\mathrm{type}} = 6$ (gravity anomaly, *ksi*, *eta* and *height* anomaly are calculated), $i_{\mathrm{kind}} = 1, 2, 3$ or 4 (*topo*, *topoiso*, *tc* and *RTM* effects are calculated), $i_{\mathrm{z\ code}} = 0$ (in the internal zone, the spline densifying is made, and the stations are moved to the model height), $i_{\mathrm{g\ typ1}} = 0$ (digital models are given in standard format), $r_1 = 20$ km (the distance up to which fine digital elevation model is used, $r_2 = 1000$ km (the distance up to which rough digital models are used). It is important to mention that vertical deflections are calculated on the surface of the Earth.

4.2 Topographic reduction and relief correction

The term topography implies in the classical approach the part of the Earth crust from geoid to the physical surface of the Earth (Fig. 4 and Fig. 5). Unlike the land, the topography extends at seas or oceans from geoid that is very often identified with the mean sea or ocean level (bathymetry). The topographic reduction is called in literature the Bouger reduction because it consists of the influence of Bouger plate and the irregularities of topography in relation to the Bouger plate. The topographic or complete Bouguer reduction implies the elimination of the influences of visible topography where the density of 2670 $\mathrm{kg/m^3}$ is usually used. This value is typical for granites and many types of sediments of Palaeozoic and Precambrian era [9].

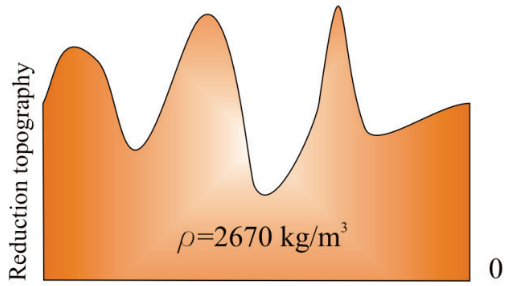

Figure 4 Reduction for the influence of topography [7]

In the topographic reduction, it is possible to distinguish the Bouguer element $2\pi G\rho h_{\mathrm{p}}$, the effect of the infinite plate and the correction of relief *tc* that includes in itself the irregularity of actual topography [9].

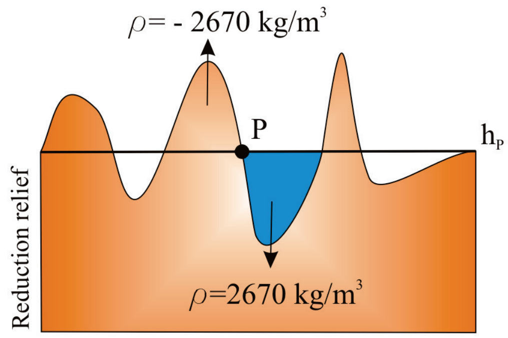

Figure 5 Reduction for the relief influence [7]

For the vertical deflections, the topographic effect is equal to the correction effect for relief because the effect of Bouguer plate is then zero. It should be mentioned that Bouguer anomalies in the mountain areas adopt large and negative values. Thus, there must be a lack of masses below the mountains, which means that the topographic masses are in some way compensated and that actual deflections are smaller than those calculated from the visible topography.

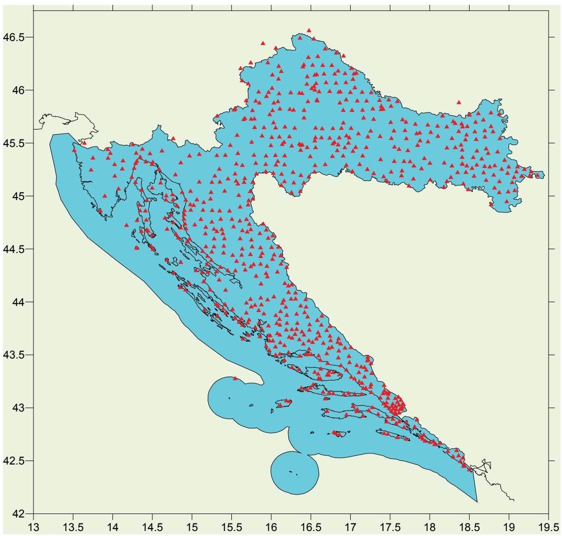

Figure 6 The arrangement of points at the territory of the Republic of Croatia where the vertical deflections have been calculated [7]

The points from the 10-km GPS network, the total of 724 points covering the entire area of the Republic of Croatia, have been chosen for the purpose of test calculation, Fig. 6.

The calculation of the values of vertical deflection because of the influence of topography has been carried out by means of defining the parameters in the programme TC ($i_{type} = 2$, $i_{kind} = 0$, $i_{z\ code} = 1$ and $i_{g\ typ1} = 1$).

The Figs. 7 and 8 show clearly a strict correlation of the vertical deflections with the actual topography. The minimal values of the vertical deflection components in the direction of the meridian and the prime meridian are -38,08 and −29,28 arc seconds, and the maximum values of the vertical deflection in the direction of the meridian plane and the prime vertical are +12,28 and +8,69 arc seconds (Tab. 3).

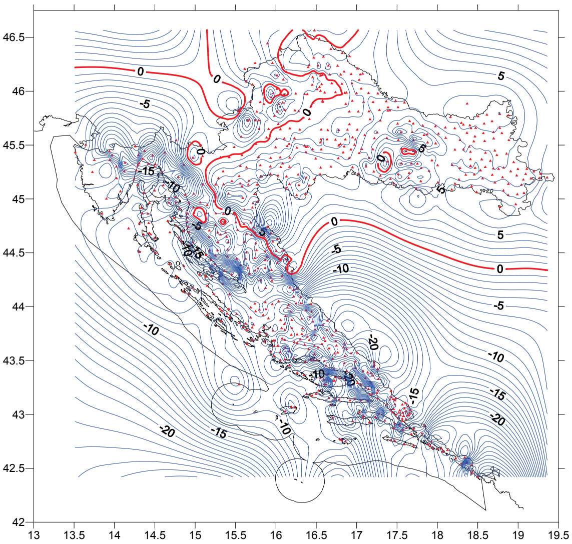

Figure 7 Vertical deflections components ξ'' due to the influence of topography-contour lines [7]

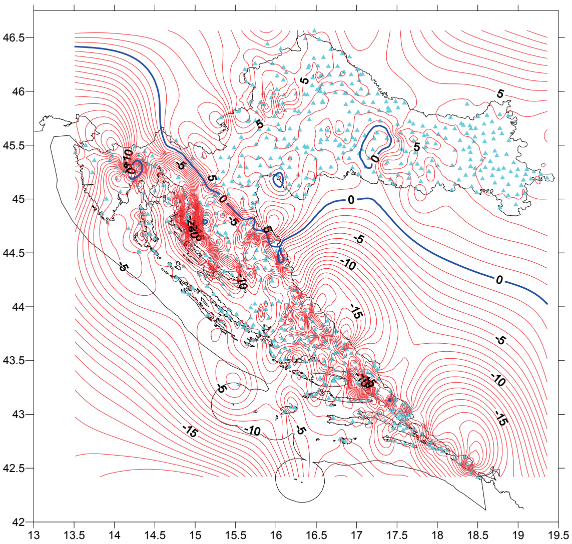

Figure 8 Vertical deflections components η'' due to the influence of topography – contour lines [7]

The calculation of vertical deflection values for the influence of relief has been made by means of defining the parameters in the programme TC ($i_{type} = 2$, $i_{kind} = 3$, $i_{z\ code} = 1$ and $i_{g\ typ1} = 1$).

Table 3 Statistical indicators of vertical deflection for the influence of topography [7]

Statistics	φ / °	λ / °	H / m	ξ (arc sec.)	η (arc sec.)
Minimal value	42,420400	13,514230	41,95	−35,0775	−29,2822
Maximal value	46,565880	19,359220	1702,03	12,2721	8,6930
Standard deviation				8,6214	7,6531
Mean	44,956905	16,444435	225,46	−4,2139	−3,6133

Table 4 Statistical indicators of vertical deflection for the relief influence [7]

Statistics	φ / °	λ / °	H / m	ξ (arc sec.)	η (arc sec.)
Minimal value	42,420400	13,514230	41,9500	−11,7341	−12,7011
Maximal value	46,565880	19,359220	1702,0300	30,0312	28,7960
Standard deviation				7,9909	7,5606
Mean	44,956905	16,444435	225,4600	5,0716	2,3784

Minimal value of the components of vertical deflection in the direction of a meridian is −11,73 arc seconds, and in the direction of the prime vertical −12,70 arc seconds, while the maximal values of vertical deflection in the direction of meridian plane and the prime vertical are +30,03 and +28,79 arc seconds, Tab. 4.

Fig. 9 and Fig. 10 show reduced negative amounts and significantly larger positive amounts for the vertical deflections due to the influence of the relief.

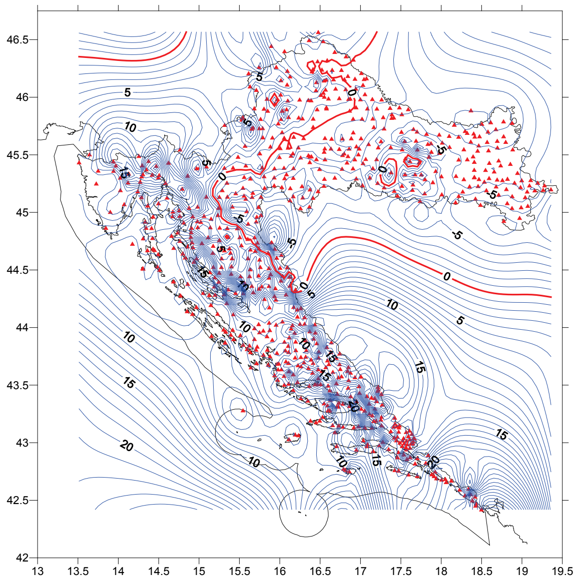

Figure 9 Vertical deflections ξ'' due to relief influence – contour lines [7]

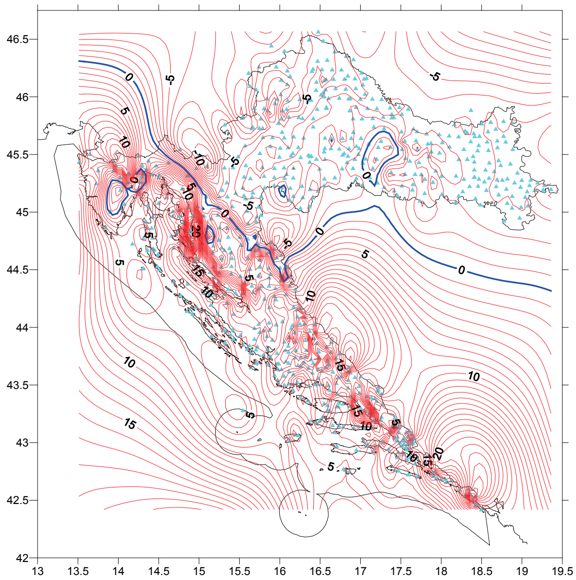

Figure 10 Vertical deflections η'' due to the relief influence – contour lines [7]

4.3 Isostatic reduction

Isostasy is the balance between the neighbouring parts (blocks) of the crust and mantle parts below them. The idea is based on the principle of hydrostatic balance. Since the crust rocks are lighter than the mantle rocks, we can say that the crust 'floats' on the mantle. The parts of the crust rise or sink as long as the isostatic balance is established, i.e. as long as the weight of the suppressed mantle part is not equal to the crust block weight. Such vertical movements of the crust are also called isostatic adjustment.

In the 1950's, two different theories of topographic mass compensation were developed that are applied on local level. According to Pratt [4], the mountains rise from the underground layers similarly to fermenting dough. Pratt-Hayford system of compensation is illustrated on Fig. 11. Below the compensation level, the density is constant, and above the level, the mass equality of the columns with the same cross section is implied. If D is the depth of the compensation level calculated from the mean sea level (MSL), and ρ_0 is the density of the height column D where h is the topography height, it is in accordance with the following equation:

$$(D+h)\rho = D\rho_0, \tag{11}$$

that expresses the equality of masses. According to Eq. (11), the density ρ different for each column is smaller than the normal value ρ_0. The difference in density indicates the lack of mass. At sea and on the ocean, the following relation is valid:

$$(D-h')\rho + h'\rho_w = D\rho_0, \tag{12}$$

where h' is the sea or ocean depth, and ρ_w is the density of sea water. The surplus of mass in the column under the sea is given by the difference of the densities ρ and ρ_w. For normal density ρ_0, we take the value 2670 kg/m^3, and for D, the values of about 100 km are presumed [4].

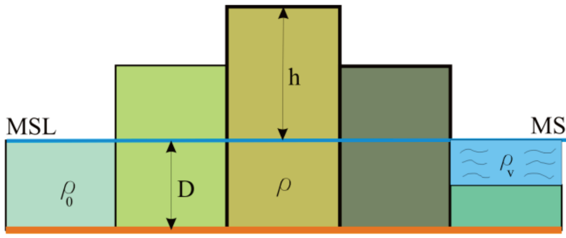

Figure 11 Isostasy according to Pratt for local model with the density change and constant column height [7]

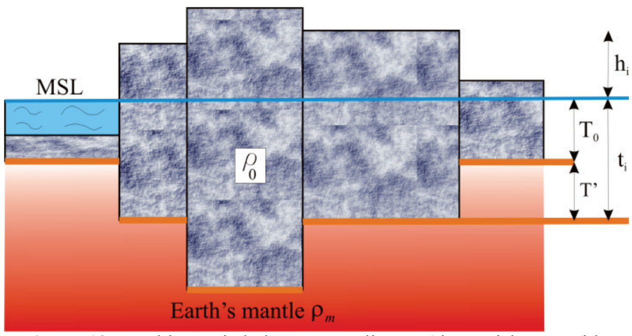

Figure 12 Local isostatic balance according to Airy-Heiskanen with constant mass density [7]

On the other hand, according to Airy [4], the mountain massifs of constant density ρ_0 float on liquid lava of high density ρ_m, hence, the higher the mountain, the deeper it is sinking, Fig. 12.

Unlike the Pratt model, the model of compensation depths is here not constant, but the density of the Earth's crust is constant, which causes the variability of depth compensation. The idea is based on the principle of static balance for any depth column t_i that follows directly from the topography height h_i. With normal crust thickness T_0 = 35 km [7], the thickness of anti-root t' and the mantle density ρ_m, we obtain the following equation:

$$t_i(x,y) = -T_0 - \underbrace{\frac{\rho_0}{\rho_m-\rho_0} h_i(x,y)}_{t'}. \tag{13}$$

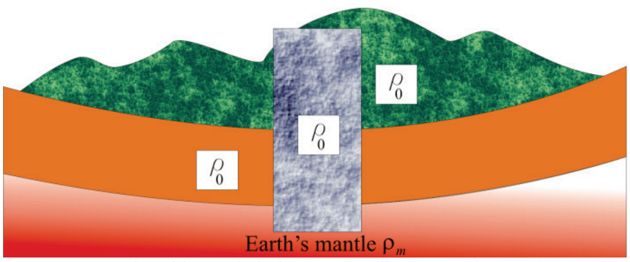

Figure 13 Regional isostatic balance according to Vening-Meinesz with constant density of surface and elastic masses of the Earth's crust [7]

Airy-Heiskanen principle presumes local (vertical) isostatic balance of the topographic column having infinitely small cross section as the condition, independent of the environment. The isostasy is usually regarded as regional phenomenon, and Vening-Meinesz [4] therefore introduced regional instead of local compensation.

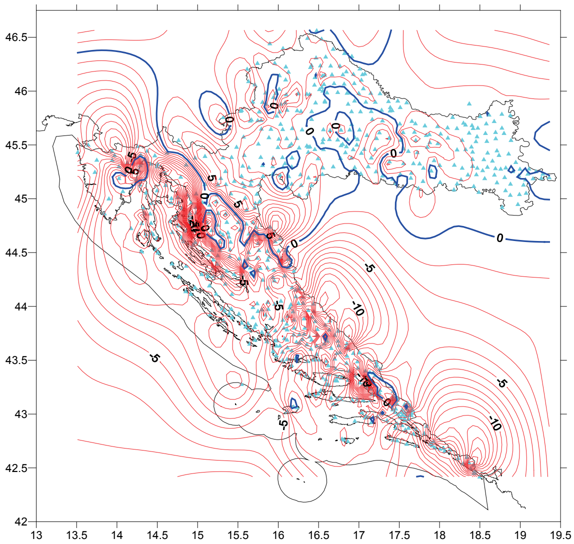
Figure 14 Isostatic components of vertical deflection η'' according to Airy-Heiskanen model-contour lines

According to this hypothesis, Fig. 13, every topographic column of infinitely small cross section is regarded as compensated with the mass derived from the local compensation of this element, but spread horizontally according to Vening-Meinesz modified curve [12].

The computation of vertical deflection values for the influence of isostasy has been made by means of defining the parameters in the programme TC ($i_{\text{type}} = 2$, $i_{\text{kind}} = 2$, $i_{\text{z code}} = 1$ and $i_{\text{g typ1}} = 1$).

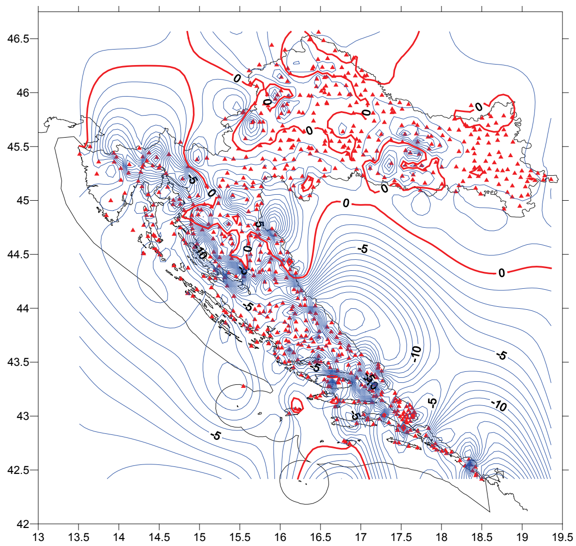
Figure 15 Isostatic components of vertical deflection ξ'' according to Airy-Heiskanen model-contour lines [7]

Figs. 14 and 15 show the reduced amounts of vertical deflection. The minimal value of vertical deflection components in the direction of a meridian is −21,42 arc seconds, and in the direction of the prime vertical −25,13 arc seconds, while the maximal values of vertical deflection in the direction of the meridian plane and the prime vertical are +11,89 and +8,78 arc seconds, Tab. 5.

Table 5 Statistical indicators of vertical deflection for isostatic influence [7]

Statistics	φ / °	λ / °	H / m	ξ (arc sec.)	η (arc sec.)
Minimal value	42,420400	13,514230	41,9500	−21,4173	−25,1332
Maximal value	46,565880	19,359220	1702,0300	11,8934	8,7801
Standard deviation				4,5080	4,4639
Mean	44,956905	16,444435	225,4600	−1,1683	−0,5659

4.4 Influence of residual terrain model -RTM

RTM reduction was introduced by Forsberg and Tscherning in 1981. In the event of *RTM* reduction, only shortwave topography effect is taken into account. It is achieved by selecting the smooth reference surface of mean heights, and by taking the masses above the reference surface, i.e. by filling up the valleys below that surface. Special selection of the surface of mean heights href offers the possibility to achieve complete agreement of *RTM* with the isostatic reduction [5]. RTM correction of the disturbance potential related to the surface of mean heights is given in the expression [13]:

$$h_{\text{ref}}(P) = \frac{T}{2\pi}\int_{\pi} \frac{h(Q)}{[r^2+T^2]^{\frac{3}{2}}}\mathrm{d}\pi_Q, \quad r = \left|\vec{r}_P - \vec{r}_Q\right|, \tag{14}$$

with $r = \sqrt{x^2+y^2+z^2}$ has form [5]:

$$T_{RTM} = G\rho \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{h_{\text{ref}}}^{h} \cdot \tag{15}$$

$$\cdot \frac{1}{\left[\left(x_Q - x_P\right)^2 + \left(y_Q - y_P\right)^2 + \left(z_Q - z_P\right)^2\right]^{\frac{3}{2}}} \mathrm{d}x_Q \mathrm{d}y_Q \mathrm{d}z_Q$$

Special selection of the surface of mean heights h_{ref} offers the possibility to achieve complete agreement of *RTM* with the isostatic reduction [9]. The computation of vertical deflection for *RTM* influence has been made by means of defining the parameters in the programme TC ($i_{\text{type}} = 2$, $i_{\text{kind}} = 4$, $i_{\text{z code}} = 1$ and $i_{\text{g typ1}} = 1$).

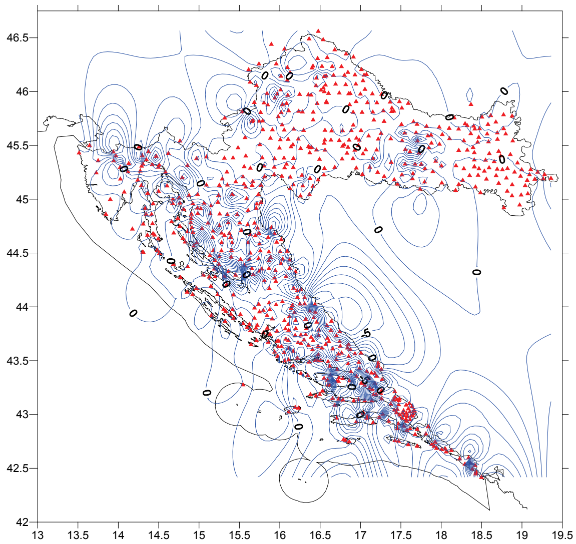

Figure 16 Vertical deflections ξ'' due to RTM influence-contour lines [7]

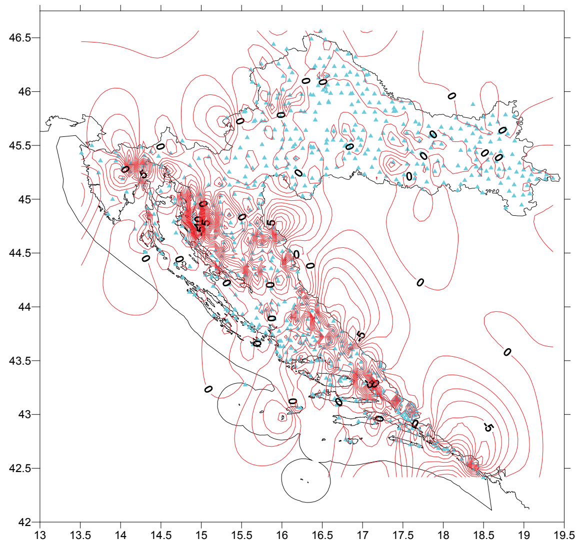

Figure 17 Vertical deflections η'' due to RTM influence-contour lines [7]

Adequate presentation of RTM effects on vertical deflections is given in Fig. 17, where, if compared with Fig. 16, it can be noticed that they agree with the isostatic effects to a great extent. Minimal value of vertical deflection in the direction of a meridian is −12,44 arc seconds, and in the direction of the prime vertical −15,92 arc seconds.

Maximal values of vertical deflections in the direction of the meridian plane and the prime vertical are +6,67 and +12,62 arc seconds, Tab. 6.

Table 6 Statistical indicators of vertical deflection for RTM influence [7]

Statistics	φ / °	λ / °	H / m	ξ (arc sec.)	η (arc sec.)
Minimal value	42,420400	13,514230	41,9500	−12,4374	−15,9204
Maximal value	46,565880	19,359220	1702,0300	6,6715	12,6189
Standard deviation				2,2475	2,5028
Mean	44,956905	16,444435	225,4600	−0,0444	−0,0638

5 Conclusion

The computations made in this paper indicate clearly the correlation of astrogeodetic vertical deflection with the relief configuration of the Republic of Croatia. The minimal values of vertical deflection for topography influence that make −35" in the direction of a meridian and −29" in the direction of the prime vertical should be specially emphasized, as well as maximal values for relief influence of 30" in the direction of a meridian and 29" in the direction of the prime vertical. Statistical indicators of the vertical deflection for isostatic influence are of the same amount as the statistical indicators for the topography influence with maximal and minimal values of 12" to −21" in the direction of a meridian, i.e. 9" to −25" in the direction of the prime vertical. It is recommended to continue with the research on isostatic reduction introducing various depths of isostasy of 5 to 50 kilometres.

6 References

[1] Špoljarić, D. Automatizacija simultanog određivanja astronomskih koordinata na geoidnim točkama (Automation of Simultaneous Determination of Astronomic Coordinates on Geoid Points). // Doctoral thesis, p. 110, University of Zagreb, Faculty of Geodesy, Zagreb, 2009.

[2] Torge, W. Geodesy. // Walter de Gruyter, Berlin, New York, 1980.

[3] Torge, W. Geodesy. // Walter de Gruyter, Berlin, New York, 2001.

[4] Heiskanen, W. A.; Moritz, H. Physical Geodesy. // Reprint, Institute of Physical Geodesy, Technical University Graz, Austria, 1996.

[5] Forsberg, R. A Study of Terrain Reductions. Density Anomalies and Geophysical Inversion Methods in Gravity Field Modelling. // Report No. 355, Department of Geodetic Science, Ohio State University, Columbus, 1984.

[6] Frančula, N. Digitalna kartografija (Digital Cartography). // Course material, University of Zagreb, Faculty of Geodesy, Zagreb, 2001.

[7] Rezo, M. Značenje i primjena fizikalnih parametara u modernom pristupu geodetskim radovima državne izmjere (The Meaning and the Application of Physical Parameters in Modern Approach in State Survey). // Doctoral thesis, p. 266, University of Zagreb, Faculty of Geodesy, Zagreb, 2010.

[8] Bašić, T. Regionalno istraživanje oblika Zemlje (Regional Research of the Earth's Shape). // Course material, University of Zagreb, Faculty of Geodesy, p. 60, Zagreb, 2002.

[9] Brkić, M. Poboljšana metoda modeliranja masa Zemljine kore za potrebe geodezije i geofizike (Improved Method of Mass Modelling for the Earth's Crust Needed in Geodesy and Geophysics). // Master thesis, University of Zagreb, Faculty of Geodesy, Zagreb, 1994.

[10] Forsberg, R.; Tscherning, C. C. The Use of Height Data in Gravity Field Approximation by Collocation. // Journal Gephys. Res. 86, B9(1981), pp. 7843-7854.

[11] Čolić, K.; Pribičević, B.; Ratkajec, M.; Stopar, R.; Šumanovac, F.; Velić, J. Početni digitalni model gustoće

površinskih masa Zemljine kore u test području "Slovenija i okolni dio Hrvatske za regionalne potrebe fizikalne geodezije" (Initial Digital Model of the Surface Mass Density of the Earth's Crust in the Test Area "Slovenia and the Surrounding Part of Croatia for the Purpose of Physical Geodesy"). // Geodetski list. 3(1992), pp. 263-281.

[12] Abd-Elmotaal, H. A-E. H. Gravity Anomalies Based on the Vening Meinesz Isostatic Model and their Statistical Behaviour. // Der Geodätischen Institute der Technischen Universität Graz, 1991.

[13] Moritz, H. Geodetic Reference System 1980. // Bull. Geod. 58, 3(1980), pp. 388-398.

Authors' addresses

Milan Rezo, doc. dr. sc.
Sveučilište u Zagrebu
Geotehnički fakultet
Hallerova aleja 7
42000 Varaždin, Croatia
Tel: +385 42 408 950
E-mail: mrezo@gfv.hr

Danko Markovinović, dr. sc.
Sveučilište u Zagrebu
Geodetski fakultet
Kačićeva 26
10000 Zagreb, Croatia
Tel: +385 (01) 4639 222
E-mail: danko.markovinovic@geof.hr

Marko Šljivarić, dr. sc.
Geodezija d.o.o.
Preradovićeva 4
10000 Zagreb, Croatia
Tel: +385 (91) 4444 998
E-mail: marko.sljivaric@gmail.hr