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OPTIMIZATION OF HCR GEARING GEOMETRY USING GENERALIZED PARTICLE SWARM OPTIMIZATION ALGORITHM

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Special kind of basic involute profile of non-standard gearing is called high contact ratio (HCR) gearing, when the contact ratio is higher and there are always at least two pairs of teeth in contact ($\varepsilon_\alpha \geq 2$) and where unit addendum height is not equal one like for standard gearing, so the tooth height is increased and it is bigger than one $h_a^* > 1$. When HCR gearing is used, it is not necessary to achieve a greater gear load capacity, but nevertheless there is a greater risk of interference due to a greater height of tooth. Advantage of the HCR gearing is also a higher resistance (load distribution is shared on the more pairs of teeth at the same time) and lower relative noise level of gearing, which can be significantly reduced by using integer HCR factor ε_α. HCR profiles are more complicated than standard involute profiles, they have greater predisposition for occurring interference, pointed tip thickness, but also undercut of teeth during the production (primary production interference). Due to increased addendum height, there is larger possibility of occurring some interference or pointed tooth tip. Therefore it should prevent these errors and check if all equation and constraints are satisfied. The given method of finding optimal solutions for h^*_{a1}, h^*_{a2} and x_1 uses Generalized Particle Swarm Optimization Algorithm and MATLAB as a program for optimization. This GPS optimization is a very fast and reliable method.

Keywords: *gears, geometry, GPSO algorithm, HCR gearing, optimization*

Optimizacija geometrije HCR-ozubljenja uporabom generaliziranog algoritma optimizacije roja čestica

Izvorni znanstveni članak

Posebna vrsta osnovnog evolventnog profila nestandardnog ozubljenja zove se ozubljenje s visokim stupnjem prekrivanja (HCR-ozubljenje), kada je stupanj prekrivanja veći uvijek su barem dva para zubi u dodiru ($\varepsilon_\alpha \geq 2$) i gdje jedinica dopune visine nije jednaka onoj za standardno ozubljenje, tako da se visina zuba povećava i veća je od jedan $h_a^* > 1$. Kada se rabi HCR-ozubljenje, nije potrebno postići veći kapacitet opterećenja zupčanika, ali ipak postoji veći rizik od smetnji zbog veće visine zuba. Prednost HCR-ozubljenja je veća otpornost (razdioba opterećenja se dijele na više parova zuba u isto vrijeme) i niža razina relativne buke ozubljenja, koja se može značajno smanjiti uporabom stupnja prekrivanja ε_α koji je cijeli broj. HCR profili su složeniji od standardnih evolventnih profila, oni imaju veću sklonost za pojavu smetnji, istaknut vrh debljine, ali i potkopavju zub tijekom proizvodnje (primarna proizvodna smetnja). Zbog povećanog dodatka visine, postoji veća mogućnost da se javljaju neke smetnje ili zašiljeni vrh zuba. Zato treba spriječiti ove pogreške i provjeriti jesu li sve jednadžbe i ograničenja zadovoljeni. Dana metoda pronalaženja optimalnih rješenja za h^*_{a1}, h^*_{a2} i x_1 koristi generalizirani algoritam optimizacije roja čestica (GPSO algoritam) i MATLAB kao program za optimizaciju. Ova optimizacija roja čestica je vrlo brz i pouzdan način.

Ključne riječi: *geometrija, GPSO algoritam, HCR-ozubljenje, optimizacija, zupčanici*

1 Introduction

Gears are the oldest mechanical parts in mechanisms and power transmissions are among the oldest mechanisms used in mechanical engineering. They are used every time when a man wanted to transfer mechanical energy to a working machine. However, after a long period of using old-time power transmissions, modern gear technology was developed in the 20th century [1].

Standard gear transmissions are used with normal contact ratio, i.e. contact ratio between 1 and 2. There is a possibility of load sharing among the teeth, but there is a time during the mesh when one pair of teeth takes the entire load. To ensure smooth and continuous operation, a minimum contact ratio of 1,4 is preferred, and larger is better. Contact ratios for conventional gearing are generally in the range 1,4 to 1,6 [2].

High contact ratio (HCR) gear pair is a contact between gears with at least two pairs of teeth in contact. High contact ratio is obtained with increased addendum height, so larger than in standard gearing. Proposed geometry of HCR gearings is much complicated due to the fact there is larger possibility of occurring meshing and during the production interference, much larger than interference happening in standard involute profiles. Also here there is a higher risk of too small thickness of a tooth tip and significantly less favourable values of specific slips into the flanks [1].

It is well known that increasing the average number of teeth in contact leads to excluding or reduction of the vibration amplitude. First, it was established experimentally that dynamic loads decrease with increasing contact ratio in spur gearing [2]. Moreover, in order to get a further reduction of the vibration, HCR gear profiles can be optimized. Sato et al. [3] found that HCR gears are less sensitive with respect to manufacturing errors. In particular, such kind of gears allows larger tolerance in the tip relief length. Moreover, they found that, in the absence of pressure angle error, the best contact ratio should be about 2; otherwise, it is better to have a contact ratio about 1,7 or higher than 2,3. Kahraman and Blankenship [4] published an experimental work on HCR gear vibration; they found that the best behaviour is obtained with an integer contact ratio, even though other specific non integer (rational) contact ratios can minimize the amplitude of some specific harmonics of the static transmission error. It is important to note that in Ref. [4] HCR gears were obtained by modifying the outside diameter; the other macro-geometric parameters, e.g. the number of teeth, were left unchanged.

Contact ratio is increased by increasing tooth height. Dynamic loads and noise are reduced by using high contact ratio gears. According to results of different measurements of gear pair, reduction of noise proved to be the best using HCR gearing with the value of contact ratio $\varepsilon_\alpha = 2$. Decrease in noise is caused by $\varepsilon_\alpha = 2$ because there are always two pairs of teeth in contact, which means when one pair of teeth go out from the contact, another pair of teeth is coming in contact and applied force is considerably smaller since it is divided on two pairs of teeth. Therefore, gearing in automotive industry should be done with $\varepsilon_\alpha = 2$ in order to reduce the noise and dynamic forces [5].

2 High contact ratio (HCR) of involute gear profile

The correct working of the gear will be assured if the value of the path of contact is higher than that of the base pitch (Fig.1). The ratio between the length of contact g_a, and pitch on the base cylinder p_{bt} is given by the following formula [7]:

$$\varepsilon_\alpha = \frac{\textit{length of contact}}{\textit{base pitch}} = \frac{g_a}{p_{bt}}, \tag{1}$$

$$\varepsilon_\alpha = \frac{\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a_w \cdot \sin\alpha_{wt}}{p_{bt}}, \tag{2}$$

is called the transverse contact ratio. So, contact ratio can be defined as the average number of teeth in contact at one time.

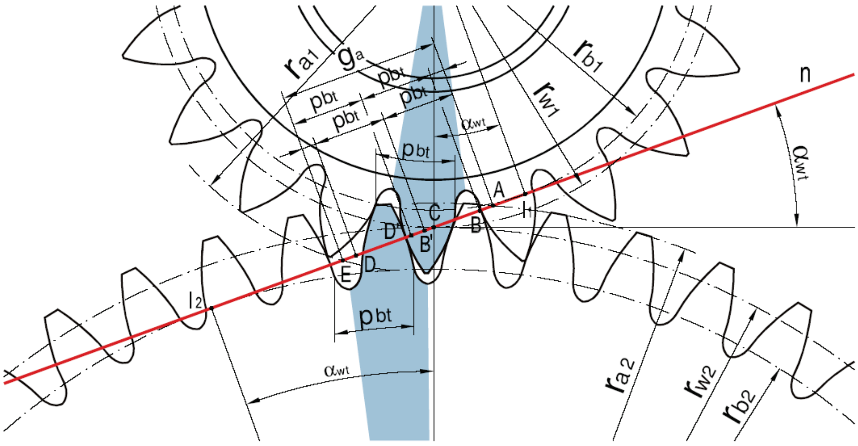

Figure 1 Geometry of involute HCR gearing [6]

The minimum acceptable contact ratio for smooth operation is 1,2. Gears should not generally be designed having contact ratios less than about 1,2, because inaccuracies in mounting might reduce the contact ratio even more, increasing the possibility of impact between the teeth as well as an increase in the noise level. To ensure smooth and continuous operation, the contact ratio must be as high as possible, which the limiting factors permit. A minimum contact ratio of 1,4 is preferred, and larger is better. Contact ratios for conventional gearing are generally in the range 1,4 to 1,6 [8, 9]; so the number of tooth engagements is either one or two. For example, contact ratio of 1,6 means two pairs of teeth are in contact 60 % of the time and one pair carries the load 40 % of the time.

Special kind of basic involute profile of non-standard gearing is called high contact ratio (HCR) gearing, when the contact ratio is higher and there are always at least two pairs of teeth in contact ($\varepsilon_\alpha \geq 2$) and where unit addendum height is not equal one like for standard gearing, so the tooth height is increased and it is bigger than one $h_a^* > 1$.

When HCR gearing is used, it is not necessary to achieve a greater gear load capacity, but nevertheless there is a greater risk of interference due to a greater height of tooth. Advantage of the HCR gearing is also a higher resistance (load distribution is shared on the more pairs of teeth at the same time) and lower relative noise level of gearing, which can be significantly reduced by using integer HCR factor ε_α.

HCR profiles are more complicated than standard involute profiles, they have greater predisposition for occurring interference, pointed tip thickness, but also undercut of teeth during the production (primary production interference).

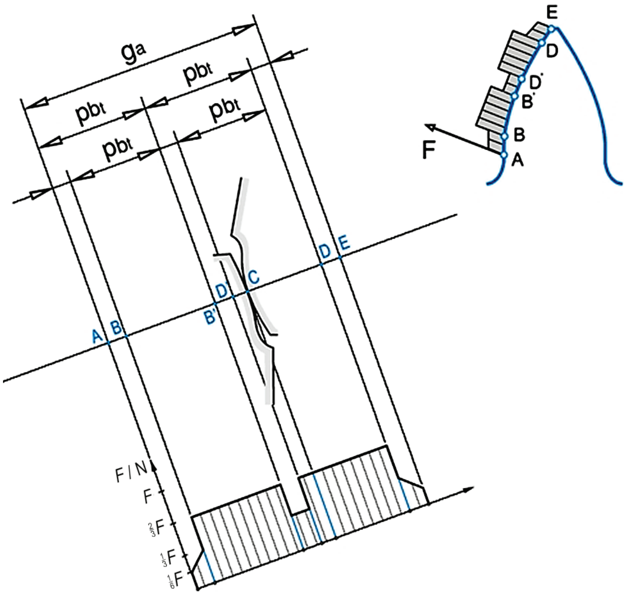

Figure 2 Distribution of tooth load during high contact ratio (HCR) [10]

The coefficient of contact ratio is the main indicator of HCR gearing, which differs from the commonly used standard profiles. The geometry of involute HCR gearing is presented on Fig. 2.

General distribution of applied force in characteristic points of HCR gear tooth flank is shown in Fig. 2. When comparing these both kind of gearing, it is clearly obvious that in the case of involute gearing the maximum force is applied when one pair of teeth is in contact (between points BD, Figs. 1 and 2), which can be considered 100 % of the value of the force F. The biggest applied force in the HCR gearing (between points BB' and DD') can be considered about 50 %, when two pairs of teeth are in contact. Consequently, the size of the applied force is decreased when three pairs of teeth are in contact. So, the value of involute gearing $2/3F$ is decreased to $1/3F$, and value $1/3F$ is decreased to $1/6F$, which means the load distribution is more favourable in the HCR gearing.

The term high-contact-ratio (HCR) applies to gearing that has at least two tooth pairs in contact at all times, i.e., a contact ratio of two or more. As the percentage change in mesh stiffness for HCR meshes is lower than the percentage change in mesh stiffness for normal-contact-ratio (NCR) meshes, one can expect high-quality HCR gear meshes to have lower mesh-induced vibration and noise than NCR gear meshes [11]. A ratio between 2 and 3 means 2 or 3 pairs of teeth are always in contact.

HCR gearing with the length of the line of action $g_a \geq 2\ p_{bt}$ is assumed more favourable when compared with line of action of standard involute gearing. When $\varepsilon_\alpha > 2$, it occurs three pairs of teeth in contact along the line of action and it is triple tooth contact. Triple tooth contact has generally higher load capacity and lower gearing noise. Also favourable properties of this HCR gearing are increasing the resistance of fatigue damage.

In high precision and heavily loaded spur gears, the effect of gear errors is negligible, so the periodic variation of tooth stiffness is the principal cause of noise and vibration. High contact ratio spur gears could be used to exclude or reduce the variation of tooth stiffness.

3 HCR gearing and possibilities of optimization of its geometrical parameters

The main indicator of HCR gearing, which differs from the commonly used standard involute profiles, is higher contact ratio, at least two pair of teeth in contact.

According to results of different measurements of gear pair, reduction of noise proved to be the best using HCR gearing with the value of contact ratio $\varepsilon_\alpha = 2$. Decrease in noise is caused by $\varepsilon_\alpha = 2$ because there are always two pairs of teeth in contact, which means when one pair of teeth goes out from the contact, another pair of teeth is coming in contact and applied force is considerably smaller since it is divided on two pairs of teeth.

Although another parameters can influence on operation noise level, such as module, rack shift factor of gearing, gear ratio, gear manufacturing deviations, and also the gear lubrication.

Contact ratio is better to be as large as possible because HCR gears are less sensitive with respect to manufacturing errors, the vibration and gear noise are less, the load capacity is higher and the load distribution is more favourable in the HCR gearing.

$$\varepsilon_\alpha = f(h_a, \alpha). \tag{3}$$

The increase in contact ratio can be implemented in two ways: by decreasing pressure angle and by increasing tooth height. Obviously, the use of a standard pressure angle and standard tools is preferable [12]. Therefore, the most favourable solution is obtained by increasing addendum height, but however there are a lot of geometrical and manufacturing constraint that has to be satisfied which limits increasing the contact ratio.

The HCR gearing with path of contact length $g_a \geq 2\ p_{bt}$ has more favourable contact length than standard involute gearing. When $\varepsilon_\alpha > 2$ gearing works in triple tooth contact which means that the traction along the lines come into contact with a three pair of teeth. Triple tooth contact has generally a higher capacity and lower noise gearing. These favourable properties of HCR gearing is therefore also influenced by increasing the resistance of fatigue contact damage - pitting, which is one of the main requirements in the design of gears used in automotive transmissions.

Origin of fatigue damage tooth flanks pitting directly depends on the load transfer. The workload at triple tooth contact spread over two or three pairs of tooth flanks instead of one or two pairs of tooth flanks (standard gearing), so its value proportionally decreases (Fig. 2).

The Eq. (2) shows that $\varepsilon_\alpha = f(g_a, p_{bt})$. Tooth pitch on the base circle of standard involute gearing is equal to base pitch on HCR gearing, and it is considered as constant. This means that achieving the greatest value of the contact ratio ε_α has to be obtained by the greatest possible increase of length of line of action g_a. The length of line of action g_a is calculated in Eq. (4):

$$g_a = \sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a_w \sin \alpha_{wt}, \tag{4}$$

where the tip diameters of pinion and gear wheel are as follows:

$$r_{a1} = r_1 + \left(h_a + x_1 \cdot m_n\right), \tag{5}$$

$$r_{a2} = r_2 + \left(h_a + x_2 \cdot m_n\right). \tag{6}$$

From equations (5) and (6), it is clear that the length of the line of action g_a is directly dependent on the addendum height h_{a1}, h_{a2} and factors of correction x_1, x_2. Optimization of geometric parameters of HCR gearing can be based on aim function to get the maximum value of contact ratio ε_α for a given centre distance a_w. The main optimization parameter at this level can be addendum heights of teeth h_{a1}, h_{a2}, and factors of correction x_1 and x_2. For a given distance between centres of the wheels, x_c can be defined as relationship between x_1 and x_2.

Addendum heights h_{a1} and h_{a2} can be found from following equations:

$$h_{a1} = h_{a1}^* \cdot m_n, \tag{7}$$

$$h_{a2} = h_{a2}^* \cdot m_n, \tag{8}$$

where h^*_{a1} and h^*_{a2} are addendum heights for module equal to one.

Further it follows:

$$x_2 = x_c - x_1. \tag{9}$$

So, that means contact ratio is the aim function of both addendum heights and correction factor of pinion $\varepsilon_\alpha = f(h^*_{a1}, h^*_{a2}, x_1)$ = max, i.e. optimization parameters h^*_{a1}, h^*_{a2}, x_1 make a nonlinear optimization of triple constraint, with limitations requirements defined for [6, 13]:

- removal of meshing interference
- minimum arc thickness of the tooth tip $s_{a1,2}$
- distribution x_c to x_1, x_2 has to be done through balancing specific slips, strength, or a particular condition, respectively compromise their combinations.

Such optimization is directly influenced by the resistance of teeth against the occurrence of pitting damage, which is mainly in the automotive practice, one of the critical factors in the design of the transmission gears.

4 Interference during the production

This interference occurs in the production process of gear forming when the tooth of the rack tool is in collision with a produced transition curve of the gear wheel, resulting in a so-called undercut tooth.

This phenomenon largely depends on the method of manufacturing process. Unfavourable conditions arising with manufacturing by tool rack, so if it is not known in advance the means of production, always should check the production interference of gearing for production by tool rack.

Interference during the production will not occur if following condition is satisfied [14]:

$$g_{F_{z1}} \geq 0, \tag{10}$$

where the expression is also valid for the length $g_{F_{z2}}$ (Fig. 3):

$$g_{F_{z1}} = I_1C - F_{z1}C = r_{b1} \cdot \tan\alpha_t - \frac{h_{ev}}{\sin\alpha_t}, \tag{11}$$

where:

$$h_{ev1} = m_n \cdot \left(h^*_{a1} - x_1\right). \tag{12}$$

Substituting relation (12) into equation (11) it follows:

$$g_{F_{z1}} = r_{b1} \cdot \tan\alpha_t - \frac{m_n}{\sin\alpha_t} \cdot \left(h^*_{a1} - x_1\right). \tag{13}$$

For boundary condition $g_{F_{z1}} = 0$, it can be found maximum values of the parameters h^*_{a1} and h^*_{a2} in the case that interference during production doesn't occur, from relation:

$$0 \leq r_{b1} \cdot \tan\alpha_t - \frac{m_n}{\sin\alpha_t} \cdot \left(h^*_{a1} - x_1\right), \tag{14}$$

from which condition the parameter h^*_{a1} is expressed:

$$h^*_{a1} \leq \frac{r_{b1} \cdot \sin^2\alpha_t}{m_n \cdot \cos\alpha_t} + x_1 . \tag{15}$$

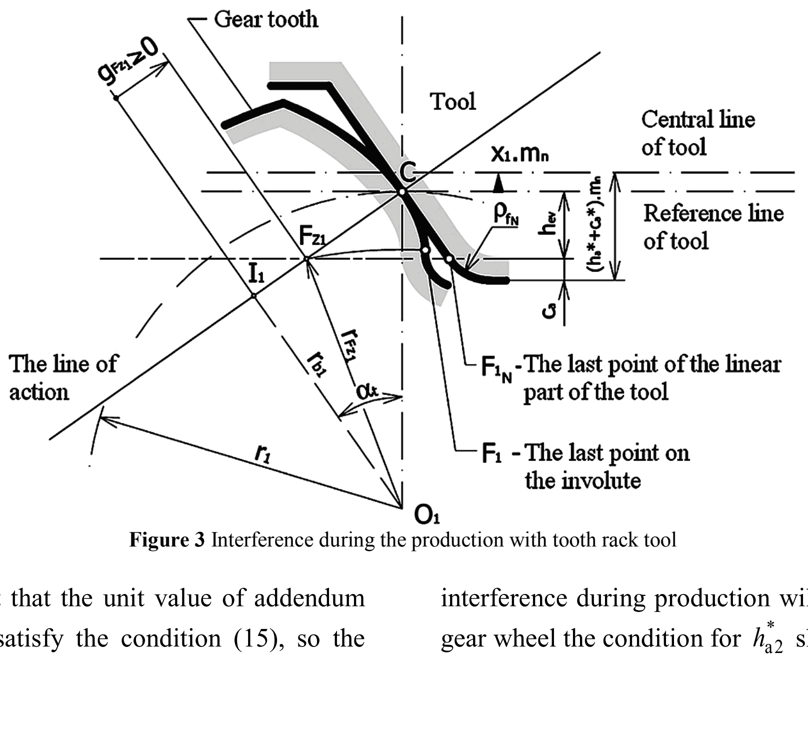

Figure 3 Interference during the production with tooth rack tool

It is very important that the unit value of addendum height h^*_{a1} for pinion satisfy the condition (15), so the interference during production will not occur. Similar for gear wheel the condition for h^*_{a2} should be satisfied:

$$h_{a2}^{*} \le \frac{r_{b2} \cdot \sin^2\alpha_t}{m_n \cdot \cos\alpha_t} + x_2 . \tag{16}$$

5 Meshing interference

Meshing interference is referred in the case of a collision between curves of teeth profiles as interference between these curves. It means that the meshing interference may occur as a collision of head of gear and the transition curve of pinion (Fig. 4) and/or head of pinion and the transition curve of gear wheel [14].

According to Fig. 4 it is possible to assess the interference between meshed head of gear wheel and transition curve of pinion, where the correct image in this case is only possible if the head wheel comes into engagement with the pinion of the involute part which is bordered with the meshing line and point F_{z1} (last point on the pinion involute).

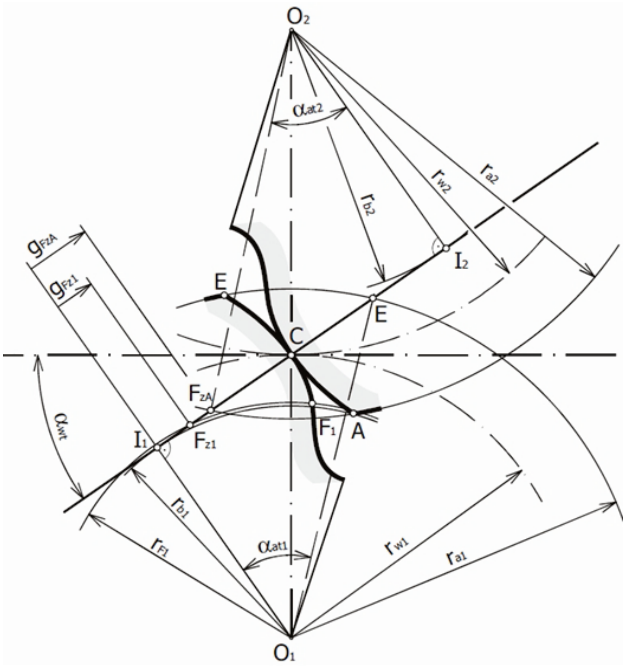

Figure 4 Meshing interference between head of gear wheel and transition curve of pinion

According to Fig. 4, meshing interference will not occur only if distance g_{FzA} is bigger than distance g_{Fz1}, so only if engagement takes place on involute part first of all on the root of pinion tooth and on the addendum tooth of gear wheel. The position of the point F_{z1} was described by Eq. (13).

$$g_{F_{zA}} = a_w \cdot \sin\alpha_{wt} - r_{a2} \cdot \sin\alpha_{at2} . \tag{17}$$

Distance g_{FzA} should be bigger than distance g_{Fz1}, so:

$$g_{F_{zA}} \ge g_{F_{z1}} . \tag{18}$$

Then, when substituting the positions of point A and F_{z1} in Eq. (18), we obtain:

$$a_w \cdot \sin\alpha_{wt} - r_{a2} \cdot \sin\alpha_{at2} \ge r_{b1} \cdot \tan\alpha_t - \frac{m_n}{\sin\alpha_t} \cdot \left(h_{a1}^{*} - x_1\right), \tag{19}$$

the expression values r_{a2} applies:

$$r_{a2} \le a_w \cdot \frac{\sin\alpha_{wt}}{\sin\alpha_{at2}} - r_{b1} \cdot \frac{\tan\alpha_t}{\sin\alpha_{at2}} + \frac{m_n}{\sin\alpha_t \cdot \sin\alpha_{at2}} \cdot \left(h_{a1}^{*} - x_1\right), \tag{20}$$

and after substituting the relation for tip circle r_{a2}, the expression becomes:

$$\begin{aligned} r_2 + \left(h_{a2}^{*} + x_2\right) \cdot m_n \le a_w \cdot \frac{\sin\alpha_{wt}}{\sin\alpha_{at2}} - r_{b1} \cdot \frac{\tan\alpha_t}{\sin\alpha_{at2}} + \\ + \frac{m_n}{\sin\alpha_t \cdot \sin\alpha_{at2}} \cdot \left(h_{a1}^{*} - x_1\right). \end{aligned} \tag{21}$$

On the basis of Eq. (21) addendum height h_{a2}^{*} can be expressed:

$$h_{a2}^{*} \le \frac{1}{m_n} \cdot \left[\begin{array}{l} a_w \cdot \dfrac{\sin\alpha_{wt}}{\sin\alpha_{at2}} - r_{b1} \cdot \dfrac{\tan\alpha_t}{\sin\alpha_{at2}} + \\ + \dfrac{m_n}{\sin\alpha_t \cdot \sin\alpha_{at2}} \cdot \left(h_{a1}^{*} - x_1\right) - r_2 \end{array} \right] - x_2 , \tag{22}$$

where:

$$\cos\alpha_{at2} = \frac{r_{b2}}{r_{a2}}, \quad \alpha_{at2} = \arccos\frac{r_{b2}}{r_{a2}},$$

$$\alpha_{at2} = \arccos\frac{r_{b2}}{r_2 + \left(h_{a2}^{*} + x_2\right) \cdot m_n} . \tag{23 a, b, c}$$

Since the magnitude of the angle α_{at2} is a function of the addendum unit coefficient, i.e. $\alpha_{at2} = f\,(h^{*}_{a2})$, the maximum value of h^{*}_{a2} for which meshing interference will not occur, so in which the tooth will not be shortened due to meshing interference, it is necessary to solve the transcendent Eq. (22). A similar relationship is also used for the parameter h^{*}_{a1}:

$$h_{a1}^{*} \le \frac{1}{m_n} \cdot \left[\begin{array}{l} a_w \cdot \dfrac{\sin\alpha_{wt}}{\sin\alpha_{at1}} - r_{b2} \cdot \dfrac{\tan\alpha_t}{\sin\alpha_{at1}} + \\ + \dfrac{m_n}{\sin\alpha_t \cdot \sin\alpha_{at1}} \cdot \left(h_{a2}^{*} - x_2\right) - r_1 \end{array} \right] - x_1 . \tag{24}$$

One condition more to avoid occurrence of these interferences is a common condition $g_{E2} \ge g_{F2} \ge 0$.

6 Minimum thickness of the tooth head circle

Changing addendum height, h^{*}_{a1} will certainly influence the total thickness of the tooth on tip circle. Greater tooth height, as well as a positive correction factor, may affect the thickness of the tooth on tip circle under the permissible value.

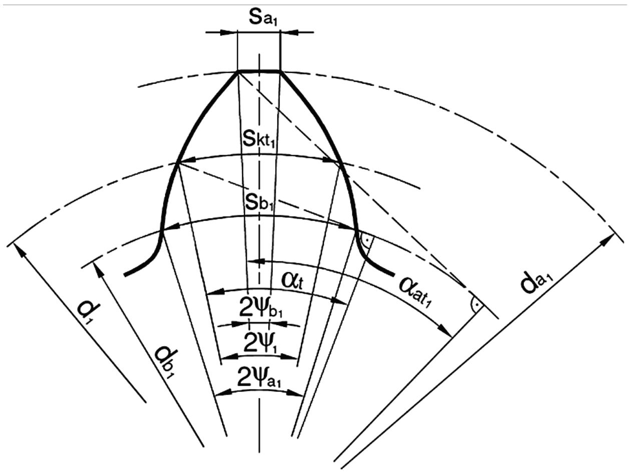

Figure 5 Determination of the tooth thickness on the tip circle d_a

Tooth tip thickness according to Fig. 5 can be expressed [10, 13]:

$$s_{a1} = 2\,\hat{\psi}_{a1} \cdot r_{a1}\,, \tag{25}$$

$$2\,\hat{\psi}_{a1} = 2\,\hat{\psi}_{b1} - 2\,\mathrm{inv}\alpha_{at1}\,, \tag{26}$$

$$2\,\hat{\psi}_{b1} = 2\,\hat{\psi}_1 + 2\,\mathrm{inv}\alpha_t\,, \tag{27}$$

$$2\,\hat{\psi}_1 = \frac{2\,s_{kt1}}{d_1}. \tag{28}$$

According to Eqs. (25) to (28) tooth thickness on the tip circle is given by:

$$s_{a1} = d_{a1} \cdot \left(\frac{s_{kt1}}{d_1} + \mathrm{inv}\alpha_t - \mathrm{inv}\alpha_{at1} \right), \tag{29}$$

while the tooth thickness on the pitch circle s_{kt1} is:

$$s_{kt1} = \frac{p_t}{2} + 2 \cdot x_1 \cdot m_n \cdot \tan\alpha_t\,. \tag{30}$$

After modifying and substituting Eq. (30) into Eq. (29) the expression becomes:

$$s_{a1} = d_{a1} \cdot \left(\frac{p_t + 4x_1 \cdot m_n \cdot \tan\alpha_t}{2d_1} + \mathrm{inv}\alpha_t - \mathrm{inv}\alpha_{at1} \right). \tag{31}$$

The same as for all conventional involute gearing, the thickness of HCR teeth on the tip circle is given by condition $s_a \geq (0{,}25 \div 0{,}4) \cdot m_n$. For case-harden teeth this value should be $s_a \geq 0{,}4 \cdot m_n$, due to lower risk of abruption the tip of tooth.

As mentioned earlier in relation (23c), the size α_{at1} angle is also a function of the unit addendum height $\alpha_{at1} = f(h^*_{a1})$. Then for case-harden teeth, the inequality becomes:

$$0{,}4 \cdot m_n \leq \left(d_1 + 2 \cdot \left(h^*_{a1} + x_1\right) \cdot m_n\right) \cdot \left(\frac{p_t + 4x_1\, m_n\, \tan\alpha_t}{2\,d_1} + \mathrm{inv}\alpha_t - \mathrm{inv}\alpha_{at1} \right), \tag{32}$$

and after arranging:

$$h^*_{a1} \leq \frac{0{,}2}{\dfrac{p_t + 4x_1 \cdot m_n \cdot \tan\alpha_t}{d_1} + 2 \cdot \left(\mathrm{inv}\alpha_t - \mathrm{inv}\alpha_{at1}\right)} - \frac{d_1}{2m_n} - x_1. \tag{33}$$

A similar pattern is valid for the unit addendum height h^*_{a2}.

$$h^*_{a2} \leq \frac{0{,}2}{\dfrac{p_t + 4x_2 \cdot m_n \cdot \tan\alpha_t}{d_2} + 2 \cdot \left(\mathrm{inv}\alpha_t - \mathrm{inv}\alpha_{at2}\right)} - \frac{d_2}{2m_n} - x_2. \tag{34}$$

7 Optimization of HCR geometry

For optimization of HCR geometry following data is used: teeth number of pinion – $z_1 = 21$, teeth number of gear wheel – $z_2 = 51$, module – $m_n = 4$ mm, modified centre distance – $a_w = 144$ mm, pressure angle – $\alpha_n = 20°$ $= \pi/9$ rad and helix angle – $\beta = 0°$.

The goal is to achieve that contact ratio has value two ($\varepsilon_\alpha = 2$), so that always two pairs of gears are engaged. With this value of contact ratio it is expected to reduce the vibration and gear noise. In order to achieve high contact ratio, addendum height is made higher in order to obtain larger line of action.

So, there are several variable parameters:

- addendum height reduced by module on pinion teeth:

$h^*_{a1} \in$ <1, 1,5>

- addendum height reduced by module on gear wheel teeth:

$h^*_{a2} \in$ <1, 1,5>

- rack shift factor of pinion:

$x_1 \in$ <–1, 1>,

and their value should be obtain in order to achieve contact ratio equals two. However, there are several constraints that should be satisfied (interference during the production, meshing interference, minimum thickness of the tooth head circle, slide-conditions in the HCR involute gearing).

Before limitation conditions are given, tooth parameters should be calculated:

- transverse pressure angle: $\alpha_t = \arctan\left(\dfrac{\tan\alpha_n}{\cos\beta}\right)$, (35)

- reference radius: $r_1 = \dfrac{z_1 \cdot m_n}{2 \cdot \cos\beta}$, $r_2 = \dfrac{z_2 \cdot m_n}{2 \cdot \cos\beta}$, (36)

- base radii: $r_{b1} = r_1 \cdot \cos\alpha_t$, $r_{b2} = r_2 \cdot \cos\alpha_t$, (37)

- working pressure angle:

$$\alpha_{wt} = \arccos\left(\frac{m_n}{\cos\beta} \cdot \frac{z_1 + z_2}{2} \cdot \frac{\cos\alpha_t}{a_w}\right), \quad (38)$$

- involute value of angle α_t:

$$\mathrm{inv}\alpha_t := \tan\alpha_t - \widehat{\alpha}_t, \quad (39)$$

- involute value of angle α_{wt}:

$$\mathrm{inv}\alpha_{wt} := \tan\alpha_{wt} - \widehat{\alpha}_{wt}, \quad (40)$$

- the sum of rack shift factor:

$$x_c = \frac{z_1 + z_2}{2 \cdot \tan\alpha_n} \cdot (\mathrm{inv}\alpha_{wt} - \mathrm{inv}\alpha_t) = 0, \quad (41)$$

- rack shift factor of gear wheel:

$$x_2 = x_c - x_1 = 0 - x_1 = -x_1, \quad (42)$$

- gear ratio: $i = -\dfrac{z_2}{z_1}$, (43)

- tip radii:

$$r_{a1} = r_1 + m_n \cdot (h_{a1}^* + x_1), \ r_{a2} = r_2 + m_n \cdot (h_{a2}^* + x_c - x_1), \quad (44)$$

- the value of the tip thickness:

$$\begin{aligned} s_{kt1} &= \frac{\pi \cdot m_n}{2 \cdot \cos\beta} + 2 \cdot x_1 \cdot m_n \cdot \tan\alpha_t, \\ s_{kt2} &= \frac{\pi \cdot m_n}{2 \cdot \cos\beta} + 2 \cdot (x_c - x_1) \cdot m_n \cdot \tan\alpha_t, \end{aligned} \quad (45)$$

- the angle corresponding to tip diameter:

$$\alpha_{at1} = \arccos\frac{r_{b1}}{r_{a1}}, \quad \alpha_{at2} = \arccos\frac{r_{b2}}{r_{a2}}, \quad (46)$$

- involute value of angle α_{at}:

$$\mathrm{inv}\alpha_{at1} := \tan\alpha_{at1} - \widehat{\alpha}_{at1}, \ \mathrm{inv}\alpha_{at2} := \tan\alpha_{at2} - \widehat{\alpha}_{at2}. \quad (47)$$

After calculating these parameters, several constraints are given. These constraints came from limitation condition of:

- interference during the production:

$$\begin{aligned} g_{F1} &= r_{b1} \cdot \tan\alpha_t - \frac{(h_{a1}^* - x_1) \cdot m_n}{\sin\alpha_t} \geq 0, \\ g_{F2} &= r_{b2} \cdot \tan\alpha_t - \frac{(h_{a2}^* - x_c + x_1) \cdot m_n}{\sin\alpha_t} \geq 0, \end{aligned} \quad (48)$$

- meshing interference:

$$\begin{aligned} B_1 &= \sqrt{r_{b1}^2 + (a_w \cdot \sin\alpha_{wt} - g_{F2})^2} \geq r_{a1}, \\ B_2 &= \sqrt{r_{b2}^2 + (a_w \cdot \sin\alpha_{wt} - g_{F1})^2} \geq r_{a2}, \end{aligned} \quad (49)$$

- minimum thickness of the tooth head circle:

$$\begin{aligned} s_{a1} &= 2r_{a1} \cdot \left(\frac{s_{kt1}}{2r_1} + \mathrm{inv}\alpha_t - \mathrm{inv}\alpha_{at1}\right) \geq 0{,}4m_n, \\ s_{a2} &= 2r_{a2} \cdot \left(\frac{s_{kt2}}{2r_2} + \mathrm{inv}\alpha_t - \mathrm{inv}\alpha_{at2}\right) \geq 0{,}4m_n. \end{aligned} \quad (50)$$

Further, there is an equation of contact ratio. It represents goal function with the aim it should have the value of two.

$$\varepsilon_\alpha = \frac{z_1}{2\pi}(\tan\alpha_{at1} - \tan\alpha_{wt} - i \cdot \tan\alpha_{at2} - i \cdot \tan\alpha_{wt}) = 2. \quad (51)$$

8 Generalized particle swarm optimization algorithm

The optimization method used in this paper is quite a new optimizing method called Generalized Particle Swarm Optimization algorithm (GPSO), which is a modification of original version of Particle Swarm Optimization algorithm (PSO). The fundamentals and outlines of this algorithm will be presented in the sequel.

It is well-known that the field of global optimization has prospered much from nature-inspired techniques, such as Genetic Algorithms (GAs) [15], Simulated Annealing (SA) [16], Ant Colony Optimization (ACO) [17] and others. Among these search strategies, the Particle Swarm Optimization (PSO) algorithm is relatively novel, yet well studied and proven optimizer based on the social behavior of animals moving in large groups (particularly birds) [18]. Compared to other evolutionary techniques, PSO has only a few adjustable parameters, and it is computationally inexpensive and very easy to implement [19, 20], which is a reason why it was used in many engineering applications.

PSO uses a set of particles called swarm to investigate the search space. Each particle is described by its position (x) and velocity (v). The position of each particle is a potential solution, and the best position that each particle achieved during the entire optimization process is memorized (p). The swarm as a whole memorizes the best position ever achieved by any of its particles (g). The position and the velocity of each particle in the k-th iteration are updated as

$$v[k+1] = w \cdot v[k] + cp \cdot rp[k] \cdot (p[k] - x[k]) + \\ + cg \cdot rg[k] \cdot (g[k] - x[k]) \\ x[k+1] = x[k] + v[k+1] \qquad (52)$$

Acceleration factors cp and cg control the relative impact of the personal (local) and common (global) knowledge on the movement of each particle. Inertia factor w keeps the swarm together and prevents it from diversifying excessively and therefore diminishing PSO into a pure random search. Random numbers rp and rg are mutually independent and uniformly distributed on the range [0, 1].

Despite many good features, the original version of PSO algorithm has also some flaws, mainly concerning its inability to independently control various aspects of the search [25]. In the recent literature, there are many modifications of this algorithm which overcome these flaws. One of them is a generalization of PSO that is based on the general linear discrete second-order model, well known from control theory [22]. This algorithm, named Generalized PSO (GPSO), was proposed in [23] and analyzed in detail in [27]. As a basis of this algorithm, the following second-order PSO model is used

$$x[k+1] - (1 + w - cp \cdot rp[k] - cg \cdot rg[k]) \cdot x[k] + \\ + w \cdot x[k-1] = cp \cdot rp[k] \cdot p[k] + cg \cdot rg[k] \cdot g[k], \qquad (53)$$

which is equivalent to the model described by (52). The idea of this algorithm is in fact to address PSO in a new and conceptually different fashion, i.e., to consider each particle within the swarm as a second-order linear stochastic system with two inputs and one output. The output of such a system is the current position of the particle (x), while its inputs are personal and global best positions (p and g, respectively). Such systems are extensively studied in engineering literature [22]. The stability and response properties of such a system can be directly related to its performance as an optimizer, i.e., its explorative and exploitative properties. Thus, one can overcome the main flaw of the PSO algorithm, its inability to independently control various aspects of the search, such as stability, oscillation frequency and the impact of personal and global knowledge [25].

Equation (53) can be interpreted as a difference equation describing the motion of a stochastic, second-order, discrete-time, linear system with two external inputs. In general, a second-order discrete-time system can be modeled by the recurrent relation

$$x[k+1] + a_1 \cdot x[k] + a_0 \cdot x[k-1] = b_p \cdot p[k] + b_g \cdot g[k], \qquad (54)$$

where some or all of the parameters a_1, a_0, b_p and b_g are stochastic variables with appropriate, possibly time-varying probability distributions. There are several restrictions that should be imposed on these parameters in order to make (54) a successful optimizer. First, the system should be stable (in the sense of control theory [22]), and its stability margins should grow during the optimization process. The swarm should explore the search space first, and the particles should initially be allowed to move more freely. As the search process approaches its final stages, the optimizer should exploit good solutions that were previously found, and the effort of the swarm should be concentrated in the vicinity of known solutions. Second, the response of the system to the perturbed initial conditions should be oscillatory in order for the particles to overshoot or fly over their respective attractor points. Further, if both external inputs approach the same limit value as k grows, the particle position x should also converge to this limit. Finally, in the early stages of the search, the system should primarily be governed by the cognitive input p, allowing particles to behave more independently in these stages. In later stages, the social input g should be dominant because the swarm should become more centralized, and global knowledge of the swarm as a whole should dominate the local knowledge of each individual particle. All of these requirements, as explained in detail in [27], can easily be satisfied if system (54) is rewritten in the following canonical form, often used in control theory [22]:

$$x[k+1] - 2\zeta\rho x[k] + \rho^2 x[k-1] = \\ = (1 - 2\zeta\rho + \rho^2)(c \cdot p[k] + (1-c) \cdot g[k]). \qquad (55)$$

In (55), ρ is the eigenvalues module, and ζ is the cosine of their arguments. Parameter c is introduced to replace both b_p and b_g. The primary idea of GPSO algorithm is to use (55), instead of (52) or (53), in optimizer implementation. The parameters in this equation allow a more direct and independent control of the various aspects of the search procedure.

It is known that proper parameter selection is crucial for the performance of PSO. Therefore, based on numerous researches published in the literature [19 ÷ 21]. and thorough empirical analysis, some recommendation for the choice of GPSO parameters are proposed in [27]. According to these recommendations, parameter ρ should linearly decrease from 0,95 to 0,6 and parameter c should linearly decrease from 0,8 to 0,2. The proper selection of ζ proved to be the most difficult task, as this factor has no direct analogy to any of the parameters of the classical PSO. The ζ parameter equals the cosine of particle eigenvalues in the dynamic model. Because this is a discrete system, the eigenvalue argument equals the discrete circular characteristic frequency of particle motion. Thus, ζ is directly related to the ability of particles to oscillate around attractor points (i.e., global and personal best) and, therefore, has a crucial impact on the exploitative abilities of the algorithm. If ζ equals 1, this would prevent particles from flying over the attractor points; however, if ζ is close to -1, this would result in the desultory movement of the particles. In both cases, particles cannot explore the vicinity of the attractor points. It is shown that the most appropriate and robust choice is to adopt ζ as a stochastic parameter with uniform distribution ranging from −0,9 to 0,2.

GPSO algorithm was already used to solve many practical problems [23, 27, 28] which confirmed that it is a very suitable optimization tool for engineering applications.

In the present research, GPSO algorithm was implemented in MATLAB, a high-performance language for technical computing, which integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. The algorithm was implemented as a function, which allows to user to specify the values of all adjustable parameters and to define many available algorithm options. The input parameters are the name of the function which is optimized, number of variables and algorithm parameters options, while output parameters are the optimal function value, its coordinates and some other optional parameters. The algorithm was also slightly modified in order to be able to deal with constraints expressed by (48 ÷ 50), which are implemented in the form of penalty functions, as addition to main optimization criterion defined by (51).

9 Obtained results using GPSO algorithm

Using GPSO algorithm, a solution of HCR value is obtained. This solution is very accurate, and it goes till 10^{-15} accuracy. Obtained results calculated by this method are presented further:

Given data:
z1 = 21
z2 = 51
mn = 4 mm
aw = 144 mm
alfan = 0.349065850398866 rad, beta = 0

Calculated optimal values for 100 iterations:
ha1dot = 1.184548908194918
ha2dot = 1.313253162560083
x1 = 0.174169864574093
Main function:
epsilonalfa = 2.000000052991015

Calculated optimal values for 200 iterations:
ha1dot = 1.18219030491075
ha2dot = 1.31311869844519
x1 = 0.16471064649626
Main function:
epsilonalfa = 2.00000000000000

Calculated optimal values for 500 iterations:
ha1dot = 1.17779276410901
ha2dot = 1.31951635807650
x1 = 0.17860257195356
Main function:
epsilonalfa = 2.00000000000000

10 Conclusion

High contact ratio (HCR) gear pair is a contact between gears with at least two pairs of teeth in contact. High contact ratio is obtained with increased addendum height, so larger than in standard gearing. Proposed geometry of HCR gearings is much complicated due to the fact there is larger possibility of occurring meshing and during the production interference, much larger than interference happening in standard involute profiles. Also, there is a higher risk of too small thickness of a tooth tip and significantly less favourable values of specific slips into the flanks.

Contact ratio is increased by increasing tooth height. Dynamic loads and noise are reduced by using high contact ratio gears. According to results of different measurements of gear pair, reduction of noise proved to be the best using HCR gearing with the value of contact ratio $\varepsilon_\alpha = 2$. Decrease in noise is caused by $\varepsilon_\alpha = 2$ because there are always two pairs of teeth in contact, which means when one pair of teeth go out from the contact, another pair of teeth is coming in contact and applied force is considerably smaller since it is divided on two pairs of teeth. Therefore, gearing in automotive industry should be done with $\varepsilon_\alpha = 2$ in order to reduce the noise and dynamic forces.

Due to increased addendum height, there is larger possibility of occurring some interference or pointed tooth tip. Therefore it should prevent these errors and check if all equation and constraints are satisfied. Both conditions for teeth on pinion and gear wheel in order not to occur interference during the production – equations (15) and (16), conditions not to occur meshing interference – equations (22) and (24), conditions for minimal thickness of the tooth head circle of both gears – equations (33) and (34).

Optimal values of parameters h^*_{a1}, h^*_{a2} and x_1 are determined using Generalized Particle Swarm Optimization (GPSO), a relatively novel, robust an efficient global optimization procedure based on linear control theory. This algorithm, implemented in MATLAB, provided multiple very precise solutions of given transcendent equation found by this method, and they all give solution for contact ratio $\varepsilon_\alpha = 2$.

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