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The New Normalized Subband Adaptive Filter Algorithms with Variable Step-Size

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Original scientific paper

This paper presents a new variable step-size normalized subband adaptive filter (VSS-NSAF) algorithm. In the proposed VSS-NSAF, the step-size changes in order to have largest decrease in the mean square deviation (MSD) for sequential iterations. To reduce the computational complexity of VSS-NSAF, the variable step-size selective partial update normalized subband adaptive filter (VSS-SPU-NSAF) is proposed. In this algorithm the filter coefficients are partially updated in each subband at every iteration. Simulation results show the good performance of the proposed algorithms in convergence speed and steady-state MSD.

Key words: Adaptive filter, Variable step-size (VSS), Selective partial update (SPU), Subband adaptive filter (SAF)

Novi normirani pojasni adaptivni filtar s promjenjivom duljinom koraka. U ovom radu prikazan je novi algoritam za normirani adaptivni filtar s promjenjivim korakom. Kod predloženog filtra, veličina koraka mijenja se kako bi se dobilo najveće smanjenje srednje vrijednosti odstupanja za uzastopne iteracije. Kako bi se smanjila računska složenost filtra, predložen je normirani pojasni adaptivni filtar s promjenjivim korakom i selektivnim parcijalnim osvježavanjem. Kod tog algortima koeficijenti filtra parcijalno se osvježavaju u svakom pojasu i pri svakoj iteraciji. Simulacijski rezultati pokazuju dobru brzinu konvergencije i malu srednju vrijednost odstupanja u stacionarnom stanju za predloženi filtar.

Ključne riječi: adaptivni filtar, promjenjiva duljina koraka, selektivno parcijalno osvježavanje, pojasni adaptivni filtar

Adaptive filtering has been, and still is, an area of active research that plays an active role in an ever increasing number of applications, such as noise cancellation, channel estimation, channel equalization and acoustic echo cancellation [1], [2], [3], [4], [5]. The least mean squares (LMS) and its normalized version (NLMS) are the workhorses of adaptive filtering. In the presence of colored input signals, the LMS and NLMS algorithms have extremely slow convergence rates. Adaptive filtering in subbands has been proposed to improve the convergence behavior of the LMS algorithm [6]. The normalized subband adaptive filter (NSAF) was proposed in [7]. In [8], the selective partial update NSAF (SPU-NSAF) was proposed to reduce the computational complexity. In this algorithm, the filter coefficient are partially updated in each subband at every iteration. This feature leads to the reduction in computational complexity.

In above mentioned algorithms, the selected fixed step-size can change the convergence and the steady-state mean square error (MSE). It is well known that the steady-state MSE decreases when the step-size decreases, while the convergence speed increases when the step-size increases. By optimally selecting the step-size during the adaptation, we can obtain both fast convergence rates and low steady-state MSE. In [9], a new variable step-size NLMS (VSS-NLMS) algorithm was proposed. In this algorithm, the step-size changes to obtain the largest decrease in MSD during the iterations [9]. In this paper, we extend the approach in [9] to NSAF, and SPU-NSAF algorithms and VSS version of these algorithms are proposed. We demonstrate the good performance of the presented algorithms through several simulation results in a system identification scenario. We have organized our paper as follows. In Section 2, we briefly review NSAF, and SPU-NSAF algorithms. In Section 3, the proposed VSS adaptive algorithms is established. Finally, before concluding the paper, we demonstrate the usefulness of these algorithms by presenting several experimental results.

Throughout the paper, the following notations are used:

$\vert . \vert$	Norm of a scalar.
$\Vert . \Vert^2$	Squared Euclidean norm of a vector.
$\Vert \mathbf{t} \Vert^2_{\mathbf{\Sigma}}$	$\mathbf{\Sigma}$ Weighted Euclidean norm of a column vector $\mathbf{t}$ defined as $\mathbf{t}^T\mathbf{\Sigma}\mathbf{t}$.
$\mathrm{vec}(\mathbf{T})$	Creates an $M^2 \times 1$ column vector $\mathbf{t}$ through stacking the columns of the $M \times M$ matrix $\mathbf{T}$.
$\mathrm{vec}(\mathbf{t})$	Creates an $M \times M$ matrix $\mathbf{T}$ from the $M^2 \times 1$ column vector $\mathbf{t}$.
$\mathbf{A} \otimes \mathbf{B}$	Kronecker product of matrices $\mathbf{A}$ and $\mathbf{B}$.
$\mathrm{Tr}(.)$	Trace of a matrix.
$(.)^T$	Transpose of a vector or a matrix.
$\lambda_{\max}$	The largest eigenvalue of a matrix.
$\Re^+$	The set of positive real numbers.
$E\{\cdot\}$	Expectation operator.
$\mathrm{diag}(.)$	Has the same meaning as the MATLAB operator with the same name: If its argument is a vector, a diagonal matrix with the diagonal elements given by the vector argument results. If the argument is a matrix, its diagonal is extracted into a resulting vector.

1 BACKGROUND ON NSAF, AND SPU-NSAF ALGORITHMS

In this sectin we briefly review NSAF, and SPU-NSAF algorithms.

1.1 NSAF Algorithm

Fig. 1 shows the structure of NSAF [7]. In this figure, $\mathbf{f}_0, \mathbf{f}_1, \cdots, \mathbf{f}_{N-1}$, are analysis filter unit impulse responses of an N channel orthogonal perfect reconstruction critically sampled filter bank system. $x_i(n)$ and $d_i(n)$ are nondecimated subband signals. It is important to note that n refers to the index of the original sequences and k denotes the index of the decimated sequences. Similar to the NLMS algorithm, NSAF can be established by the solution of the following optimization problem

$$\min \|\mathbf{w}(k+1) - \mathbf{w}(k)\|^2 \tag{1}$$

subject to the set of N constraints imposed on the decimated filter output

$$d_{i,D}(k) = \mathbf{x}_i^T(k)\mathbf{w}(k+1) \quad for \;\; i = 0, \cdots, N-1 \tag{2}$$

where

$$\mathbf{x}_i(k) = [x_i(kN), x_i(kN-1), \cdots, x_i(kN-M+1)]^T \tag{3}$$

By solving this optimization problem based on the method of Lagrange multipliers, the filter update equation for NSAF can be stated as [7]

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \sum_{i=0}^{N-1} \frac{\mathbf{x}_i(k)e_{i,D}(k)}{||\mathbf{x}_i(k)||^2} \tag{4}$$

where $e_{i,D}(k) = d_{i,D}(k) - \mathbf{w}^T(k)\mathbf{x}_i(k)$ is the decimated subband error signal, and μ is the step size which is chosen in the range $0 < \mu < 2$ [7]. We also assumed a linear data model for the desired signal as

$$d_{i,D}(k) = \mathbf{x}_i^T(k)\mathbf{w}^o + v_{i,D}(k) \tag{5}$$

where $\mathbf{w}^o$ is the true unknown filter vector, and $v_{i,D}(k)$ is partitioned and decimated additive noise with zero mean and variance, $\sigma^2_{v_{i,D}}$. We also assume that $v(n)$ is identically and independently distributed (i.i.d.) and statistically independent of the input data $\mathbf{x}(n)$.

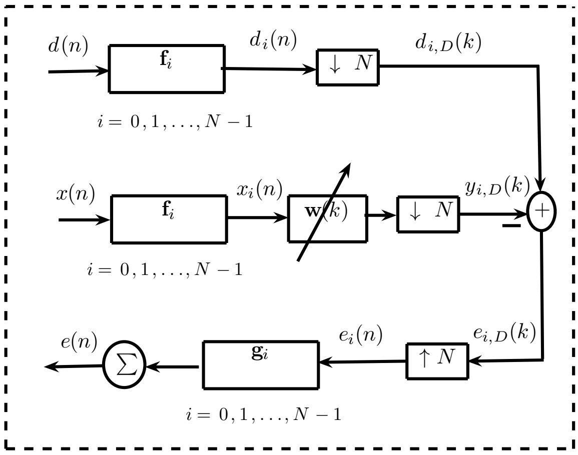

Fig. 1. Structure of NSAF algorithm.

1.2 SPU-NSAF Algorithm

To reduce the computational complexity of NSAF, SPU-NSAF algorithm was proposed in [8]. Partition $\mathbf{x}_i(k)$ for $0 \le i \le N-1$ and $\mathbf{w}(k)$ into B blocks each of length L which are defined as

$$\mathbf{x}_i(k) = [\mathbf{x}^T_{i,1}(k), \mathbf{x}^T_{i,2}(k), \ldots, \mathbf{x}^T_{i,B}(k)]^T \tag{6}$$

$$\mathbf{w}(k) = [\mathbf{w}^T_1(k), \mathbf{w}^T_2(k), \ldots, \mathbf{w}^T_B(k)]^T. \tag{7}$$

Suppose we want to update S blocks out of B blocks in each subband at every adaptation. Let $F = \{j_1, j_2, \ldots, j_S\}$ denote the indexes of the S blocks out of B blocks. In this case, the optimization problem is defined as

$$\min_{\mathbf{w}_F(k+1)} \|\mathbf{w}_F(k+1) - \mathbf{w}_F(k)\|^2, \tag{8}$$

subject to (2). By using the Lagrange multipliers approach, the filter vector update equation is given by

$$\mathbf{w}_F(k+1) = \mathbf{w}_F(k) + \mu \sum_{i=0}^{N-1} \frac{\mathbf{x}_{i,F}(k)e_{i,D}(k)}{\|\mathbf{x}_{i,F}(k)\|^2 + \epsilon} \tag{9}$$

where $\mathbf{x}_{i,F}(k) = [\mathbf{x}^T_{i,j_1}(k), \mathbf{x}^T_{i,j_2}(k), \ldots, \mathbf{x}^T_{i,j_S}(k)]^T$. To reduce the computational complexity associated with the selection of the blocks to update, two alternative simplified criteria were proposed: 1) In the first approach, we compute the following values

$$\sum_{i=0}^{N-1} \|\mathbf{x}_{i,b}(k)\|^2 \ \text{ for } \ 1 \leq b \leq B. \tag{10}$$

The indexes of the set F correspond to the indexes of the S largest values of (10). 2) In the second approach, we identify a set of indexes, correspond to the S smallest values of (11) [8].

$$j = \arg \min_{1 \leq b \leq B} \{ \sum_{i=0}^{N-1} \frac{|e_{i,D}(k)|^2}{\|\mathbf{x}_{i,b}(k)\|^2} \}. \tag{11}$$

2 DERIVATION OF VSS-NSAF AND VSS-SPU-NSAF ALGORITHMS

In this section, we establish the family of VSS-NSAF algorithms based on [9].

2.1 VSS-NSAF Algorithm

By defining the weight error vector as, $\tilde{\mathbf{w}}(k) = \mathbf{w}^o - \mathbf{w}(k)$, the weight error vector update equation for NSAF algorithm is given by

$$\tilde{\mathbf{w}}(k+1) = \tilde{\mathbf{w}}(k) - \mu \sum_{i=0}^{N-1} \frac{\mathbf{x}_i(k) e_{i,D}(k)}{||\mathbf{x}_i(k)||^2} \tag{12}$$

By taking the squared Euclidean norm and expectation for both sides of (12), we obtain

$$E\|\tilde{\mathbf{w}}(k+1)\|^2 = E\|\tilde{\mathbf{w}}(k)\|^2 + \mu^2 E\left[\sum_{i=0}^{N-1} \frac{e^2_{i,D}(k)}{\|\mathbf{x}_i(k)\|^2}\right] - 2\mu \left\{ E\left[\sum_{i=0}^{N-1} \frac{e^T_{i,D}(k)\mathbf{x}^T_i(k)\tilde{\mathbf{w}}(k)}{\|\mathbf{x}_i(k)\|^2}\right]\right\} \tag{13}$$

Equation (13) can be written as

$$E\|\tilde{\mathbf{w}}(k+1)\|^2 = E\|\tilde{\mathbf{w}}(k)\|^2 - \Delta(\mu) \tag{14}$$

where

$$\Delta(\mu) = 2\mu \left\{ E\left[\sum_{i=0}^{N-1} \frac{e^T_{i,D}(k)\mathbf{x}^T_i(k)\tilde{\mathbf{w}}(k)}{\|\mathbf{x}_i(k)\|^2}\right]\right\} - \mu^2 E\left[\sum_{i=0}^{N-1} \frac{e^2_{i,D}(k)}{\|\mathbf{x}_i(k)\|^2}\right] \tag{15}$$

The optimum step size will be found with derivation of $\Delta\mu$ with respect to μ, $\frac{\partial \Delta(\mu)}{\partial \mu} = 0$,

$$\frac{\partial \Delta(\mu)}{\partial \mu} = 2\left\{ E\left[\sum_{i=0}^{N-1} \frac{e^T_{i,D}(k)\mathbf{x}^T_i(k)\tilde{\mathbf{w}}(k)}{\|\mathbf{x}_i(k)\|^2}\right]\right\} - 2\mu E\left[\sum_{i=0}^{N-1} \frac{e^2_{i,D}(k)}{\|\mathbf{x}_i(k)\|^2}\right] = 0 \tag{16}$$

Therefore

$$\mu^o(k) = \frac{E\left[\sum_{i=0}^{N-1} \frac{e^T_{i,D}(k)\mathbf{x}^T_i(k)\tilde{\mathbf{w}}(k)}{\|\mathbf{x}_i(k)\|^2}\right]}{E\left[\sum_{i=0}^{N-1} \frac{e^2_{i,D}(k)}{\|\mathbf{x}_i(k)\|^2}\right]} \tag{17}$$

From (5), $e_{i,D}(k)$ is obtained by

$$e_{i,D}(k) = \mathbf{x}^T_i(k)\tilde{\mathbf{w}}(k) + v_{i,D}(k) \tag{18}$$

Using again the approximation for $v(k)$, and neglecting the dependency of $\tilde{\mathbf{w}}(k)$ on past noises, the optimum step-size is given by

$$\mu^o(k) = \frac{E\|\tilde{\mathbf{w}}(k)\|^2_\Sigma}{E\|\tilde{\mathbf{w}}(k)\|^2_\Sigma + \sum_{i=0}^{N-1} \sigma^2_{v^2_{i,D}} E\left[\frac{1}{\|\mathbf{x}_i(k)\|^2}\right]} \tag{19}$$

where

$$E\|\tilde{\mathbf{w}}(k)\|^2_\Sigma = E\left\{\sum_{i=0}^{N-1} \frac{\tilde{\mathbf{w}}^T(k)\mathbf{x}_i(k)\mathbf{x}^T_i(k)\tilde{\mathbf{w}}(k)}{\|\mathbf{x}_i(k)\|^2}\right\} \tag{20}$$

By defining the vector $\mathbf{p}(k)$ as

$$\mathbf{p}(k) = \sum_{i=0}^{N-1} \frac{\mathbf{x}_i(k)\mathbf{x}^T_i(k)\tilde{\mathbf{w}}(k)}{\|\mathbf{x}_i(k)\|^2} \tag{21}$$

we obtain

$$\|\mathbf{p}(k)\|^2 = \sum_{i=0}^{N-1} \frac{\tilde{\mathbf{w}}^T(k)\mathbf{x}_i(k)\mathbf{x}^T_i(k)\tilde{\mathbf{w}}(k)}{\|\mathbf{x}_i(k)\|^2} \tag{22}$$

Therefore the optimum step-size becomes

$$\mu^o(k) = \frac{E\|\mathbf{p}(k)\|^2}{E\|\mathbf{p}(k)\|^2 + \sum_{i=0}^{N-1} \sigma^2_{v_{i,D}} E\left[\frac{1}{\|\mathbf{x}_i(k)\|^2}\right]} \tag{23}$$

By taking the expection from both sides of (21), and using (18), the following relation is obtained

$$E\left[\mathbf{p}(k)\right] = E\left[\sum_{i=0}^{N-1} \frac{\mathbf{x}_i(k)e_{i,D}(k)}{\|\mathbf{x}_i(k)\|^2}\right] \tag{24}$$

Motivated by these facts, we propose to estimate $\mathbf{p}(k)$ by time averaging as follows:

$$\hat{\mathbf{p}}(k) = \alpha\hat{\mathbf{p}}(k-1) + (1-\alpha)\sum_{i=0}^{N-1}\frac{\mathbf{x}_i(k)e_{i,D}(k)}{\|\mathbf{x}_i(k)\|^2} \quad (25)$$

with a smoothing factor $0 < \alpha < 1$. Using $\|\hat{\mathbf{p}}(k)\|^2$ instead of $E\|\mathbf{p}(k)\|^2$ in (23), the proposed variable step-size for NSAF algorithm becomes

$$\mu(k) = \mu_{max}\frac{\|\hat{\mathbf{p}}(k)\|^2}{\|\hat{\mathbf{p}}(k)\|^2 + C} \quad (26)$$

where C is a positive constant and is related to $\sum_{i=0}^{N-1}\sigma_{v_{i,D}}^2 E\left[\frac{1}{\|\mathbf{x}_i(k)\|^2}\right]$. This parameter can be estimated by $C \approx \frac{N}{M.SNR}$[1]. Also, μ_{max} is introduced in (26) to guarantee the stability bound of VSS-NSAF algorithm[2]. Table I summarizes the VSS-NSAF algorithm.

2.2 VSS-SPU-NSAF Algorithm

By defining $\tilde{\mathbf{w}}_F(k) = \mathbf{w}_F^o - \mathbf{w}_F(k)$, where $\mathbf{w}_F^o = [\mathbf{w}_{j_1}^{o^T}, \mathbf{w}_{j_2}^{o^T}, \ldots, \mathbf{w}_{j_S}^{o^T}]^T$, equation (9) can be stated as

$$\tilde{\mathbf{w}}_F(k+1) = \tilde{\mathbf{w}}_F(k) - \mu\sum_{i=0}^{N-1}\frac{\mathbf{x}_{i,F}(k)e_{i,D}(k)}{||\mathbf{x}_{i,F}(k)||^2} \quad (27)$$

Taking the squared Euclidean norm and expectation from both sides of (27) leads to

$$E\|\tilde{\mathbf{w}}_F(k+1)\|^2 = E\|\tilde{\mathbf{w}}_F(k)\|^2 - 2\mu\left\{E\left[\sum_{i=0}^{N-1}\frac{e_{i,D}^T(k)\mathbf{x}_{i,F}^T(k)\tilde{\mathbf{w}}_F(k)}{\|\mathbf{x}_{i,F}(k)\|^2}\right]\right\} + \mu^2 E\left[\sum_{i=0}^{N-1}\frac{e_{i,D}^2(k)}{\|\mathbf{x}_{i,F}(k)\|^2}\right] \quad (28)$$

Equation (28) can be represented as

$$E\|\tilde{\mathbf{w}}_F(k+1)\|^2 = E\|\tilde{\mathbf{w}}_F(k)\|^2 - \Delta(\mu) \quad (29)$$

where

$$\Delta(\mu) = 2\mu\left\{E\left[\sum_{i=0}^{N-1}\frac{e_{i,D}^T(k)\mathbf{x}_{i,F}^T(k)\tilde{\mathbf{w}}_F(k)}{\|\mathbf{x}_{i,F}(k)\|^2}\right]\right\} - \mu^2 E\left[\sum_{i=0}^{N-1}\frac{e_{i,D}^2(k)}{\|\mathbf{x}_{i,F}(k)\|^2}\right] \quad (30)$$

[1]Appendix A presents an approximation for C.

[2]The stability bounds of NSAF, and SPU-NSAF have been presented in Appendix B.

The optimum step size for SPU-NSAF will be found with derivation of $\Delta\mu$ with respect to μ, $\frac{\partial\Delta(\mu)}{\partial\mu} = 0$

$$\frac{\partial\Delta(\mu)}{\partial\mu} = 2\left\{E\left[\sum_{i=0}^{N-1}\frac{e_{i,D}^T(k)\mathbf{x}_{i,F}^T(k)\tilde{\mathbf{w}}_F(k)}{\|\mathbf{x}_{i,F}(k)\|^2}\right]\right\} - 2\mu E\left[\sum_{i=0}^{N-1}\frac{e_{i,D}^2(k)}{\|\mathbf{x}_{i,F}(k)\|^2}\right] = 0 \quad (31)$$

Therefore, the optimum step-size is given by

$$\mu^o(k) = \frac{\left\{E\left[\sum_{i=0}^{N-1}\frac{e_{i,D}^T(k)\mathbf{x}_{i,F}^T(k)\tilde{\mathbf{w}}_F(k)}{\|\mathbf{x}_{i,F}(k)\|^2}\right]\right\}}{E\left[\sum_{i=0}^{N-1}\frac{e_{i,D}^2(k)}{\|\mathbf{x}_{i,F}(k)\|^2}\right]} \quad (32)$$

Using the approximation for $e_{i,D}(k)$ as $e_{i,D}(k) = \mathbf{x}_{i,F}^T(k)\tilde{\mathbf{w}}_F(k) + v_{i,D}(k)$, the optimum step-size can be stated as

$$\mu^o(k) = \frac{E\|\tilde{\mathbf{w}}_F(k)\|_\Sigma^2}{E\|\tilde{\mathbf{w}}_F(k)\|_\Sigma^2 + \sum_{i=0}^{N-1}\sigma_{v_{i,D}^2}^2 E\left[\frac{1}{\|\mathbf{x}_{i,F}(k)\|^2}\right]} \quad (33)$$

where

$$E\|\tilde{\mathbf{w}}_F(k)\|_\Sigma^2 = E\left\{\sum_{i=0}^{N-1}\frac{\tilde{\mathbf{w}}_F^T(k)\mathbf{x}_{i,F}(k)\mathbf{x}_{i,F}^T(k)\tilde{\mathbf{w}}_F(k)}{\|\mathbf{x}_{i,F}(k)\|^2}\right\} \quad (34)$$

Now by defining $\mathbf{p}(k)$ as

$$\mathbf{p}(k) = \sum_{i=0}^{N-1}\frac{\mathbf{x}_{i,F}(k)\mathbf{x}_{i,F}^T(k)\tilde{\mathbf{w}}_F(k)}{\|\mathbf{x}_{i,F}(k)\|^2} \quad (35)$$

we obtain

$$\|\mathbf{p}(k)\|^2 = \sum_{i=0}^{N-1}\frac{\tilde{\mathbf{w}}_F^T(k)\mathbf{x}_{i,F}(k)\mathbf{x}_{i,F}^T(k)\tilde{\mathbf{w}}_F(k)}{\|\mathbf{x}_{i,F}(k)\|^2} \quad (36)$$

Therefore, the optimum step-size becomes

$$\mu^o(k) = \frac{E\|\mathbf{p}(k)\|^2}{E\|\mathbf{p}(k)\|^2 + \sum_{i=0}^{N-1}\sigma_{v_{i,D}}^2 E\left[\frac{1}{\|\mathbf{x}_{i,F}(k)\|^2}\right]} \quad (37)$$

Taking the expectation from both sides of (35), the following relation is obtained

$$E\left[\mathbf{p}(k)\right] = E\left[\sum_{i=0}^{N-1}\frac{\mathbf{x}_{i,F}(k)e_{i,D}(k)}{\|\mathbf{x}_{i,F}(k)\|^2}\right] \quad (38)$$

The vector $\mathbf{p}(k)$ is estimated by time averaging as follows:

$$\hat{\mathbf{p}}(k) = \alpha\hat{\mathbf{p}}(k-1) + (1-\alpha)\sum_{i=0}^{N-1}\frac{\mathbf{x}_{i,F}(k)e_{i,D}(k)}{\|\mathbf{x}_{i,F}(k)\|^2} \quad (39)$$

Using $\|\hat{\mathbf{p}}(k)\|^2$ instead of $E\|\mathbf{p}(k)\|^2$ in (37), the proposed variable step-size for SPU-NSAF algorithm becomes

$$\mu(k) = \mu_{max} \frac{\|\hat{\mathbf{p}}(k)\|^2}{\|\hat{\mathbf{p}}(k)\|^2 + C} \tag{40}$$

where C is a positive constant and is related to $\sum_{i=0}^{N-1} \sigma^2_{v_{i,D}} E\left[\frac{1}{\|\mathbf{x}_{i,F}(k)\|^2}\right]$. This parameter can be approximated as $C \approx \frac{N}{SL.SNR}$. Table II summarizes the VSS-SPU-NSAF algorithm.

Table 1. Summary of VSS-NSAF algorithm

For $k = 0, 1, 2, \ldots$

$$e_{i,D}(k) = d_{i,D}(k) - \mathbf{w}^T(k)\mathbf{x}_i(k)$$
$$\hat{\mathbf{p}}(k) = \alpha\hat{\mathbf{p}}(k-1) + (1-\alpha)\sum_{i=0}^{N-1} \frac{\mathbf{x}_i(k)e_{i,D}(k)}{\|\mathbf{x}_{i,F}(k)\|^2}$$
$$\mu(k) = \frac{\|\hat{\mathbf{p}}(k)\|^2}{\|\hat{\mathbf{p}}(k)\|^2 + C}$$
$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu(k)\sum_{i=0}^{N-1} \frac{\mathbf{x}_i(k)e_{i,D}(k)}{\|\mathbf{x}_i(k)\|^2 + \epsilon}$$

end

Table 2. Summary of VSS-SPU-NSAF algorithm

For $k = 0, 1, 2, \ldots$

$$e_{i,D}(k) = d_{i,D}(k) - \mathbf{w}^T(k)\mathbf{x}_i(k)$$
$$\hat{\mathbf{p}}(k) = \alpha\hat{\mathbf{p}}(k-1) + (1-\alpha)\sum_{i=0}^{N-1} \frac{\mathbf{x}_{i,F}(k)e_{i,D}(k)}{\|\mathbf{x}_{i,F}(k)\|^2}$$
$$\mu(k) = \frac{\|\hat{\mathbf{p}}(k)\|^2}{\|\hat{\mathbf{p}}(k)\|^2 + C}$$
$$\mathbf{w}_F(k+1) = \mathbf{w}_F(k) + \mu(k)\sum_{i=0}^{N-1} \frac{\mathbf{x}_{i,F}(k)e_{i,D}(k)}{\|\mathbf{x}_{i,F}(k)\|^2 + \epsilon}$$

end

3 COMPUTATIONAL COMPLEXITY

Table III shows the number of multiplications, divisions, and comparisons of different adaptive algorithms. The computational complexity of NSAF for each input sampling period is exactly $3M + 3NK + 1$ multiplications and 1 division, where K is the length of the channel filters of the analysis filter bank, M is the number of filter coefficients, and N is the number of subbands. SPU-NSAF needs $2M + SL + 3NK + 1$ multiplications, 1 division, and $O(B) + Blog_2(S)$ comparisons when using the heapsort algorithm [10]. The proposed VSS-NSAF needs $3M$ multiplications and 1 division more than conventional NSAF. Using SPU approach in VSS-NSAF leads to the reduction in number of multiplications. The number of multiplications is $2M + 4SL + 3NK + 1$ in this algorithm. The VSS-SPU-NSAF algorithm needs also 2 divisions and $O(B) + Blog_2(S)$ comparisons.

4 SIMULATION RESULTS

We demonstrate the performance of the proposed algorithm by several computer simulations in a system identification scenario. In first simulation, we use the real acoustic impulse response with length $M = 256$ as shown in Fig. 2 (a) [11]. The same length is used for the adaptive filter. The colored Gaussian signal is used for the input signal. The input signal is obtained by filtering a white, zero-mean and unit variance Gaussian random sequence through a second-order auto regressive (AR(2)) system with transfer function $T(z) = \frac{1}{1-0.1z^{-1}-0.8z^{-2}}$. The filter bank used in NSAF was the four subband extended lapped transform (ELT) [12]. The white zero-mean Gaussian noise was added to the filter output such that the $SNR = 30dB$. In all simulations, we show the normalized MSD, $E\|\mathbf{w}^o - \mathbf{w}(k)\|^2/\|\mathbf{w}^o\|^2$ which is evaluated by ensemble averaging over 20 independent trials. Also, we assume that the noise variance, σ^2_v, is known a priori [13]. For all simulations we consider $\alpha = 0.99$, and $C = 10^{-5}$. The parameter μ_{max} was set to 1. Table IV shows the values of the parameters in simulations. In the case of VSS-SPU-NSAF with $S = 2$, the parameter μ_{max} was set to 0.8. Fig. 3 compares the convergence of NSAF algorithm with the proposed VSS-NSAF when the real unknown impulse response should be identified. In NSAF, different step sizes ($1, 0.2$ and 0.05) were chosen. As we can see, the proposed VSS-NSAF has both fast convergence and low steady-state MSD features compared with ordinary NSAF. Fig. 2 (b) shows the filter coefficients values after adaptation based on VSS-NSAF algorithm. This figure proves that the filter coefficients have been optimally converged to the impulse response of the car echo path.

Fig. 5 shows the normalized MSD curves for the proposed VSS-NSAF for $\mathbf{w}^o = e^{-j\tau}r(j),\ \ j = 0, \cdots, M-1$ where $r(j)$ is a white Gaussian random sequence with zero-mean and variance σ^2_r of 0.09. In this case, the impulse response length is $M = 200$, and the envelope decay rate τ is set to 0.04. The exponential unknown impulse response has been presented in Fig. 4 (a). The simulation results show that for low and large values for the step-size, the performance of NSAF is deviated. But the VSS-NSAF has both fast convergence speed and low steady-state MSD due to the strategy of variable step-size. Again, Fig. 4 (b) shows good estimation for the filter coefficients after adaptation.

In Fig. 7, we presented the results for random unknown impulse response in Fig. 6 (a). The parameter M is set to 50. The simulation results show that in the case of random unknown system, the performance of VSS-NSAF is again better than NSAF algorithm with different step-sizes. Fig. 6 (b) shows the filter coefficients after adaptation in this case. As we can see, the VSS-NSAF has good ability to predict the coefficients of unknown impulse response.

Table 3. Computational complexity of the Family of VSS-NSAF Algorithms

Algorithm	Multiplications	Divisions	Additional Multiplications	Comparisons
NSAF [7]	$3M + 3NK + 1$	1	-	-
SPU-NSAF [8]	$2M + SL + 3NK + 1$	1	-	$O(B) + Blog_2(S)$
Proposed VSS-NSAF	$3M + 3NK + 1$	2	$3M$	$-$
Proposed VSS-SPU-NSAF	$2M + SL + 3NK + 1$	2	$3SL$	$O(B) + Blog_2(S)$

Fig. 8 compares the MSD curves of VSS-NSAF, and VSS-SPU-NSAF algorithms when the real unknown impulse response should be identified. The number of blocks (B) was set to 4 and various values for S were selected. By increasing the parameter S, the performance of VSS-SPU-NSAF will be closed to the VSS-NSAF algorithm. Furthermore, the computational complexity of VSS-SPU-NSAF is lower than VSS-NSAF due to partial updates of filter coefficients.

Table 4. The Values of the Parameters in Simulations

Input signal	M	SNR	μ_{max}	α	C
Gaussian AR(2)	$256, 200, 50$	$30dB$	1	0.99	10^{-5}

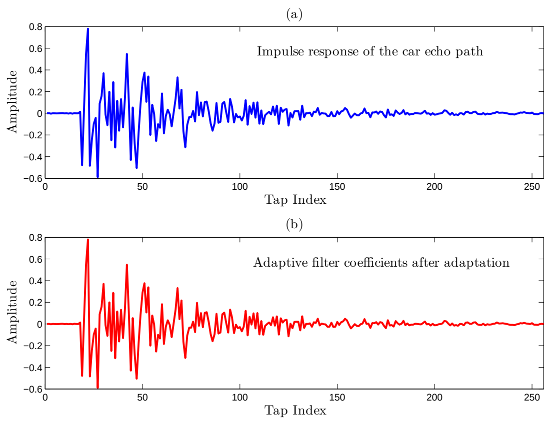

Fig. 2. (a) The impulse response of the car echo path (b) The filter coefficients after adaptation.

4.1 Simulation results for mean-square stability

Table V shows the stability bounds of SPU-NSAF algorithm for colored Gaussian input signal. These values have been obtained from Equations (61) and (63) (Appendix B). We justified these values by presenting some simulation results. Fig. 9 shows the simulated steady-state MSE curves of SPU-NSAF algorithm as a function of the step-size for

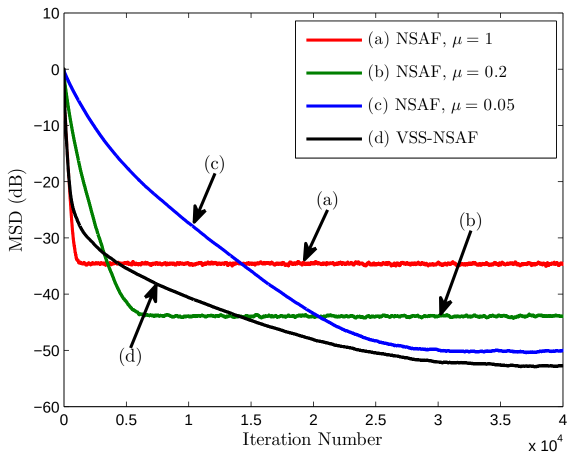

Fig. 3. The MSD curves of VSS-NSAF and conventional NSAF for real unknown impulse response.

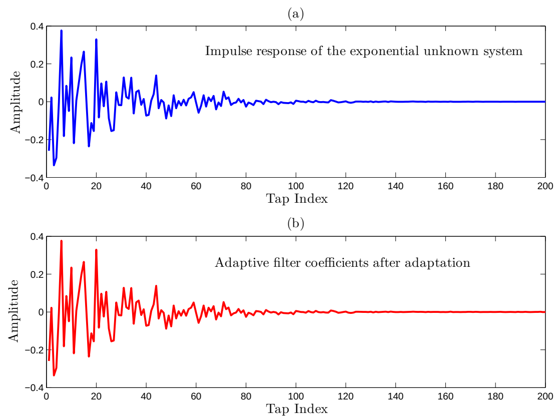

Fig. 4. (a) The impulse response of exponential unknown system (b) The filter coefficients after adaptation.

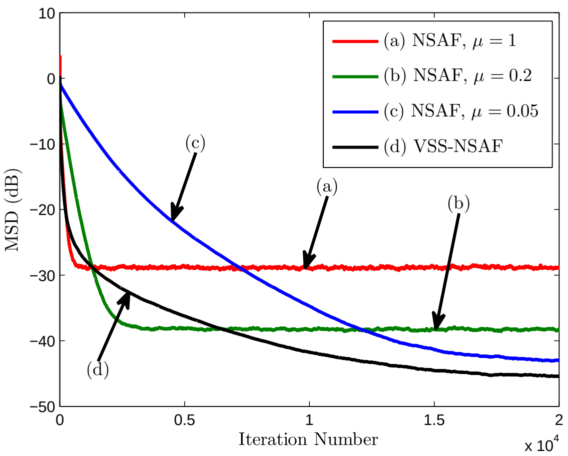

Fig. 5. The MSD curves of VSS-NSAF and conventional NSAF for exponential unknown impulse response.

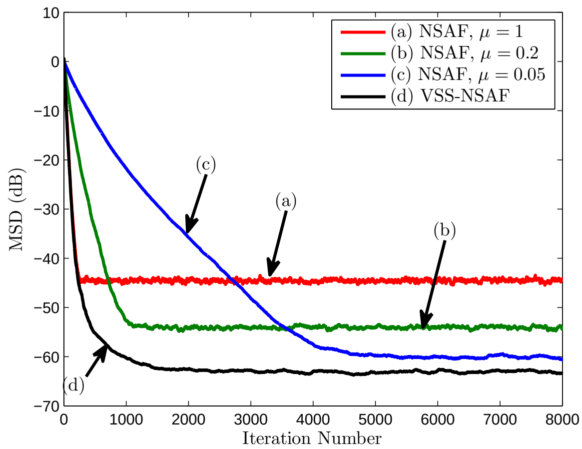

Fig. 7. The MSD curves of VSS-NSAF and conventional NSAF for random unknown impulse response.

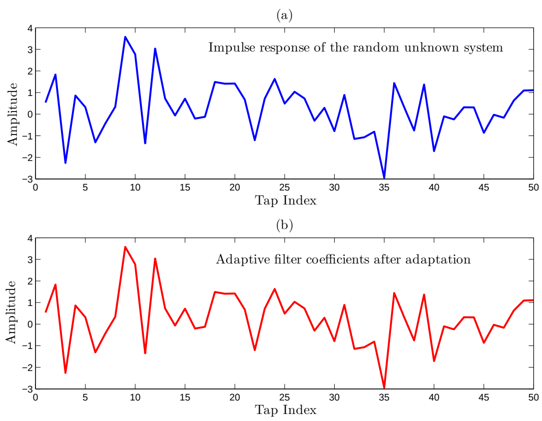

Fig. 6. (a) The impulse response of the random unknown system (b) The filter coefficients after adaptation.

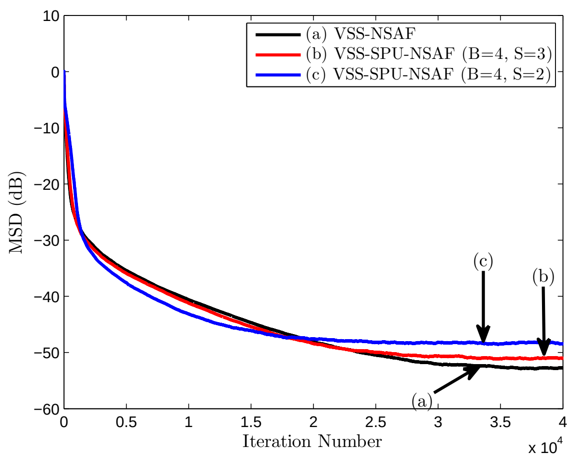

Fig. 8. The MSD curves of VSS-SPU-NSAF with B=4 and S=2, 3, and 4 for real unknown impulse response.

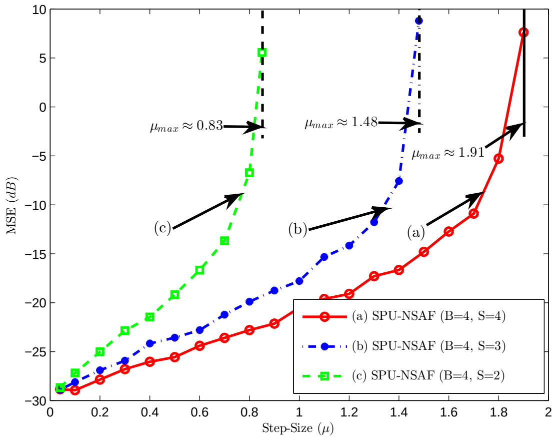

Fig. 9. Simulated steady-state MSE of SPU-NSAF algorithm with $B = 4$ and $S = 2, 3, 4$ as a function of the step-size for colored Gaussian input signal (AR(2)).

colored Gaussian input. The parameter B was set to 4 and different values for S (2, 3 and 4) were selected. The step-size changes from 0.04 to μ_{max} for each parameter adjustment. By increasing the parameter S, the stability bounds of SPU-NSAF will be increased. As we can see the theoretical values from Table V are good estimation for stability bounds of NSAF, and SPU-NSAF algorithms.

5 CONCLUSION

In this paper we presented the new variable step-size NSAF algorithm. This algorithm had fast convergence speed and low steady-state MSD compared with ordinary NSAF algorithm. To reduce the computational complexity of VSS-NSAF, the VSS-SPU-NSAF was proposed. We demonstrated the good performance of the presented VSS adaptive algorithms in system identification scenario by several simulation results.

APPENDIX A FINDING AN APPROXIMATION FOR C

In proposed VSS-NSAF, positive constant C is related to $\sum_{i=0}^{N-1} \sigma^2_{v_{i,D}} E\left[\frac{1}{\|\mathbf{x}_i(k)\|^2}\right]$. For a high-order adaptive filter, the fluctuations of $\|\mathbf{x}_i(k)\|^2$ from one iteration to the next can be assumed to be small, so the following approximation can be acceptable:

$$E\left[\frac{1}{\|\mathbf{x}_i(k)\|^2}\right] \approx \frac{1}{E\left[\|\mathbf{x}_i(k)\|^2\right]} \tag{41}$$

and

$$E\left[\|\mathbf{x}_i(k)\|^2\right] \approx \|\mathbf{x}_i(k)\|^2 \tag{42}$$

We know that $\|\mathbf{x}_i(k)\|^2 = \mathbf{x}_i^T(k)\mathbf{x}_i(k)$ and $\mathbf{x}_i^T(k)\mathbf{x}_i(k) \approx M\sigma^2_{x_i}$ for $M \gg 1$. Therefore

$$C \approx \frac{N\sigma^2_{v_{i,D}}}{M\sigma^2_{x_i}} \approx \frac{N}{M.SNR} \tag{43}$$

Following the same approach for the parameter C in VSS-SPU-NSAF algorithm leads to $C \approx \frac{N}{SL.SNR}$.

APPENDIX B MEAN-SQUARE STABILITY ANALYSIS OF THE FAMILY OF SPU-NSAF

Now, we introduce the general filter vector update equation to analyze the mean-square stability of the family of SPU-NSAF. The general filter vector update equation to establish the family of SPU-NSAF is introduced as:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\mathbf{C}(k)\mathbf{X}(k)\mathbf{Z}(k)\mathbf{e}(k). \tag{44}$$

where $\mathbf{C}(k)$, and $\mathbf{Z}(k)$ matrices are obtained from Table VI and $\mathbf{e}(k) = \mathbf{d}(k) - \mathbf{X}^T(k)\mathbf{w}(k)$. In (44), $\mathbf{F}$ is the $K \times N$ matrix whose columns are the unit responses of the channel filters of the analysis filter bank, where N is the number of subbands and K is the length of the channel filters. The matrix $\mathbf{d}(k)$ is defained as $\mathbf{d}(k) = [d(kN), d(kN-1), \ldots, d(kN-(P-1))]^T$. Also, the matrix $\mathbf{X}(k)$ is obtained by $\mathbf{X}(k) = [\mathbf{x}(kN), \mathbf{x}(kN-1), \ldots, \mathbf{x}(kN-(P-1))]$ where $\mathbf{x}(kN) = [x(kN), x(kN-1), \ldots, x(kN-M+1)]^T$. In Table V, the matrix $\mathbf{A}(k)$ is the $M \times M$ diagonal matrix with the $\mathbf{1}$ and $\mathbf{0}$ blocks each of length L on the diagonal and the positions of 1's on the diagonal determine which coefficients should be updated in each subband at every adaptation. These positions are obtained by indexes of (10) or (11). To find the theoretical stability bound, we first study the transient behavior of the adaptive algorithms. The transient behavior of an adaptive filter algorithm is determined by the evolution of the expected squared *a priori* error in time n, i.e. $E\{e_a^2(k)\}$, which is:

$$E\{e_a^2(k)\} = E\{\tilde{\mathbf{w}}^T(k)\mathbf{x}(k)\mathbf{x}^T(k)\tilde{\mathbf{w}}(k)\}. \tag{45}$$

where $\tilde{\mathbf{w}}(k) = \mathbf{w}^\circ - \mathbf{w}(k)$ is the weight-error vector. Employing the common *independence assumption* [2], we have:

$$E\{e_a^2(k)\} = E\{\tilde{\mathbf{w}}^T(k)\mathbf{R}\tilde{\mathbf{w}}(k)\} = E\{\|\tilde{\mathbf{w}}(k)\|^2_{\mathbf{R}}\}, \tag{46}$$

where the autocorrelation matrix is $\mathbf{R} = E\{\mathbf{x}(k)\mathbf{x}^T(k)\}$. Thus, to obtain the learning curve, we need to find $E\{\|\tilde{\mathbf{w}}(k)\|^2_{\mathbf{R}}\}$ as a function of k. We can recursively obtain $E\{\|\tilde{\mathbf{w}}(k)\|^2_{\mathbf{\Sigma}}\}$, where $\mathbf{\Sigma}$ is a positive definite symmetric matrix whose dimension is commensurate with that of $\tilde{\mathbf{w}}(k)$. If we substitute (5) into $\mathbf{e}(k) = \mathbf{d}(k) - \mathbf{X}^T(k)\mathbf{w}(k)$,

Table 5. Stability bounds of SPU-NSAF algorithm with different parameters for colored Gaussian input

Algorithm	$\frac{2}{\lambda_{\max}(E\{\mathbf{D}^T(n)\mathbf{X}^T(n)\})}$	$\frac{1}{\lambda_{\max}(\mathbf{M}^{-1}\mathbf{N})}$	$\frac{1}{\max(\lambda(\mathbf{H})\in\Re^+)}$	$\mu_{\max}$
SPU-NSAF ($B=4, S=1$)	3.6393	0.2443	1.3811	0.2443
SPU-NSAF ($B=4, S=2$)	3.5167	0.8374	2.4469	0.8374
SPU-NSAF ($B=4, S=3$)	3.1530	1.4823	2.4278	1.4823
SPU-NSAF ($B=4, S=4$)	3.0538	1.9109	2.4833	1.9109

Table 6. NSAF, and SPU-NSAF Algorithms

Algorithm	$\mathbf{C}(k)$	$\mathbf{Z}(k)$
NSAF	$\mathbf{I}$	$\mathbf{F}[\epsilon\mathbf{I}+\text{diag}\{\text{diag}\{\mathbf{F}^T\mathbf{X}^T(k)\mathbf{X}(k)\mathbf{F}\}\}]^{-1}\mathbf{F}^T$
SPU-NSAF	$\mathbf{A}(k)$	$\mathbf{F}[\epsilon\mathbf{I}+\text{diag}\{\text{diag}\{\mathbf{F}^T\mathbf{X}^T(k)\mathbf{A}(k)\mathbf{X}(k)\mathbf{F}\}\}]^{-1}\mathbf{F}^T$

the relation between output estimation error vector, a priori error vector and the noise vector is:

$$\mathbf{e}(k)=\mathbf{e}_a(k)+\mathbf{v}(k) \tag{47}$$

where $\mathbf{e}_a(k)=\mathbf{X}^T(k)\tilde{\mathbf{w}}(k)$ is the a priori error vector. The generic weight error vector update equation can be stated as:

$$\tilde{\mathbf{w}}(k+1)=\tilde{\mathbf{w}}(k)-\mu\mathbf{C}(k)\mathbf{X}(k)\mathbf{Z}(k)(\mathbf{X}^T(k)\tilde{\mathbf{w}}(k)+\mathbf{v}(k)). \tag{48}$$

By defining $\mathbf{D}(k)=\mathbf{Z}^T(k)\mathbf{X}^T(k)\mathbf{C}^T(k)$, the $\mathbf{\Sigma}$ weighted norm of both sides of (48) is:

$$\begin{aligned}\|\tilde{\mathbf{w}}(k+1)\|^2_{\mathbf{\Sigma}}&=\|\tilde{\mathbf{w}}(k)\|^2_{\mathbf{\Sigma}'}+\mu^2\mathbf{v}^T(k)\mathbf{X}^{\mathbf{\Sigma}}(k)\mathbf{v}(k)\\&+\{\text{Cross terms involving one instance of }\mathbf{v}(k)\}\end{aligned} \tag{49}$$

where

$$\begin{aligned}\mathbf{\Sigma}'=\quad&\mathbf{\Sigma}-\mu\mathbf{\Sigma}\mathbf{D}^T(k)\mathbf{X}^T(k)-\mu\mathbf{X}(k)\mathbf{D}(k)\\&+\mu^2\mathbf{X}(k)\mathbf{X}^{\mathbf{\Sigma}}(k)\mathbf{X}^T(k)\end{aligned} \tag{50}$$

and

$$\mathbf{X}^{\mathbf{\Sigma}}(k)=\mathbf{D}(k)\mathbf{\Sigma}\mathbf{D}^T(k). \tag{51}$$

Taking the expectation from both sides of (49) yields:

$$\begin{aligned}E\{\|\tilde{\mathbf{w}}(k+1)\|^2_{\mathbf{\Sigma}}\}&=E\{\|\tilde{\mathbf{w}}(k)\|^2_{\mathbf{\Sigma}'}\}\\&+\mu^2E\{\mathbf{v}^T(k)\mathbf{X}^{\mathbf{\Sigma}}(k)\mathbf{v}(k)\}.\end{aligned} \tag{52}$$

We now obtain the time evolution of the weight-error variance. The expectation of $\|\tilde{\mathbf{w}}(k)\|^2_{\mathbf{\Sigma}'}$ is difficult to calculate because of the dependency of $\mathbf{\Sigma}'$ on $\mathbf{C}(k)$, $\mathbf{Z}(k)$, $\mathbf{X}(k)$, and of $\tilde{\mathbf{w}}(k)$ on prior regressors. To solve this problem, we need to use the following independence assumptions [14]:

1. $\mathbf{X}(k)$ is independent and identically distributed sequence matrix. This assumption guarantees that $\tilde{\mathbf{w}}(k)$ is independent of both $\mathbf{\Sigma}'$ and $\mathbf{X}(k)$.
2. $\tilde{\mathbf{w}}(k)$ is independent of $\mathbf{D}^T(k)\mathbf{X}^T(k)$.

Using these assumptions, the final result is

$$\begin{aligned}E\{\|\tilde{\mathbf{w}}(k+1)\|^2_{\mathbf{\Sigma}}\}&=E\{\|\tilde{\mathbf{w}}(k)\|^2_{\mathbf{\Sigma}'}\}\\&+\mu^2E\{\mathbf{v}^T(k)\mathbf{X}^{\mathbf{\Sigma}}(k)\mathbf{v}(k)\},\end{aligned} \tag{53}$$

where

$$\begin{aligned}\mathbf{\Sigma}'=\quad&\mathbf{\Sigma}-\mu\mathbf{\Sigma}E\{\mathbf{D}^T(k)\mathbf{X}^T(k)\}-\mu E\{\mathbf{X}(k)\mathbf{D}(k)\}\mathbf{\Sigma}\\&+\mu^2E\{\mathbf{X}(k)\mathbf{X}^{\mathbf{\Sigma}}(k)\mathbf{X}^T(k)\}.\end{aligned} \tag{54}$$

Looking only at the second term of the right hand side of (53) we write

$$\begin{aligned}E\{\mathbf{v}^T(k)\mathbf{X}^{\mathbf{\Sigma}}(k)\mathbf{v}(k)\}&=E\{\text{Tr}(\mathbf{v}(k)\mathbf{v}^T(k)\mathbf{X}^{\mathbf{\Sigma}}(k))\}\\&=\text{Tr}(E\{\mathbf{v}(k)\mathbf{v}^T(k)\}E\{\mathbf{X}^{\mathbf{\Sigma}}(k)\}).\end{aligned} \tag{55}$$

Since $E\{\mathbf{v}(k)\mathbf{v}^T(k)\}=\sigma_v^2\mathbf{I}$, equation (53) can be stated as

$$\begin{aligned}E\{\|\tilde{\mathbf{w}}(k+1)\|^2_{\mathbf{\Sigma}}\}&=E\{\|\tilde{\mathbf{w}}(k)\|^2_{\mathbf{\Sigma}'}\}\\&+\mu^2\sigma_v^2\text{Tr}(E\{\mathbf{X}^{\mathbf{\Sigma}}(k)\}),\end{aligned} \tag{56}$$

Applying the $\text{vec}(.)$ operator [15] on both sides of (54) yields:

$$\begin{aligned}\text{vec}(\mathbf{\Sigma}')=\quad&\text{vec}(\mathbf{\Sigma})-\mu\text{vec}(\mathbf{\Sigma}E\{\mathbf{D}^T(k)\mathbf{X}^T(k)\})\\&-\mu\text{vec}(E\{\mathbf{X}(k)\mathbf{D}(k)\}\mathbf{\Sigma})\\&+\mu^2\text{vec}(E\{\mathbf{X}(k)\mathbf{X}^{\mathbf{\Sigma}}(k)\mathbf{X}^T(k)\}).\end{aligned} \tag{57}$$

Since in general, $\text{vec}(\mathbf{P}\mathbf{\Sigma}\mathbf{Q})=(\mathbf{Q}^T\otimes\mathbf{P})\text{vec}(\mathbf{\Sigma})$ [15], equation (57) can be written as:

$$\begin{aligned}\sigma'&=\sigma-\mu(E\{\mathbf{X}(k)\mathbf{D}(k)\}\otimes\mathbf{I}).\sigma\\&-\mu(\mathbf{I}\otimes E\{\mathbf{X}(k)\mathbf{D}(k)\}).\sigma\\&+\mu^2(E\{(\mathbf{X}(k)\mathbf{D}(k))\otimes(\mathbf{X}(k)\mathbf{D}(k))\}).\sigma,\end{aligned} \tag{58}$$

where $\sigma' = \mathrm{vec}(\boldsymbol{\Sigma}')$ and $\sigma = \mathrm{vec}(\boldsymbol{\Sigma})$. By defining the $M^2 \times M^2$ matrix $\mathbf{G}$ as:

$$\mathbf{G} = \mathbf{I} - \mu E\{\mathbf{X}(k)\mathbf{D}(k)\} \otimes \mathbf{I} - \mu \mathbf{I} \otimes E\{\mathbf{X}(k)\mathbf{D}(k)\} \\ + \mu^2 E\{(\mathbf{X}(k)\mathbf{D}(k)) \otimes (\mathbf{X}(k)\mathbf{D}(k))\}, \tag{59}$$

equation (58) becomes:

$$\sigma' = \mathbf{G}.\sigma. \tag{60}$$

The second term of the right hand side of (56) is

$$\mathrm{Tr}(E\{\mathbf{X}^{\boldsymbol{\Sigma}}(k)\}) = \mathrm{Tr}(E\{\mathbf{D}^T(k)\mathbf{D}(k)\}.\boldsymbol{\Sigma}). \tag{61}$$

Defining γ as

$$\gamma = \mathrm{vec}(E\{\mathbf{D}^T(k)\mathbf{D}(k)\}), \tag{62}$$

we have:

$$\mathrm{Tr}(E\{\mathbf{D}^T(k)\mathbf{D}(k)\}.\boldsymbol{\Sigma}) = \gamma^T.\sigma. \tag{63}$$

From the above, the recursion of (56) is

$$E\{\|\tilde{\mathbf{w}}(k+1)\|^2_{\sigma}\} = E\{\|\tilde{\mathbf{w}}(k)\|^2_{\mathbf{G}\sigma}\} + \mu^2\sigma_v^2\gamma^T\sigma. \tag{64}$$

Equation (64) is stable if the matrix $\mathbf{G}$ is stable [14]. From (59), we know that $\mathbf{G} = \mathbf{I} - \mu\mathbf{M} + \mu^2\mathbf{N}$, where $\mathbf{M} = E\{\mathbf{X}(k)\mathbf{D}(k)\} \otimes \mathbf{I} + \mathbf{I} \otimes E\{\mathbf{X}(k)\mathbf{D}(k)\}$, and $\mathbf{N} = E\{(\mathbf{X}(k)\mathbf{D}(k)) \otimes (\mathbf{X}(k)\mathbf{D}(k))\}$. The condition on μ to guarantee the convergence in the mean-square sense of the adaptive algorithms is:

$$0 < \mu < \min\{\frac{1}{\lambda_{\max}(\mathbf{M}^{-1}\mathbf{N})}, \frac{1}{\max(\lambda(\mathbf{H}) \in \Re^+)}\}. \tag{65}$$

where $\mathbf{H} = \begin{bmatrix} \frac{1}{2}\mathbf{M} & -\frac{1}{2}\mathbf{N} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$. Taking the expectation from both sides of (48) yields:

$$E\{\tilde{\mathbf{w}}(k+1)\} = [\mathbf{I} - \mu E\{\mathbf{D}^T(k)\mathbf{X}^T(k)\}]E\{\tilde{\mathbf{w}}(k)\}. \tag{66}$$

From (66), the convergence to the mean of the adaptive algorithm in (44) is guaranteed for any μ that satisfies:

$$\mu < \frac{2}{\lambda_{\max}(E\{\mathbf{D}^T(k)\mathbf{X}^T(k)\})}. \tag{67}$$

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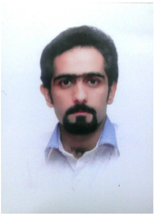

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