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COMPARISON OF THE FORMING - LIMIT DIAGRAM (FLD) MODELS FOR DRAWING QUALITY (DQ) STEEL SHEETS

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In this paper a comparative investigation of three mathematical models (Marciniak - Kuczynski model, Hill - Swift model and Sing - Rao model) as well as an empirical model proposed by the North American Deep Drawing Research Group (NADDRG) has been carried out. The yield criterion (1993), recently proposed by Hill, is used for the calculation of the limit strains in connection with the Swift's instability condition for diffuse necking and with the Marciniak - Kuczynski analysis. The emphasis of this investigation is to consider these different approaches to predicting the FLD. Experimental results have been obtained for different low carbon steel sheets of drawing quality - FeP06 G and ZStE 220P as well as rephosphorised and micro-alloyed steel. It was compared, which theoretical model showing good correlation with experiment is suitable for materials mentioned above.

Key words: *sheet steel forming, drawing quality, forming - limit diagrams (FLD), prediction models*

Usporedba oblikovno - graničnog dijagrama (FLD) modela izvlačenja čeličnih limova. U ovom radu je provedeno usporedno ispitivanje tri matematička modela (model Marciniak - Kuczynskog, Hill-Swiftov i Sing-Rao) kao i empirijskih modela kojeg je predložila Sjeverno američka grupa za istraživanje dubokog izvlačenja (NADDRG). Granični kriterij (1993.) koji je nedavno predložio Hill koristi se za izračun graničnih deformacija u vezi s Swiftovim uvjetom nestabilnosti difuzije pri sužavanju i korištenjem analize Marciniak - Kuczynski. Naglasak ovog istraživanja je u različitim pristupima predmnjevanja FLD. Dobiveni su eksperimentalni rezultati za različite niskougljične čelične limove za izvlačenje u kakvoći limova FeP06 G i ZStE 220P kao i refosforiziranih i mikrolegiranih čelika. Usporedbom, teorijski model je bio u dobroj podudarnosti s praksom.

Ključne riječi: *oblikovanje čeličnih limova, kakvoća izvlačenja, oblikovno - granični dijagram (FLD), predmnijevanje modela*

INTRODUCTION

The concept of forming limit diagrams (FLD) was introduced by Keeler (1964) and Goodwin (1968) and represents the first safety criterion for deep drawing operations. Marciniak and Kuczynski (M-K) have proposed a mathematical model for the theoretical determination of FLD that supposes an infinite sheet metal to contain a region of local imperfection where heterogeneous plastic flow develops and localizes.

The implementation of different yield criteria in the M-K model has been investigated by several authors [1 - 4]. Because of the complexity of the experimental determination of the FLD, a number of theoretical calculating models have been set up on the basis of the classical or modified Swift and Hill instability criteria [5 - 8]. In recent years, the knowledge and principles of damage mechanics, plastic mechanics of porous materials, and microscopic materials science combined with the finite-element method (FEM) have also been introduced into the theoretical predictions of the FLD [9 - 11]. These results have significantly enriched and improved the understanding and application of the FLD.

However, there has not been any general model that can be applied for various steel sheets until now and, furthermore, the still-too-complex calculations for predicting the FLD will limit their use in practical applications.

This investigation was carried out for a better understanding of the forming behaviour of selected steel sheets by means of the experimental determination and theoretical predication of the FLD.

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THEORETICAL MODELS

The Marciniak-Kuczynski model

The M-K model assumes that the strain localization appears in the region of a material or geometrical inhomogeneity. The initial groove or trough is assumed to develop when proportional loading is applied outside the groove. The force equilibrium ensures that the strain level within the groove grows faster than the strain outside, until eventually a plane strain condition is reached with in the groove [12]. At this point, the material is assumed to lose its capability for carrying additional load, and localized necking occurs. The M-K method has been used widely in predicting forming limits of sheet metals (e.g., [13 - 15]).

The model presented in this paper assumes the existence of a geometric non-homogeneity in the form of notch (zone b) perpendicular to the direction of the maximum principal stress σ_1. The initial thickness of the sheet metal t^a_0 is greater than the initial thickness in the region which contains an imperfection t^b_0 (see Figure 1.). The sheet-metal is stretched the principal stresses σ_1 and σ_2. The current value of the inhomogeneity coefficient is expressed by the relationship:

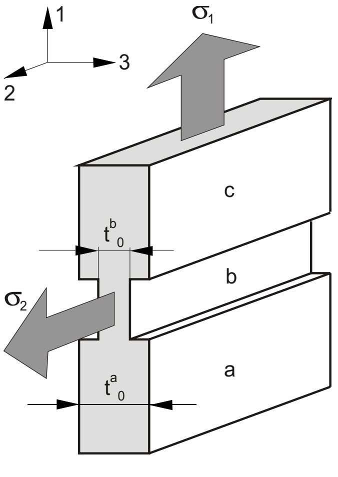

Figure 1. **Geometrical M-K model**
Slika 1. **Geometrijski model M-K**

$$f_0 = \frac{t_b}{t_a} \tag{1}$$

where t_a and t_b are the current values of the thickness in the regions a and b, respectively.

For each of the two regions of the sheet are valid the following Levy-Mises equations and Hollomon's equation, respectively.

The model is completed with two equations the link between regions a and b. Equation expressing the equilibrium of the interface of the two regions:

$$\sigma_{1a} \cdot t_a = \sigma_{1b} \cdot t_b \tag{2}$$

Equation expressing the fact that the strains parallel to the notch are equal in both regions:

$$d\varepsilon_{2a} = d\varepsilon_{2b} \tag{3}$$

The Swift-Hill model

It has been proven that a good simulation of the forming limit strains can be provided on the basis of the Swift diffuse instability theory and the Hill localized instability theory [16, 17], and where Swift's and Hill's theories are used to calculate the forming limit strains on the left and the right side, respectively, of the FLD. Assuming that the stress-strain relationship of sheets can be expressed by Hollomon's equation.

According to Swift's and Hill's criterion, the formulae calculating the forming-limit strains can be written as follows, with $\alpha = \sigma_2/\sigma_1$

- for $\varepsilon_2 < 0$:

$$\varepsilon_1 = \frac{1+(1-\alpha)r_m}{1+\alpha}n \tag{4}$$

$$\varepsilon_2 = \frac{\alpha-(1-\alpha)r_m}{1+\alpha}n \tag{5}$$

- for $\varepsilon_2 > 0$:

$$\varepsilon_1 = \frac{\left[1+r_m\left(1-\alpha\right)\right]\cdot\left[1-\frac{2r_m}{1+r}\alpha+\alpha^2\right]}{\left(1+\alpha\right)\left(1+r_m\right)\left[1-\frac{1+4r_m+2r_m^2}{\left(1+r\right)^2}\alpha+\alpha^2\right]}n \tag{6}$$

$$\varepsilon_2 = \frac{\left[\left(1+r_m\right)\alpha-r_m\right]\cdot\left[1-\frac{2r_m}{1+r}\alpha+\alpha^2\right]}{\left(1+\alpha\right)\left(1+r_m\right)\left[1-\frac{1+4r_m+2r_m^2}{\left(1+r\right)^2}\alpha+\alpha^2\right]}n \tag{7}$$

The Sing-Rao model

According to the original Sing-Rao proposition the FLSC could be obtained using the linear regression technique based on the results of calculation using below mentioned scheme taking into account mean plastic anisotropy ratio [18, 19].

On the base of flow rule the surface limit strains for different stress (or strain) ratio could be calculated as:

$$\varepsilon_1 = \left[\left(1+2r_m\right)\left(\sigma_1-\sigma_2\right)+\left(\sigma_1+\sigma_2\right)\right]\lambda \tag{8}$$

$$\varepsilon_2 = \left[-\left(1+2r_m\right)\left(\sigma_1-\sigma_2\right)+\left(\sigma_1+\sigma_2\right)\right]\lambda \tag{9}$$

where

$$\lambda = \frac{\varepsilon_e}{2\left(1+r_m\right)\sigma_e} \tag{10}$$

The NADDRG model

For simplifying the experimental and theoretical determination of the FLD and utilizing the FLD more easily in the press workshop, the North American Deep Drawing Research Group (NADDRG) introduced an empirical equation for predicting the FLD in practise. [20] According to this model, the FLD is composed of two lines through the point ε_{10} in the plane-strain state. The slopes of the lines located on the left and right side of FLD are about 45° and 20°. The equation for calculation the forming limit strain ε_{10} in term of engineering strain can be expressed as

- Keeler:

$$\varepsilon_{10} = \frac{\left(23,3+14,13t_0\right)n}{0,21} \tag{11}$$

- Bethlehem Steel:

$$\varepsilon_{10} = 2,78+3,244t_0+0,892A \tag{12}$$

where t_0 is the sheet thickness in mm, A is drawability.

EXPERIMENTAL WORK

The materials used in the present investigation are listed in Table 1. along with their mechanical properties, thickness and flow of curve parameters applied in the theoretical calculations of FLD.

Table 1. **Mechanical properties, thickness and flow curve parameters of the steels of drawing quality**

Tablica 1. **Mehanička svojstva, debljina i krivulja protoka parametara za čelike**

Steel	Thickness / mm	(R_e) YS / MPa	(R_m) UTS / MPa	A_{80} / %	R	n	A_g
DX 54D	0,8	172	305	41,8	2,385	0,215	22,0
DX 53D	1,0	232	324	36,7	1,515	0,154	15,0
ZStE220P	0,8	234	380	34,2	1,676	0,197	20,4
ZStE 340	1,25	386	483	25,6	1,076	0,165	

DX 54D G and DX 53D are low carbon deep drawable steels. Both are hot dip galvanized steel sheets. ZStE 220 P is the rephosphorised drawing quality steel. ZStE 340 is high-strength low alloy steel (HSLA). All of the steels have been produced industrially.

The experiments determining the FLD for all the sheets studied have been carried out in Erichsen 145-60 universal materials testing machine with specimens of different width and shape. Specimens were deformed by a rigid punch with hemispherical shape. The limit strains have been determined from a circular grid pattern. The flow curves have been determined by means of the conventional tensile test. In this investigation, all tests were carried out at room temperature.

RESULTS AND DISCUSSION

Comparison between the theoretical and experimental FLD

Figures 2. - 5. show the comparisons between the theoretical predictions based on the different models mentioned above and the experimental FLD, for all the steel sheets studied. Generally speaking, there exists no one model that can be used for every material. Because of adequacy examination, the FLD obtained by predicted mathematical models have been necessary compared with experiment.

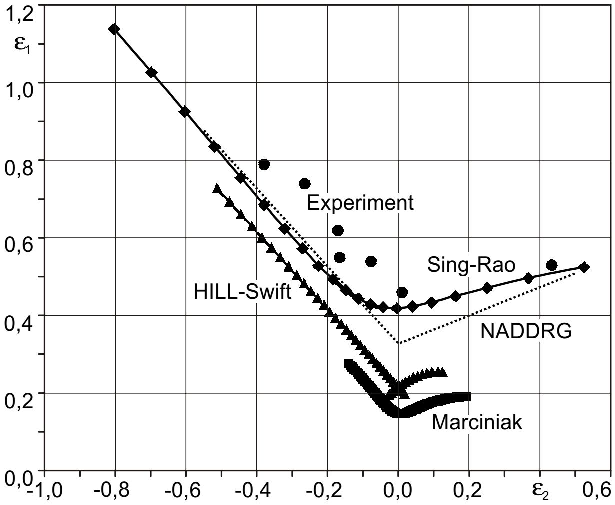

Figure 2. **The theoretical and experimental FLD for DX 54D**

Slika 2. **Teorijski i praktični FLD za DX 54D**

The simple empirical model developed by NADDRG gives very good predications of the FLD for DX 54D steel as well as for DX 53D, ZStE 220P and ZStE 340 steel which belong to the group of deep-drawable or high strength ferritic steel. For DX 54D and DX 53D this model has upper boundary and for ZStE 220P and ZStE 340 just lower boundary than those experimental results.

The predictions by the Hill-Swift model based on Hollomon's equation seem to give a lower boundary of the FLDs for DX 54D and DX 53D. The FLC calculated according to the method proposed by Hill and Swift seemed to be in good correlation with experimental results only on left-hand side for ZStE 220P. The right-hand side is much lower against the experiment. This prediction seems to be able for micro-alloyed steel ZStE 340.

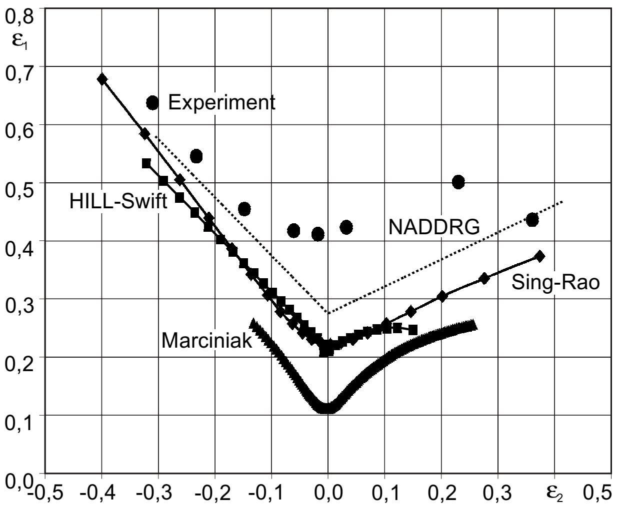

Figure 3. **The theoretical and experimental FLD for DX 53D**
Slika 3. **Teorijski i praktični FLD za DX 53D**

The method proposed by Sing - Rao seems to be in good correlation with experimental results for DX 54D and ZStE 340. However, for materials such as DX 53D and ZStE 220P, the predictions by this model have slightly lower and upper boundaries respectively, than those in experiment.

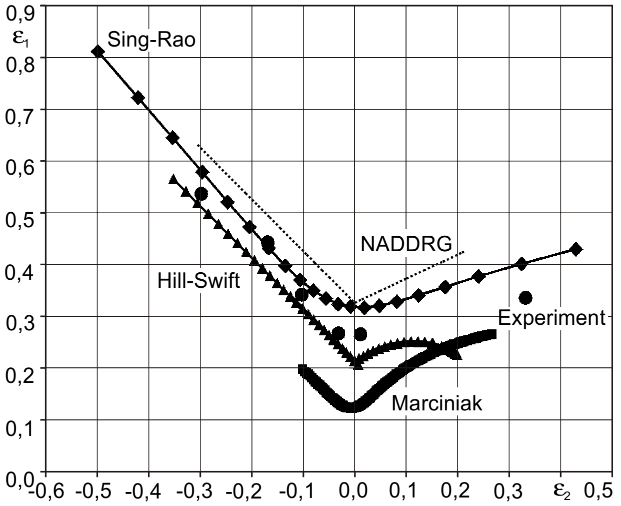

Figure 4. **The theoretical and experimental FLD for ZStE 220P**
Slika 4. **Teorijski i praktični FLD za ZStE 220P**

The worst correlation between calculated and experimental results was obtained in the case of calculation according to Marciniak - Kuczynski method. The predictions by this model do not display a good coincidence with the experimental results. The predicted FLC are much lower than the measured values. Only in case of the ZStE 220P steel and the ZStE 340 steel a satisfactory agreement with the experimental FLDs was observed, especially at greater deformations.

Figures 2. - 5. show that the major limit strains based on M-K method increase rapidly and monotonically from the plane-strain state up to the equi-biaxial stress state, whereas the major limit strains based on the Hill-Swift method increase somewhat slowly and finally decrease near to equi-biaxial stress state.

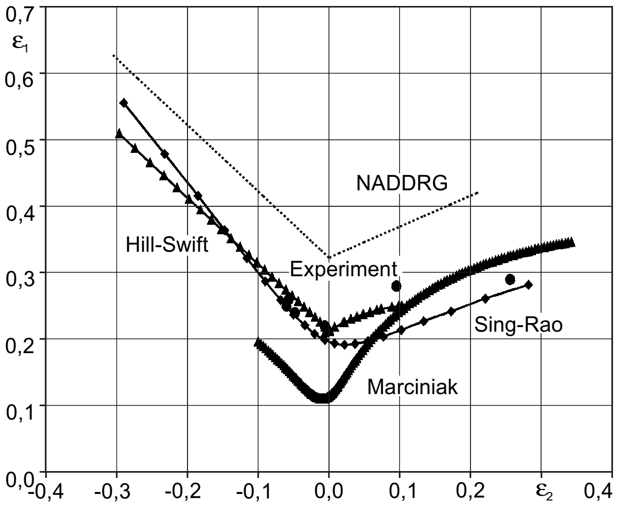

Figure 5. **The theoretical and experimental FLD for ZStE 340**
Slika 5. **Teorijski i praktični FLD za ZStE 340**

The major difference between the models lies in the applied strain-hardening models for material. It is obvious that the theoretical FLD differ greatly with the strain-hardening model, and for the same model the predicted accuracy varies with different materials. On the other hand, although the flow rule can represent very well the stress-strain relationships in uniaxial tension materials, the theoretical predictions still show large deviations from the experimental FLD. This implies that an appropriate calculating method depends only on the understanding of the flow behaviour of materials, but also on the assumptions for instability criteria and perhaps on further material properties and experimental factors.

Comparison of the theoretical FLDs and Sing-Rao method

It is generally recognized that the sheet metal formability increases with increasing value of strain hardening exponent n. The value of *R* has practically no effect in the plane strain region and a maximum effect in the equi-biaxial stress region, where higher values of *R* result in de-

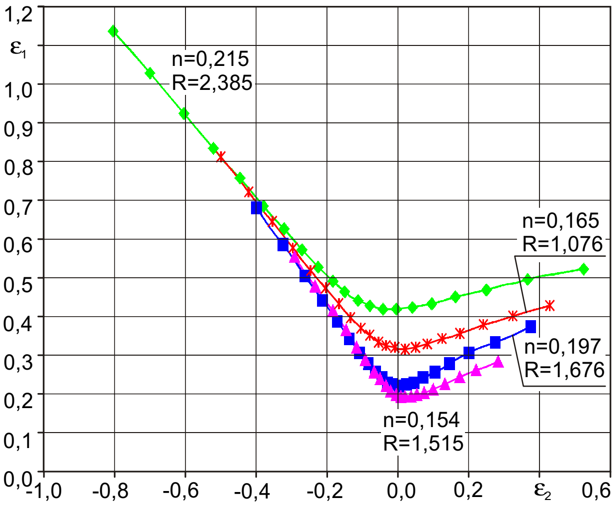

Figure 6. **The Influence of strain hardening exponent and *R*-value of plastic anisotropy respectively on forming limits obtained by Sing - Rao method**

Slika 6. **Utjecaj eksponenata otvrdnjavanja i veličine *R* plastične anizotropije na graničnu deformaciju dobivenu metodom Sing - Rao**

creased levels of forming limits [21]. Figure 6. shows the predicted FLC for various steel sheets with differnt strain hardening exponents and R-values of plastic anisotropy obtained by Sing - Rao method.

CONCLUSIONS

In this paper a comparative investigation of three mathematical models (Marciniak - Kuczynski model, Hill - Swift model, Sing - Rao model) as well as an empirical model proposed by the North American Deep Drawing Research Group (NADDRG) and experimental results has been carried out for different steel sheets.

None of the models can predict the forming-limit diagram reliably. The FLD_0 value is met by the empirical NADDRG model and the modified Hill - Swift model with sufficient accuracy for ferritic steels. The classical Hill - Swift and M-K models deliver too-small FLD_0 values. The method proposed by Sing - Rao seems to be in good correlation with experimental results for some steel sheets.

The forming-limit diagram is affected by the thickness, the yield and tensile strength, and the strain hardening and plastic anisotropy.

In order to understand the influence of the material properties and make effective use of the materials, there is a need for adopting mathematical models for analysis the interacting factors as a whole, with due consideration to the practical manufacturing process constrains.

Based on the formability prediction models, the analytical influence of the basic material properties have been investigated, and have been compared with experimental data.

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