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SOME ASPECTS OF MODELLING OF LINE SCREEN ELEMENT REFLECTANCE PROFILE WITHIN THE MONTE CARLO METHOD

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Original scientific paper
Better understanding of how internal structural changes affect the optical properties requires modelling the composite structure of a paper layer. We propose a Monte Carlo simulation platform, which allows different geometrical representations of the inner paper structure. Monte Carlo Modelling is a method that can be used to simulate photon interaction with substrate (in our case paper). The model can be built such that increasing complexity can be added as the model is being developed. Thus, the model is developed from a simple concept of random number generation to a very accurate interpretation of photon propagation in paper. We applied the model to the analysis of the effect of dot gain. The above model was used to analyse the effect of weight gain dot gain of printed lines of different thicknesses. The simulation results are in good agreement with measurements of reflection after having additionally introduced a "filter" function that permanently removes photons thereby adapting simulated values to the measured ones. Within the framework of the simulation, we can estimate the number of photons that ends below the screen element causing a possible increase of the optical dot gain.

Keywords: *Monte Carlo simulation, optical dot gain, subsurface scattering in paper*

Neki aspekti modeliranja reflektancijskog profila linijskog rasterskog elementa u okviru Monte Carlo metode

Izvorni znanstveni članak
Bolje razumijevanje kako unutarnje strukturne promjene utječu na optička svojstva papira zahtijeva modeliranje kompozitne strukture njegovih slojeva. Predložena je simulacija u okviru Monte Carlo platforme, koja omogućuje različite geometrijske reprezentacije unutarnje strukture papira. Monte Carlo simulacija je metoda koja se može koristiti za simulaciju interakcije fotona s podlogom (u našem slučaju papirom). Model se može graditi tako da se povećanje složenosti može dodati kako se model razvija. Dakle, model se razvija od jednostavnog koncepta generacije slučajnih brojeva u vrlo točno tumačenje širenja fotona u papiru. Navedeni model primijenjen je na analizu učinka prirasta rastertonske vrijednosti otisnutih linija raznih debljina. Rezultati simulacije dobro se slažu s mjerenjima refleksije nakon što je dodatno uvedena funkcija "filtar" koja trajno uklanja fotone čime smo prilagodili simulirane vrijednosti mjerenim. U okviru simulacije, možemo procijeniti broj fotona koji završava ispod rasterskog elementa i uzrokuje mogući optički prirast rastertonske vrijednosti.

Ključne riječi: *Monte Carlo simulacija, optički prirast rastertonske vrijednosti, podpovršinsko raspršenje u papiru*

1 Introduction

The motivation of this simulation was to describe and anticipate the transport of light through a medium such as paper and to see the impact the dispersion has on the optical dot gain. We considered paper as multi-layered inhomogeneous optical medium where coating and paper base are treated as two separate, well-defined layers, excluded from the calculation of the reflection and refraction of light on boundaries, where there is a change in local conditions of absorption and scattering. Inside the paper, as an extremely complex medium, photons experience multiple scattering while the phase and polarization are rapidly randomized, so that they initially do not significantly affect the energy transport alone.

Identification and characterization of all the sub-processes in the whole system is the basis for complete control and calibration systems Print quality (system response such as tone resolution, reproduction characteristic, etc.) for each printing press and the general process of transferring information to establish a graphic communication depends on the interaction of light and substrate, in our case paper. Realistic description of the material depends on the knowledge of transport of light in the medium. The applied approach to the propagation of photons within the Monte Carlo method frame is based on the work Phral et al. [1].

The possibility of "experimenting" with different combinations of component of paper without the actual realization of the paper and followed by the application of screen element on the paper allows us to control some of the ideas and to avoid lengthy and costly production of real paper of interest. The scattering of light within paper can affect the tone characteristics of a printed halftone image. A halftone image is formed by variation in the average reflectance, which is determined by the size of screen dots. Some photons end up below the screen element and are absorbed by the ink. This lack of reflected photons is manifested as an increase in the screen element dimension (screen element appears effectively larger than their original size). This effect is known as optical dot gain [2, 3]. Dot gain is a phenomenon in some forms of printing which causes printed material to look darker than intended. The previous researches showed that the optical dot gain depends on two different factors, namely the properties of the materials such as paper and ink and the geometrical distribution of ink such as resolution, location, size and shape [4, 5].

The radiation transfer theory describes the interaction of radiation with the media that scatter and absorb light. The solutions of radiation transport equation are applied to a wide range of problems such as, for example, neutron diffusion, optical tomography, heat transfer and scattering of visible light in the atmosphere and to the study of prints on paper. Paper, as the basis on which we focus in our work, is stochastic three-dimensional structure, which is mainly composed of cellulose fibres, interconnected via hydrogen bonding at the molecular level, and substances that are used as fillers and various additives on the macroscopic scale. The properties of the fibres depend on the type of wood from which they originate, and that is during the production further treated mechanically and chemically. Some scatterers (fibres and particles added) in a sheet of paper are so close to each other that multiple scattering becomes important. In the scattering theory we begin with a single scattering, where we define ensembles with small enough (order of hundred) number of particles whose mutual travel distance is large enough so none of the particles is experiencing interaction with the neighbouring particles. Qualitative criteria for assuming one scattering is that on every particle in the ensemble acts the same input electric field, which is relatively unaffected by scattered fields from the other particles in the ensemble. The ensemble of scatterers is treated in a way that scattering ensemble characteristic is simply the sum of individual contributions of individual scatterers [6]. Paper is, in principle, an inappropriate medium for research due to the fact that the optical properties of its components are completely different.

2 The aim of research

In this paper, we approached the problem of light scattering in the paper in a way that physically more accurately describes the real situation. Our simulation provides a flexible and, at the same time, a rigorous approach to the problem of transport of light in a medium such as paper with stochastic interaction which takes photon-component of paper. This means that if the paper has 30 % volume fraction of some component then the same percentage of photons will interact with that component. The method describes the local rules of propagation of photons, which are expressed in the simplest case, as a probability distribution that describes the step size shift of photons between two photons interacting with components of paper and corners directions trajectory of photons in the case when it comes to scattering.

On the other hand, it should be noted that the method is statistical in nature and is based on the calculation of the trajectories of a large number of photons, which results in a large consumption of computer time. If we recall that the energy of one photon is hc/λ, where h = $6{,}626\times10^{-34}$ J·s is Planck's constant, c = (speed of light) = 3×10^{8} m/s and λ = 550 nm (wavelength of light), then we get the solar flux Φ at the top of the atmosphere to the value of $2{,}55\times10^{15}$ photons/(cm^2·s). This number of photons per unit area per unit time allows us to model the radiation field statistical analysis of traveling photons. The number of photons required for correct description of the problem is determined with required accuracy and the desired spatial resolution. For example, to get basic information about the multiple scattering of light in paper $0{,}5\times10^{6}$ photons will be enough to obtain useful information in a particular simulated case. However, in order to map the spatial distribution of photons returning to the entrance surface, 30×10^{6} photons are necessary to get a satisfactory answer, and could carry out necessary statistics calculus.

Our method is based on the macroscopic optical properties of the components of the paper for which we assume that are uniform in small volumes within the paper. The average distance (step size) between positions where there is interaction (scattering and/or absorption) is of the order within 10 ÷ 100 μm, which allows us to apply the principles of geometrical optics. Distance in question applies only to the average distance in the path of photons generated with the Monte Carlo method. Due to its stochastic nature, it is not the real distance between the constituents of paper, but the expression of its numerical value.

Our goal was to generate as much as possible physically plausible and analytically (computer assisted) model, even though a large part of the simulation was based on the knowledge of the structure and behaviour of the constituents of the paper, its confirmation could not be checked directly but indirectly by measuring the reflectance profiles of the screen elements. A direct consequence of the internal light scattering in the substrate is an optical gain - dot gain, so that without prints of individual screen elements and their image analysis would not be possible to verify the results predicted by the model. The model simulated scattering of light in the substrate, on which there is a printed line, either in the positive or in the negative. In our work, we focused on the treatment of photons that finish below the screen element. Monte Carlo model, which will be briefly presented later in the text does not take into account these photons, and we had to include a computer program so called "filter" function that all photons that finish below the screen element considered completely absorbed in the layer of deposited dye. Accordingly, the only modelling of prints by adding mechanical dot gain could verify our model as a useful tool, not only as a source and verification of the initial assumptions, but also as a possible "virtual experimental" apparatus.

3 Theory

Paper, as a substrate on which we will focus in this work, is a stochastic three-dimensional structure, which is mainly composed of fibres mutually connected by hydrogen bonds at the molecular level and substances that we use as filler on the macroscopic level. Characteristics of fibres depend on the type of wood they originated from, except that during manufacturing they are further treated mechanically and chemically. Individual scatterers in sheet of paper - fibre and added particles - are so close to each other that multiple scattering becomes important. In scattering theory we start with single scattering in which we observe ensembles with sufficiently small number of particles whose mutual distance is large enough so that each particle can achieve individual sample without influence of neighbouring particles. Qualitative criteria for assuming a single scattering is that every particle in the ensemble is affected by the same input electric field with negligible influence of scattered fields from other particles from the ensemble. Treatment of scatterer ensemble is such that the scattering properties of the ensemble are simply the sum of the contributions of each scatterer [7]. The reason for introducing scatterer ensemble lies in the need to respect the law of energy conservation.

Paper is, in principle, ungrateful media due to the fact that the optical properties of its components are quite different. The industry uses approximation of average scattering function of individual scatterer that is needed for radiation transfer models. This function is obtained by using the outdated Kubelka - Munk theory in its original form [7]. However, some time ago this model was quite improved by introducing anisotropy of intensity of propagated diffuse radiation [8]. Accurate modelling of the scattering of light by materials is fundamental for realistic image synthesis. Even the most sophisticated light transport algorithms fail to produce convincing results if the local scattering models are too simple. Therefore a great deal of research has gone into describing the scattering of light from materials.

Physical explanation begins with scattering and absorption of light (photons) on individual particles, such as fibres or fillers. Light scattering in paper is a process of multiple scattering, because the paper is a tight complex structure. The degree of complexity of the mathematical treatment of the scattering radiation on an isolated scatterer heavily affects the relationship between the wavelength of the radiation and the scatterer physical dimensions. Light scattering on the particle is described with integral equations defined throughout the volume of particles. In general, the approximations that simplify the problem can be applied in separate cases where the wavelength is at least an order of magnitude larger or smaller than the scatterer. The previous case corresponds to the well-known theory of Rayleigh scattering, which, for example, explains the blue colour of the sky on a clear day, while the latter involves the application of techniques of geometrical optics to predict scattering. Hypothesis for the application of geometrical optics, i.e. that the input beam can be split into independent rays, applies to cellulose fibres in paper the typical length of which is of the order of few millimetres with a diameter of 20 μm.

4 Monte Carlo simulation

As mentioned above, light scattering in the paper is modelled within the framework of the Monte Carlo approximations. Monte Carlo method is a universal numerical technique to solve mathematical problems. It is based on random sampling from the defined probability distribution. Monte Carlo [9, 10, 11] methods were used in the 50's of the last century while solving the problem of neutron transport [12]. The random sampling technique was widely used in the 1940's to develop the atomic bomb, for modelling a random sample of neutron diffusion in fissile material. The year of 1949 has been accepted as the year of the foundation of the Monte Carlo method when the article by Metropolis and Ulam [13] titled "The Monte Carlo Method" was published. The name Monte Carlo was coined by Metropolis (inspired by Ulam interest in poker) during the Manhattan Project in the Second World War, due to the similarity of statistical simulation to games of chance, and the famous casino in the capital of the Kingdom of Monaco. The main components of every Monte Carlo method are random numbers, or the numbers that appear in the non-correlated order. The appearance of computers has enabled the practical use of Monte Carlo techniques. But since the behaviour of the computer is deterministic, random numbers generated with the help of computers are not truly random numbers, so they are called pseudo-random numbers. In 1954 Hayward and Hubbell used Monte Carlo method to study the reflection of gamma-rays [14].

For modelling the transport of light in the visible region, Monte Carlo technique was first used in the determination of the optical properties of photographic emulsions [15], and the technique is still used today in astronomy and remote sensing. Monte Carlo method in graphics appears independently, starting with Apple [16] (1968), which calculated random images by specifying particles (photons) paths, so-called random particle tracing. The papers by Whitted [17] (1980) and Cook and others (1984) [18] lead to the first complete light transport algorithm based on Monte Carlo method proposed by Kajiya [19] (1986). He realized that the problem can be displayed using the integral equations which can be evaluated by sampling paths. Since then, much has been done to improve his path tracing technique (Arvo and Kirk [20], 1990).

The transport of photons in the paper is simulated by the Monte Carlo simulation method of propagation photons, while neglecting the wave nature of light. The method describes the local rules of propagation photons, which are expressed in the simplest case, as a probability distribution that describes the step size (mean free path) of movement of photons between the two points of interaction photon - substrate, and the angles of the deviation from the previous direction of path photons after scattering occurs in the given point [21, 22]. As this method is essentially statistical, based on calculating the propagation path of a large number of photons, it consumes a lot of computing time. This is equivalent to the analytical modelling of the transport photons with the radiation transfer equation (of RTE), which describes the motion of photons using differential equations. However, some geometric finite solutions of RTE are often not possible. Diffusion approximation can be used to simplify the RTE although it introduces a lot of inaccuracies, particularly in the vicinity of the source and boundaries. In contrast, Monte Carlo simulations can be arbitrarily accurate by increasing the number of the monitored photons. In addition, Monte Carlo simulations can simultaneously track multiple physical quantities with any desired spatial and temporal resolution. This flexibility makes the Monte Carlo modelling a really powerful tool. Thus, although computing may be ineffective, Monte Carlo methods are often considered as the standard for measuring the simulation of photon transport in many applications. In direct determination of Monte Carlo simulations of light transport in the medium that scatters and absorbs light, the method in general passes the procedure that follows a series of steps [23]. As electromagnetic rays pass through different materials, some of the photons interact with the particles of matter, whereas their energy can be absorbed or scattered. This absorption and scattering in a medium is called attenuations and total attenuations are the sum of attenuations due to different types of interactions. Linear attenuation coefficient (μ_t) describes the part of the beam of electromagnetic radiation that is absorbed or scattered per unit thickness of material. It is basically a calculation of probability that the photon is scattered or absorbed when passing through the material. For the narrow ray of mono energetic photons of for example visible light, part of the beam that passes through the material without collision can be expressed by the equation (Beer's law) [24]:

$$I = I_0 \mathrm{e}^{-\mu_t t}, \tag{1}$$

where:
I − intensity of photons transmitted over a distance t,
I_0 − initial photons intensity
t − penetration depth.

Scattered photons are described with Mie theory. But our model describes the combined effects of absorption and scattering.

1) As we have seen, it is difficult to know how a single photon will behave in a medium. What is easier to constrain is how an ensemble of N photons behaves in terms of their statistical properties. The simulations treat photons as neutral particles rather than as a wave phenomenon. The basic procedure is as follows: We choose a point of origin to start the simulation of transport of photons. The starting point of the photon is determined from the known distribution of sources and may not be random.

2) Emit N photon packets (hereafter referred to simply as photons). Each photon packet is initially assigned a weight, W, equal to unity.

We follow the history of successive scattering and/or absorption in the medium until some threshold condition (termination of photons) is satisfied. To follow the history of the photon random samples are taken from a well-defined probability distribution [25, 26] that controls different transitions or interactions that a photon experiences. This probability distribution can be derived from knowledge of the physics of the interaction or may be based on measurements. The probability density function is defined as

$$P(s) = \mu_t \mathrm{e}^{-\mu_t s}. \tag{2}$$

The step size of the photon is calculated based on sampling the probability for the photon's mean free path. The photon is moved a propagation distance Δs that is calculated based on pseudo random number ζ generated in the interval (0,1].

$$\Delta s = \frac{\ln(\zeta)}{\mu_t}, \tag{3}$$

where $\mu_t = \mu_a + \mu_s$, is linear attenuation coefficient; μ_a is the absorption coefficient of the media; μ_s is the scattering coefficient of the media.

It should be noted that if step size is too small, Monte Carlo method is inefficient, but if step size is too large, we get poor approximation of real photon travel.

The scattering and absorption of the photons are determined by sampling from the albedo

$$a = \frac{\mu_s}{\mu_a + \mu_s}, \tag{4}$$

(albedo a is part of energy which is re-emitted or more precisely the probability that a photon is re-emitted).

Photon absorption occurs in a variety of different ways (line absorption, resonant line scattering and continuum absorption). In general, the terms in Eq. (3) are dependent on incident photon wavelength. For the sake of simplicity, we consider continuum absorption only.

Once the photon has taken a step, some attenuation of the photon weight must be calculated. A fraction of the photon's current weight, W, will be deposited in the substrate. The amount of deposited photon weight, ΔW, is calculated with equation

$$\Delta W = W \frac{\mu_a}{\mu_a + \mu_s}. \tag{5}$$

The photon packet with the new weight W will suffer scattering at the next interaction site. Note that the whole photon packet experiences interaction at the end of the step, either absorption or scattering.

There will be a deflection angle, $\theta \in [0, \pi)$, and an azimuth angle, $\psi \in [0, 2\pi)$ to be sampled statistically. The probability distribution for the cosine of the deflection angle, $\cos\theta$, is described by the scattering function characterized by the Henyey-Greenstein phase function [25]

$$p(\cos\theta, g) = \frac{1}{4\pi} \cdot \frac{1-g^2}{\sqrt{(1+g^2-2g\cos\theta)^3}}, \tag{1}$$

where the parameter g is defined as the integral over all angles of the phase function multiplied by the cosine of the angle θ (angle enclose to entrance and scattered ray).

$$g = 2\pi \int_0^{\pi} p(\cos\theta) \cos\theta \, \mathrm{d}\theta. \tag{7}$$

The anisotropy factor g, determines the directionality of scattering (average forward component). The anisotropy g, equals $\langle\cos\theta\rangle$ and has a value between –1 and 1, which determines forward or backward (back scattering) scattering. The azimuth angle is uniformly distributed between [0, 2π], and may be generated by multiplying a random number $\xi \in [0, 1]$ by 2π. The Henyey-Greenstein phase function (Eq. 2.3) is also easily sampled [27]. By applying the standard integration and inversion approach to find a sampling distribution that is proportional to the function's distribution, we have (8):

$$\cos\theta = \begin{cases} \dfrac{1}{2g}\left\{1+g^2-\left[\dfrac{1-g^2}{1-g+2g\xi}\right]^2\right\} & \text{if } g \neq 0 \\ 2\xi - 1 & \text{if } g = 0 \end{cases}. \tag{2}$$

3) The photon is scattered at an angle (θ, ψ). We record or extract the parameters of interest from the photon history. There are a multitude of different parameters that can be monitored from the photon's history, such as the point of exit, the total length of the

photon path, photon output angle, its intensity when leaving substrate, etc. It is one of the great advantages of the method.

4) The above-mentioned steps are repeated for a sufficient number of times to accumulate enough data in order to implement the appropriate statistical analysis. In all the applications, the method constructs a stochastic model in which the expected value of the variable (or a combination of several variables) is equivalent to the value of physical quantities to be determined. The expected value is determined by the mean value of multiple mutually independent samples representing the given random variable. To construct a series of independent samples, randomly generated numbers are used that follow the distribution of the determined variables. The number of photon history that is required depends on the number of recorded data, the parameters and distributions that describe the scattering in the medium and the required precision of the monitored values. Depending on the problem which simulates, there is a variety of techniques to reduce the required number of photons. These methods are called variance reduction techniques.

In all the applications of Monte Carlo methods a stochastic model is constructed in which the expected value of some random variable (or a combination of several variables) is equivalent to the value of the physical size to be specified. This expected value estimates the average number of independent samples representing the random variable. For the construction of a number of independent samples the random numbers are used that follow the distribution of the variables, which are to be evaluated.

This work presents the model of photon by Monte Carlo method in an infinite homogeneous medium, but the model is easily applied to multi-layer media. For inhomogeneous media, as well as for semi-infinite media (in which the photons are considered lost if coming out of medium at its upper limit), the boundaries should be considered [28]. The problem is solved by using an infinitely small point source (analytically presented as a Dirac delta function in space and time). The response to an arbitrary geometry source can be constructed by Green's functions (or convolution, if there is a spatial symmetry). The required parameters are the absorption coefficient, scattering coefficient and scattering phase function. If the layers in the medium are well-defined, we must take into account the refractive index of each layer in the medium.

In our simplified model the following variance reduction techniques are used to reduce the computing time and to improve the efficiency of Monte Carlo simulations. This technique enables propagating a large amount of equivalent photons simultaneously as a package along the road. Any number of variables can be traced along the way, depending on the particular application. Each photon packet will experience multiple successive absorptions and/or scatterings until it is extinguished (fully absorbed), reflected or transmitted outside the media. Any number of photon packets can be launched and modified until the simulated test result has the desired signal to the noise ratio. It should be kept in mind that the Monte Carlo modelling is a process that includes statistics of random numbers, for which the variable ξ will be used as a pseudorandom number in our calculations. A detailed description of the model and the associated computer program used in the research can be found in D. Modrić's dissertation [29].

In order to simulate the transport of light in the substrate using the Monte Carlo approach, photon packets are sent to a random walk through a sample of virtual paper. Random number generators were used for the selection of step size, deflection angle and azimuthal angle for sampling from known probability density functions. The history of photon packets (their movement) is recorded until a package either leaves the medium or is absorbed in the interaction with it. A photon can leave the medium, in our case the paper, at the lower limit (light transmission) or at the upper limit. Consequently, the information on subsurface transport of light has been obtained.

5 Parameters of the paper model

If the paper itself varies in lightness or colour, this variation will be more or less visible even on prints. Most inks besides black are transparent, and even black prints will show paper reflectance in the screened halftone areas. However, since most papers do not vary very much in reflectance, this is not a dominating reason for print mottle, except for dots and specks in some mechanical or recycled grades. A special case is the use of FWAs (fluorescent whitening agents) that are inhomogeneously dispersed in the paper structure and can thus create mottle in the presence of UV-containing illumination.

Table 1 Numerical data for one of our "papers" consisting of five components: filler, mechanical pulp, chemical pulp, sizing agent (starch) and air

Component	Weight ratio, %	Asymmetry factor (g)	Scattering coefficient μ_s / m^2/kg				Absorption coefficient μ_a / m^2/kg			
			Wavelength λ / nm				Wavelength λ / nm			
			400	500	600	700	400	500	600	700
Filler	10	0,7	25	25	25	25	0,5	0,5	0,5	0,5
Mechanical pulp	10	0,5	25	75	70	70	29	6	1	0,5
Chemical pulp	59	0,75	25	108	115	110	29	6	1	0,5
Surface sizing agent	9	0,02	30	30	30	30	0,02	0,02	0,02	0,02
Air	12	0	0	0	0	0	0	0	0	0

The following results are calculated with some typical parameters, which have been found through the analysis of the literature on the typical compositions of paper, so, for our needs, we have to take compromise paper composition that meets the physical picture, and at the same time is simple enough in the sense that does not consume excessive computer time. In our calculations we used the "paper" consisting of mechanical and chemical pulp, fillers, surface sizing and air (although the number of components was not limited by the software) in the amounts represented as weight ratios shown in Tab. 1 with the corresponding absorption σ_a and scattering σ_t coefficients, and asymmetry factor g. The composition used in our paper model gave fairly good results, as demonstrated by the modelled reflectance profiles of paper prints [30].

The refractive index (n) of the surface sizing layer was chosen to be 1,47, which corresponds to starch.

6 Experiment

The beam of photon packets was perpendicular to the surface of the paper and counted a total of 20×10^6 photons. In reality there are many more photons, but in our case this number is again a compromise between realistic description and consumption of computer time. Increasing the number of photons would reduce the noise in the reflection from the surface of the paper. At the same time, reflectance line profile would not be significantly changed, but it would significantly increase the computer time consumption.

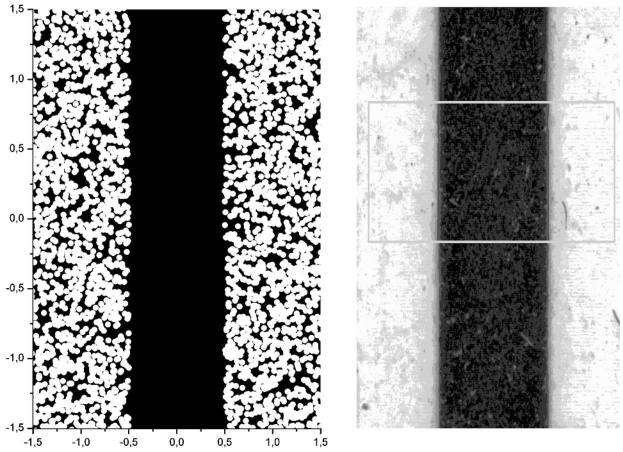

Figure 1 Illustration of random distribution of photon packets entrance over paper surface and actual printed line (d = 1 mm). The illustration was made with 2000 photon packets. Gray square on left image represents ROI (region of interest) for PIAS image analyser

The reason why we took the line as a basis for comparison of our model with the real situation lies in the fact that line is one of the basic geometric shapes used in the screen reproduction. The importance of the quality of printed lines can be seen in the fact that the line appears frequently in the business graphics as an important element of tables, graphs, images, and of course in technical illustrations. Its quality is strongly correlated with the quality of the text, as are many common desired characteristics of lines and text: density, sharpness, edge quality, etc. For many techniques printing quality of the line is the measure of the basic variables of the printer, such as the consistency of the size of the droplets of ink-jet printer. These variables affect not only the quality of the line, but also other aspects of print quality, such as the quality of the text, the uniformity of areas coverage, etc. In this way, measuring the quality of printing lines can be used to predict a much more general print quality. On the other hand, the analysis of the lines can be used to determine the colour spreading ("spilling" ink over the expected edge) either on the substrate or on the already existing layers of other inks. This is a critical factor in determining the dye-media interaction and therefore the possibility of realization of the finer details in the image. All these reasons have been the motive in choosing a line pattern as a means of comparison of our model with the real situation. Another reason for taking the line as the basic element is of a mathematical nature because that problem is reduced to one-dimensional, as it will be explained below.

Verification of the calculated data was made by measuring line samples of different thickness on the test form created at the Faculty of Graphic Arts in Zagreb. For printing of test forms was used electrophotographic digital machine Indigo Turbo Stream 1000 + with previously performed calibration. Edition of printing forms on which we exerted measurements was 20 copies of each paper used. Indigo Turbo Stream 1000 + is a standard four-color printing machine with printing speed 60 cm/s with resolution of 812 dpi.

Quantitative analysis of the quality of prints printed on coated embossed and matte paper was performed using a Personal IAS image analysis device.

The preliminary calculations showed that the resulting modelled scattering also includes photons that reach below the screen element, which does not correspond to the truth, because in reality they are lost to us, as can be seen from the fact that we perceive screen element.

Therefore, it was necessary to introduce an algorithm (which we call the "filter") that rejects photons that reach below the screen element, and considers them absorbed. So far, this "filter" is such that does not include the ink profile penetrated into the substrate.

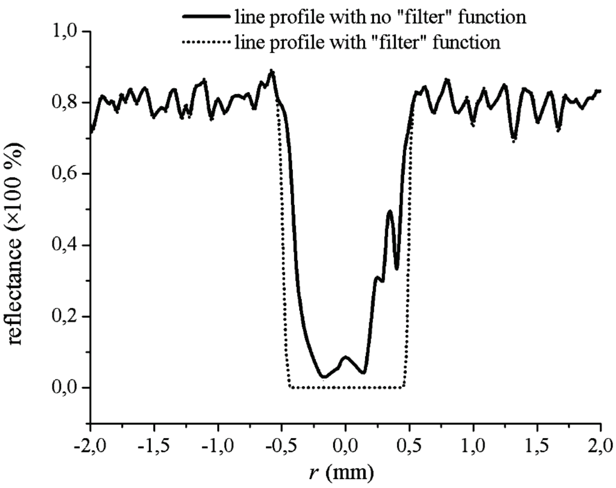

Figure 2 Figure shows the calculated profile of the above line with and without the "filter" (routine which subtracts those that finish below the line, which are lost to us).

It is important to note that it might be better to talk about the three-dimensional profile screen element. In such geometry it is natural to accept that the ink on the edges of the screen element can be in such a thin layer (due to its diffusion into the substrate) that through it some of the photons manage to penetrate.

Fig. 3 shows the modelled profile of 1,0 mm thick line with and without using the "filter" routine obtained in such a way that for every point the contribution of scattered light counted. The points on the profile are average values for the given radial distance as a result of two-dimensionality problem. However, this indicates that this approach does not provide the complete answer to the ink penetration into the substrate, which indicates the complexity of the approach to the problem solving effect of ink penetration in substrate. Specifically, modelling vertical penetration ink requires 3D convolution and detailed knowledge of the three-dimensional geometry of penetrated ink. Such modelling screen value is extremely complex, so that in our statistical model the penetration of ink into the paper is extremely simplified. The main optical effect of the ink penetration is reduced to pure absorption of photons that come under the screen element. In this way, in the model actual penetration of ink does not occur but automatically increases the number of the absorbed photons. It is important to note that, in our model, the penetration of ink into the surface does not occur, which better fits offset printing technique, with which we printed our test forms.

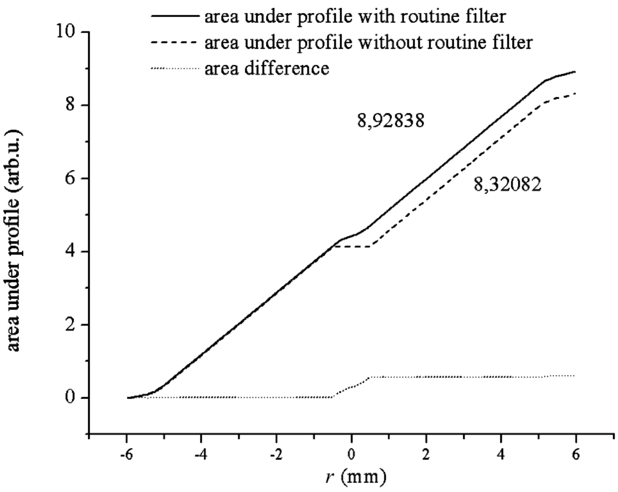

Figure 3 Graphic display of the area under the profile lines with and without the applied "filter" routines (centre of line is located at $r = 0$). The numbers next to curves are numerical values of surfaces.

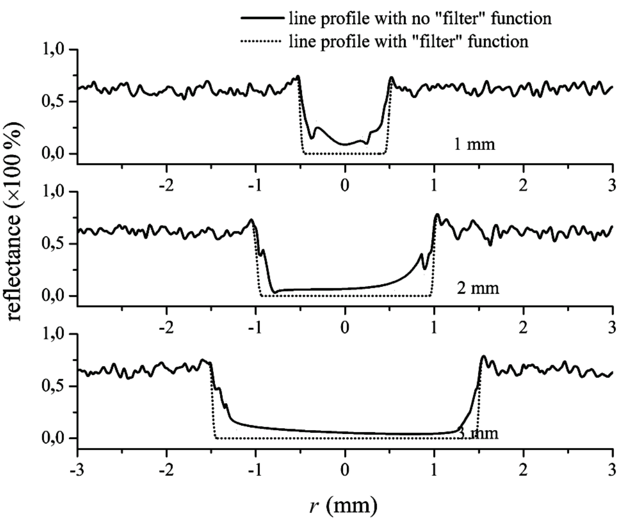

Figure 4 Comparison of the calculated profile of different line thickness with and without routine "filter"

To determine the average percentage of photons that have ended under the printed line, we compared the area under the modeled reflectances.

Accordingly, in our case we have line thickness of 1 mm and paper thickness d = 50 μm and get a value of about 7 % of photons that end under the line. These photons are lost to us and we consider them absorbed. However, this number is not a direct indicator of the size of the optical dot gain, although for this percentage the effective absorption cross section has increased, because 7 % of photon packets penetrated into the substrate, rather than the total number of incoming photon packets.

Fig. 4 shows the calculated profiles with and without the "filter" function for different thickness of the line. It is evident that the ratio of the area under each profile is closer to the unit as we increase the line width. This can be easily explained by the photons lateral scattering in the paper that has a mean free path order of 0,5 mm.

The ratio of the reflected and scattered photons is defined by Fresnell equations that must be averaged out at the angle of incidence.

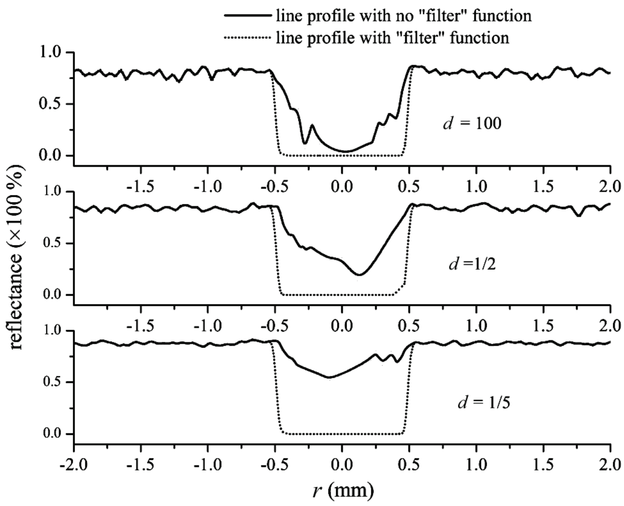

Figure 5 Comparison of the calculated profile of 1 mm thick line with and without routine "filter" for different paper thicknesses

Fig. 5 shows the comparison of the calculated profiles with and without the use of routine "filter" for different paper thicknesses. Here it is necessary to explain the fact that the thickness of the paper in the pictures is without corresponding data unit. In the calculation the dimensional structure was respected, so that all sizes are given in centimetres. To simplify, we have calculated some paper thickness as a standard (d_{actual} = 50 μm), and all other thicknesses are in fact multiple of our standard thickness. At first glance Fig. 5 shows a puzzling behaviour because it turns out that many more photons end below the line of the thinner paper than the thicker one. The reason for this lies in the fact that the thinner paper has a much higher transmission, so that the number is relative. Accordingly, the output plane of paper on which an internal reflection occurs is much closer to the line than the one for the thicker paper. The calculation of profile for d = 100 corresponds to measuring of R_∞ (R_∞ corresponds to reflectance of infinitely thick substrate) in Kubelka-Munk approach [7] should be considered. Since our model assumes that all the transmitted photons are lost forever, this means that we can say we have performed all the calculations ("measurements") with the paper on a black background. It should be noted that Kubelka-Munk method produced only one part of the optical dot gain because the calculation did not include the absorption of the substrate. This absorption plays a significant role in the appearance of the substrate itself, and can be offset by adding optically active components in the paper itself during the production.

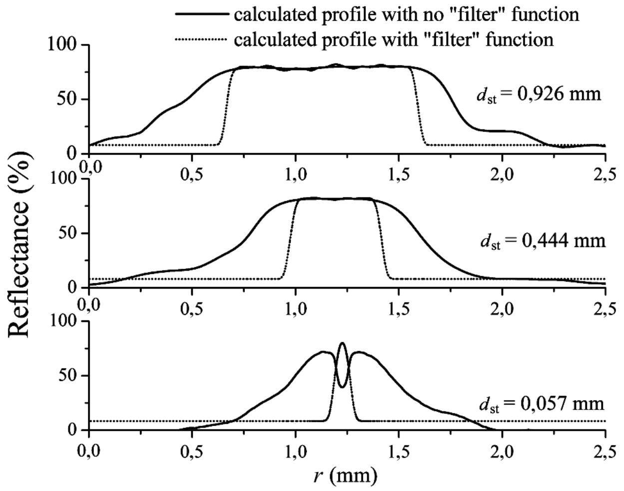

Figure 6 Comparison of the calculated profile of different lines obtained as the space between the two fields of full tone thickness with and without routine "filter"

Fig. 6 shows a much greater difference in areas (number of photons that end up below the full tone area) of the calculated profiles with and without "filter". It is completely understandable if we consider photons absorbed if they get below the surface on which ink is applied. Considering that here we have much larger surface area on which we have a full tone, compared to the surface line realized by applying a layer of ink, it is logical that more photons end up underneath them.

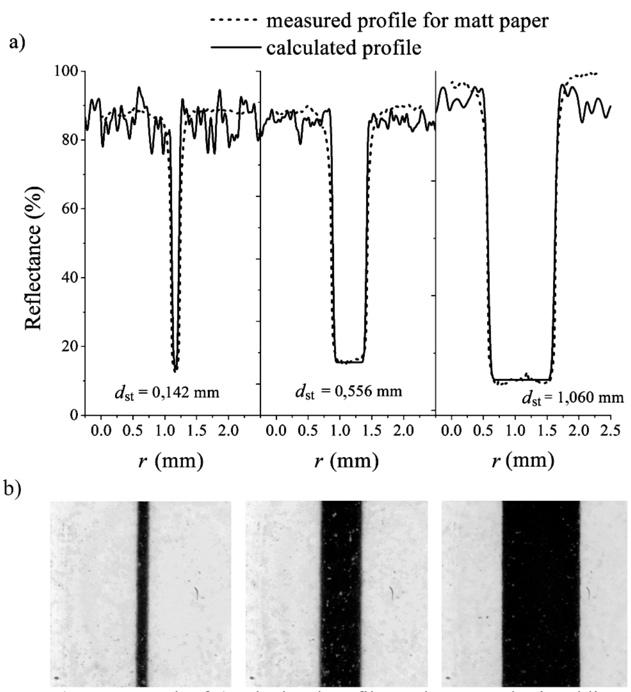

Figure 7 Result of a) calculated profiles and measured printed lines of nominal thickness d = 0,1 mm, 0,5 mm and 1,0 mm; b) images of printed lines obtained by PIAS image analyser

As can be seen from Figs. 7 and 8 modelled profiles exhibit fairly good agreement with measured ones although the "filter" function does not fully satisfy due to physical assumptions under which the "filter" is realized. It should be noted that the agreement between the calculated and measured profiles is better for a wider line, i.e. for those lines for which the ratio of line width and depth of penetration is higher, even though matching was satisfactory for narrow lines. The conclusion is valid only for printed lines. In the case of the line we get as a "gap" between the two full tone fields (i.e. "negative" of lines in Fig. 7), the discrepancy is greater for thinner lines. Fluctuations that are observed in all modelled profiles in paper region, i.e. the maximum reflection area are the result of stochastic noise that is caused by a number of incoming photons (higher number gives smaller fluctuations) and an algorithm that makes the line smoother.

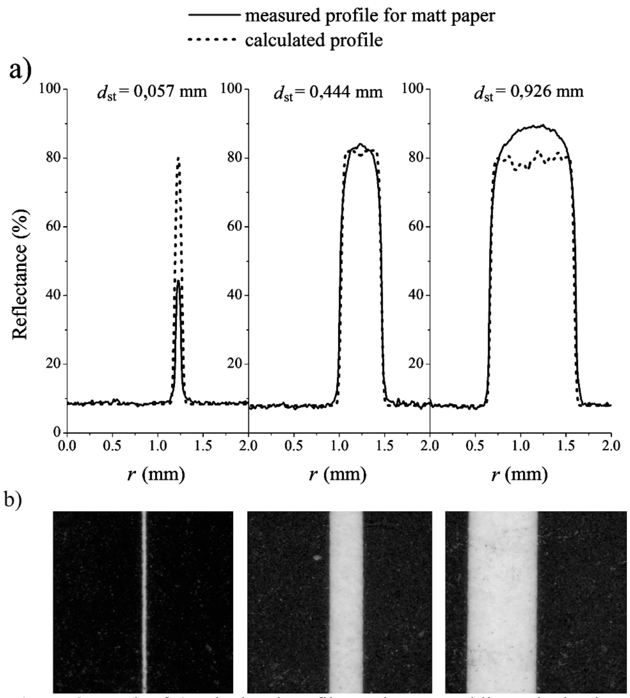

Figure 8 Result of a) calculated profiles and measured lines obtained as the space between the two fields of full tone of nominal thickness d = 0,1 mm, 0,5 mm and 1,0 mm; b) images of printed lines obtained by PIAS image analyser

7 Conclusion

In all applications of the Monte Carlo method, a stochastic model is constructed in which the expected value of a certain random variable (or of a combination of several variables) is equivalent to the value of a physical quantity to be determined. This expected value is then estimated by the average of multiple independent samples representing the random variable introduced above. For the construction of the series of independent samples, random numbers following the distribution of the variable to be estimated are used. Main advantage of the Monte Carlo model is that there is no limitation concerning boundary conditions or spatial localization of inhomogeneities in the substrate which gives great flexibility. On the other hand main disadvantage lies in the problem of getting good statistics, particularly if the attraction is located far away from the point of entry of the light and the scattering and absorption coefficients are high, causing a large consumption of computing resources. The Monte Carlo approximation is not in itself a model of the light scattering properties of paper. In order to run a simulation, the particles building a paper layer, their distributions within the sheet, and the physical models of their interaction with light must be represented by parameterised functions, whose parameters are then determined by direct or indirect measurements (parameter estimation). The model is defined so that the optimisation problem is not underdetermined, since this would lead to several solutions that are only valid for the set of samples used for the parameter estimation. Other measurable quantities that are not directly related to light scattering must also be taken into account for model consistency. There are some limitations on Monte Carlo method such as that it is based on macroscopic optical properties that are assumed to extend uniformly over the substrate volume.

The printed dots appear bigger than the dots in the original digital bitmap. This is partly because of the spreading and penetrating of the ink on and in the paper, called physical dot gain, and partly because of the diffusion of the light in paper, which is referred to as optical dot gain. The optical dot gain follows the physical dot (screen) shape (including the physical dot gain) and not the dot (screen) shape in the original bitmap. One of the most important factors that cause optical dot gain is the structure of the substrate. How the structure of paper affects dot gain is included in the Monte Carlo simulation using scattering and absorption coefficients, asymmetry coefficient g, and volume percentage representation of each component. With this model it is also possible to estimate the amount of photons that ends under the screen element, which in turn is a conditional measure of optical dot gain. This measure was obtained by introducing a routine named "filter" in Monte Carlo simulation. Therefore, it was necessary to introduce an algorithm (which we call the "filter") which discards photons that reach below the screen element, and considers them absorbed. For now, this "filter" is such that does not include the colour profile penetrated into the substrate. It is important to note that it might be better to talk about the three-dimensional profile of the screen element, because that can lead to penetration of dye into the substrate so the same can lead to falling behind thick layers of dye on the surface of the substrate. In such geometry it is natural to accept that the colour on the edges can be in so thin layer (due to its diffusion into the substrate) that however some of the photons manage to penetrate through it. Next steps in the development of our program will go in the direction of better knowing the physics of creation of the screen element on the surface of the substrate and its interaction with the ink (the definition of the edge of the screen element). Of course the "real filter" function is function of light wavelength and colour of the ink. We did not consider this dependency due to the lack of available data on the dependence of all parameters taken into account, on the wavelength.

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