### OPTIMISATION PROCESS OF STRIP COLD ROLLING

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Solutions of differential equations for smooth surfaces and transversal strip roughness with inertial lubrication forces were analyzed. Ten factors impacting the height of lubrication film at material input deformation zone were systemized through: rheologic lubrication properties, kinematics of technological processes and geometric characteristics of rolling processes. Linear optimization is based on the process of cloning, where two methods are proposed (rotary and step-like) and it contains three variants of adjustment of technological factors. The cloning process disposes of a possibility of the control of differential equations. The cloning is carried out with the Osnup tool that proposes a static and a dynamic method of transmission of similarity criterion over a singular point.

Key words: cold rolling of strip, Monte-Carlo method, multiple lineal regressions, lineal optimization

Optimiziranje procesa hladnog valjanja trake. Analizirana su rješenja diferencijalnih jednadžbi za glatke površine i poprečnu hrapavost trake sa utjecajem inercijskih sila maziva. Deset faktora koji utječu na visinu mazivog filma na ulaznom presjeku zone deformacije metala sistematizirani su kroz reološka svojstva maziva, kinematiku tehnološkog procesa i geometrijske karakteristike procesa valjanja. Linearno optimiziranje zasnovano je na procesu kloniranja, gdje su predložene dvije metode (rotacijska i stupnjevasta), i sadrži tri varijante podešavanja tehnoloških faktora. Proces kloniranja ima mogućnost kontrole riješena diferencijalnih jednadžbi kada zahvatni kutovi teže nuli. Kloniranje se vrši preko alata Osnup gdje je predložena statička i dinamička metoda prenosa kriterija sličnosti preko singularne točke.

Ključne riječi: hladno valjanje trake, metoda Monte-Carlo, višestruka linearna regresija, linearno optimiziranje

## INTRODUCTION

Lubrication flow in Descartes coordinates system for conditions of incompressibility can be described in tensor notation [1, 2]:

$$\left[\sigma_{ij}(p, t, v_i)\right], j = \rho\omega_i \tag{1}$$

where:

 $\sigma_{ii}$  - components of stress tensor,

P' - lubrication density,

 $\omega_i$  - substance derivation by speed,

*t* - temperature,

p - pressure in lubrication film,

 $v_i$  - lubrication moving speed.

form:

For Newton fluid the notation (1) takes the following

$$v_{j}v_{i,j} = -\frac{p}{\rho} + V_{k}\Delta v_{i} \tag{2}$$

where:

 $V_k$  - kinematics value,

 $\Delta\,$  - Laplas differential operator.

Through the expression (2) we reach Reynolds notation of lubrication differential equation :

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 v_x}{\partial y^2}; \quad \frac{\partial p}{\partial y} = 0$$
 (3a)

$$\frac{\partial p}{\partial z} = \mu \frac{\partial^2 v_z}{\partial v^2}; \quad \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = 0$$
 (3b)

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where:

 $\mu$  - dynamic lubrication viscosity.

Differential equation that takes into account the influence of inertial forces of lubrication at high technological speeds of metal processing has the following record:

$$\frac{\partial p}{\partial x} = 6\mu \frac{v_0 + v_R}{\varepsilon^2(x)} + C\mu \frac{1}{\varepsilon^3(x)} + + x\rho \frac{16(v_0 + v_R)^2 \varepsilon^2(x) - C^2}{120R\varepsilon^3(x)}$$
(4a)

$$C = \frac{k}{2} - \sqrt{\frac{k^2}{4} + 2(\nu_0 + \nu_R)\varepsilon_0 \left[8(\nu_0 + \nu_R)\varepsilon_0 + 3k\right]};$$

$$k = 120\nu \frac{R}{r}$$
(4b)

where:

 $v_0 + v_R$  - speed of movement of rolled strip and operating speed of rolls,

R - radius of rolls,

 $\varepsilon(x)$  - geometrical shape of lubricating wedge in front of the entrance into the deformation zone,

x - abscissa,

v - kinematics viscosity,

 $\varepsilon_0$  - the height of lubricating film at the inlet cross-section of metal zone deformation.

Differential equation that takes into account transversal strip roughness and longitudinal roughness of rolls has the following record:

$$\left\langle \frac{dp}{dx} \right\rangle = 6\mu \left( v_0 + v_{Rx} \right) \left| \left\langle \frac{1}{\varepsilon^2 \left( x_0 \right)} \right\rangle - \frac{\left\langle \frac{1}{\varepsilon_0^2} \right\rangle}{\left\langle \frac{1}{\varepsilon_0^3} \right\rangle} \left\langle \frac{1}{\varepsilon^3 \left( x_0 \right)} \right\rangle \right|$$
 (5)

where:

⟨ ⟩ - symbol for the operator of mathematical hope.

Accidental value of rolls and strip roughness at  $R_2 = 6\sigma$  according to GOST is

$$\varepsilon(x_0) = \varepsilon_N + [\sigma_y(x_0) + \sigma_{tr}(x_0)]$$

where:

 $\varepsilon_N$  - nominal height of lubrication film when the surfaces of rolls and strip are described by smoothness  $(R_2 = 0)$ .

## SOLUTIONS OF DIFFERENTIAL EQUATIONS BY MONTE-CARLO METHOD AND DISCUSSION ON THE RESULTS

## Evaluation of the roller cage rhythmicity

Lubrication viscosity depending on rolling pressure is introduced in the formula

$$\mu = \mu_0 e^{\gamma p} \tag{6}$$

where:

 $\mu_{\scriptscriptstyle 0}$  - dynamic viscosity at atmospheric pressure,

p - roll pressure on metal,

 $\gamma$  - piezocoefficient of lubrication viscosity.

Should viscosity depend on temperature then is:

$$\mu = \mu_0 e^{\gamma D} \cdot e^{\frac{-\Theta(t - t_1)}{B - 1}} \tag{7}$$

where coefficients D, B, and  $\Theta$  are defined on rotary viscosimeters.

If some reliable dynamic lubrication viscosity is available according to (6) and (7) solutions of differential quotations may be presented through technological parameters:

$$A = \frac{1 - e^{-\gamma D}}{6\mu_0 \gamma \left(\nu_0 + \nu_R\right)} \tag{8}$$

The lubrication height in the area [-a, 0] can be approximated by the following polynomial (Figure 1.):

$$\varepsilon(x) \equiv \varepsilon_N = \varepsilon_0 - \alpha x + \frac{x^2}{2R} - \alpha \frac{x^3}{2R^2}$$
 (9)

For the area of cold rolling it is sufficient to approximate  $\varepsilon(x)$  by square polynomials.

In the theoretical analysis a cage rolling composed of 11 roll pairs will be supposed inside of which technological parameter A will fall lineally from  $A_0 = 1\,965\,512~\text{m}^{-1}$  down to  $A_{10} = 898\,519~\text{m}^{-1}$  with  $\Delta A = -\,106\,700$ . In this process is:  $v_0 = 0.6v_R$  m/s, vR m/s = 10 m/s,  $\mu_0 = 0.024$  Pas,

 $\gamma$  = 0,218E–6 m<sup>2</sup>/N,  $p \cdot \gamma$  = 4,36,  $\varepsilon_a$  = 0,001 m,  $R_z$  = 0, isothermal conditions of technological process t = (20 °C).

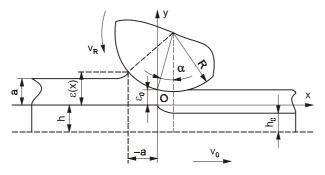


Figure 1. The scheme of rolling with lubrications Slika 1. Shema valjanja sa mazivima

Guidelines of analysis will be moving over singular point in which discriminate of square expression in (9) is equal to zero [3, 4].

The Figure 2. involves the monitoring of inertial lubrication forces according to the differential equation (4a), the influence of roughness according to (5) and normal



Figure 2. Deviation of lubrication film height  $I_{\rm R}$  / % at input cross-section of deformation zone in relation to technological parameters ratio  $\psi$ 

 $A_{10} = 898\,519\,\mathrm{m}^{-1}(10^{th}\,\mathrm{roll\,pair}); R = 0.2\,\mathrm{m}, \alpha = 0.03\,\mathrm{rad}, R_z = 6E^{-6}\,\mathrm{m}, \epsilon_a = 0.0015\,\mathrm{m},$ 

Series 1 differential equation (3a),

Series 2 differential equation (5),

Series 3 differential equation (4a)

Slika 2. Odstupanja visine mazivoga filma I<sub>R</sub>/% na ulaznome presijeku zone deformacije u odnosu na omijere tehnoloških parametara ψ

 $A_{10} = 898519 \text{ m}^{-1}(10. \text{ par valjaka}); R = 0.2 \text{ m}, \alpha = 0.03 \text{ rad},$ 

 $R_z = 6E^{-6} \text{ m}, \varepsilon_s = 0.0015 \text{ m},$ 

Serija 1 diferencialna jednadžba (3a),

Serija 2 diferencialna jednadžba (5),

Serija 3 diferencialna jednadžba (4a)

lubrication height film according to differential equation (3a). The results are obtained by Monte-Carlo numerical method for all three differential equations with the following approximation for  $\varepsilon(x)$ :

$$\varepsilon(x) = \varepsilon_0 - \alpha x + \frac{x^2}{2R} - \alpha \frac{x^3}{2R^2} + \frac{x^4}{8R^3}$$
 (10)

For roll radius R=0.2 m, disharmony is contributed by pairs of roller cages 1., 2., and 7. for the series 2. On the abscissa of Figure 2. a ratio of technological parameter in relation to the initial roll pair, that has  $A_0=1965512~{\rm m}^{-1}$ , is plotted. In addition to a graphical method of the moving of disharmonic roller cage some other criteria of this estimation can be developed. The easiest one is regression estimation through regression coefficient. So for differential equation (5) and exampled Figure 2. there is a following movement in Table 1. for linear regression analysis.

Table 1. The movement of the coefficient of linear regression depending on distancing of roller cage ordinary numbers for differential equation (5);  $\alpha=0.03$  rad

Tablica I. Kretanje koeficijenata linearne regresije u ovisnosti o udaljavanju rednih brojeva valjaka iz kaveza za diferencijalnu jednadžbu (5); α=0,03 rad

All present	Removed  1 <sup>st</sup> couple (A <sub>0</sub> )	Removed 1 <sup>st</sup> and 2 <sup>nd</sup> couple	Removed 1 <sup>st</sup> , 2 <sup>nd</sup> , and 7 <sup>th</sup> couple	-
$R^2 = 0.9882$	$R^2 = 0.9802$	$R^2 = 0.9948$	$R^2 = 0.9955$	-

The results of the Figure 2. could be presented by a multiple linear regression in a following form:

$$Y_i = 2,4628 - 0,2896X_1 - 0,7285X_2 - 0,3657X_3$$
 (11)

$$X_{i} = \frac{A_{0}}{A_{1-10}}; \quad Y_{i} = \frac{\varepsilon_{0}(A_{10}) - \varepsilon_{0}(A_{1-10})}{\varepsilon_{0}(A_{10})} \% = 1_{R}$$
 (12)

where:

 $X_1$  - influence of smooth surfaces (dif. equation (3a)),

 $\vec{X}_2$  - influence of inertial forces (dif. equation (4a)),

 $X_3$  - influence of surfaces' roughness (dif. equation (5a)).

It can be seen in (11) that the inertial forces have the greatest influence. It is shown in Figure 2., where the solutions of differential equations (4a) are the most balanced.

The coefficient of multiple digression is  $R^2 = 0.99862$ , Durbin-Watson Test = 2,15062.

According to Table 1. it is visible that with removing of the seventh roll pair there was no contribution to any great increase to the index of rythmicity for differential equation (5) which is also graphically visible in Figure 2.

### **Process optimization**

The Figure 3. presents a technological chart as regards the optimization of the results in Figure 2. It would be more appropriate to suppose that the roller cage should not be removed but their technological parameter adjusted so that

### Ordinary number of cage rollers

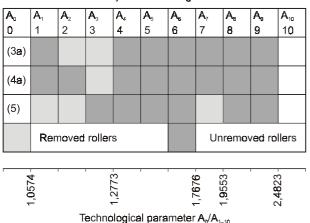


Figure 3. Optimization of index of cage rhythmicity in the area of technological parameter  $A_{\perp} \subseteq [1,2773-1,7676] \cup [1,9553-2,4823]$  (3a)-diff. equation for smooth surfaces (4a)-diff. equation for rough surfaces (5)-diff. equation for rough surfaces  $R_z = 6E^{-6}$  m Optimizacija indeksa ritmičnosti kaveza u području tehnološkog parametra  $A_1 \subseteq [1,2773-1,7676] \cup [1,9553-2,4823]$  (3a)-dif. jednadžba za glatke površine (4a)-dif. jednadžba koja uključuje inercijske sile

(5)-dif. jednadžba za hrapave površine  $R_z = 6E^{-6}$  m

they follow the general regression line (11). The separated area of optimization  $A_L \in [12773 - 1,7676]$  U[1,9553 - 2,4823] according to the technological parameter, presents a concatenated case that has no need for switching over to new values of technological parameters. Rhythmicity indices for linear optimizing may hold on their values together with unchanged value of technological parameter, inside of which there is possibility for the change of kinematical and rheological characteristics according to the law of hyperbole. Consequently for the fourth roller cage,  $A_4 = 1538$  714 m<sup>-1</sup> gives an initial product for  $\mu_0 \cdot v_R = 0,030657 \times 1,6 \times 10 = 0,49051$  Pam (values of  $A_0$  stayed on). Should the rolling speed be increased to 11 m/s, then the new value for dynamic lubrication viscosity amounts  $\mu = 0,02787$  Pas.

However, if we are outside the area  $A_r$ , then it is good to change the index of rhythmicity through the change of constructive characteristics; engaged angle or radius of rolls. Another and more difficult case of optimization proceeds by change of technological parameter  $A_2$ . According to  $A_2=1.752.113$  with change of only  $m_0$  proceeds the value  $\mu_0=0.0269231$  Pas (other parameters are equal as for  $A_0$ ). With a small addition of increment of  $\mu_0=0.027$  the height of lubrication film will increase and rythmicity index amount  $\lg=0.9869$ . In the Table 2. the review of calculation results is given.

In dynamic lubrication viscosity the result of optimization changes on the  $4^{th}$  Figure of calculation and, in this case, has a theoretic meaning for the cloning processes if they should be transmitted on engaged angles about 0,05 rad. for an differential equation (3a). The high value of correlation coefficient for regression direction (11) indicates that the cage has a good hydrodynamic work stability, so insignificant shifts on  $1_R$  require small shift on dynamic lubrication viscosity.

For the transmission of similarity criterion (cloning process)  $R^2 \rightarrow 1$  is required and will be explained later in the text.

# The scheme of the tool for determining index of rythmicity and solution of cloning of differential equations

The past analysis was made with a tool the algorithm of which will be presented. The principled scheme is given in the Figure 4. where two methods can be observed:

- I) Transfer of similarity criterion from the etalon  $A_0$  over guides  $A_{1-10}$  in relation to the fixed roller cage  $A_{10}$  is made by hyperbole,
- II) Transfer of similarity criterion in the cage is made over rotating OSNUP.

Example application for differential equation (3a) is shown in Table 3. In this connection R=0.2 cm,  $\alpha_a=0.001$  m, without influence of inertial forces.

From the Table 3. it is visible that both methods give the same result so in fact they control each other. The high value of the correlation coefficient for the tool in Figure 4. which can be seen in the equation (11) also enable some

 Table 2.
 The results of iteration for differential equation (3a), variable A

 Tablica 2.
 Rezultati iteracije za diferencijalnu jednadžbu (3a), varijabilan A

A / m <sup>-1</sup>	α* / rad	α* / m	α* / m	$\alpha = 0.03 \text{ rad}$ $\epsilon_0 / m$	Index of rhythmicity		
$A_0 = 1965512$	0,011058	12,227E <sup>-6</sup>	15,827E <sup>-6</sup>	7,223E <sup>-6</sup>			
$A_2 = 1752112$	0,011483	13,198E <sup>-6</sup>	17,083E <sup>-6</sup>	8,013E <sup>-6</sup>			
$A_{10}$ = 898519	0,0143357	29,551E <sup>-6</sup>	26,600E <sup>-6</sup>	14,471E <sup>-6</sup>			
Nev	New $A_2 = 1747122$ comes with the new $\mu_0 = 0.0267$ Pas.						
	The cons	sequence is c	orrection of	Osnup.			
$A_2 = 1747122$ <b>0,0</b> 115137 13		13,257E <sup>-6</sup>	17,157E <sup>-6</sup>	8,0338E <sup>-6</sup>	2,0853		
	There follows linear interpolation						
$I_{\mathbf{R}}$		$\mu_{o}$					
1,060		0,0269231					
	2,0853	0,027					
	0,9869	0,0026917					

Table 3. Example for the application of tool presented in the Figure 4.

Tablica 3. Primjer primjene alata sa slike 4.

				I		
$A_{\rm i}/{\rm m}^{-1}$	α* / rad	α	* / m	$\alpha_1^0 / m$	$\alpha = 0.03 \text{ rad}$	
$A_0 = 1965512$	0,0110580	12,227E <sup>-6</sup>		15,827E <sup>-6</sup>	7,223E <sup>-6</sup>	
$A_1 = 1752112$	0,0112649	12,	689E <sup>-</sup>	16,425E <sup>-6</sup>	7,5 <b>9</b> 7E <sup>-6</sup>	
$A_{10}$ = 898519	0,0143357	20,	551E <sup>-6</sup>	26,600E <sup>-6</sup>	14,471E <sup>-6</sup>	
Ev	aluation of i	ıdex	of cage	rhythmicit	У	
Method	Method I (Static)			Method II (Dynamic)		
$A_0$ : $A_1$			$A_0$ :. $A$	•1		
$\Delta \varepsilon_0 = 3.74 E^{-6}$	$\Delta_0 = 6,060 E^-$	6	$\Delta \varepsilon_0 = 3.74 E^{-7}$ $\Delta_0 = 6.060 E^{-6}$			
	$\Delta_1 = 6,476E^{-6}$			$\Delta_{1} = 6,476E^{-6}$		
	$\Delta_1 - \Delta_0 = 4.16 E^{-7}$			$\chi = (\Delta_1 - \Delta_0) = 4.16 E^{-7}$		
$\varphi_0 = (\Delta_1 - \Delta_0)/\Delta \varepsilon_0 = 1,1123$			$\chi - \chi/\phi = \Delta \varepsilon_e \longrightarrow \phi = 9.905$			
$A_{10}$ $A_1$			$A_{10}$ $\therefore$ $A_{10}$	$\mathbf{d}_1$		
$\Delta \varepsilon_0 = 6.874 E^{-6}$	$\Delta \varepsilon_0 = 6.874 E^{-6} \Delta_{10} = 1.394 E^{-5}$			$\Delta \varepsilon_{0} = 6.874 E^{-6} \Delta_{10} = 1.394 E^{-5}$		
	$\Delta_1 = 6,476E^{-1}$	6			$6,476E^{-6}$	
(Δ <sub>10</sub> -Δ	$(A_1)_2 = 7,465E^{-1}$	6		$\chi_2 = \Delta_{10} - \Delta_{10}$	$\Delta_{i} = 7,465E^{-6}$	
$\left  \left( \Delta \varepsilon_0 \right)^{\text{C}} = \left( \Delta_{10} - \Delta_{10} \right)^{\text{C}} \right $	$(\Delta_1)_2/\phi_0 = 6.71$	1E-6	$\left  (\Delta \varepsilon_0)^{c} \right $	$=\chi_2-\chi_2/\phi_0$	= <b>6</b> ,711E <sup>-6</sup>	
$\varepsilon_0 = 7.597E^{-6} + 6.711E^{-6} =$			$\varepsilon_0 = 7.597 E^{-6} + 6.711 E^{-6} =$			
Index = $1,126$	= 14,30	8E <sup>-6</sup>	Index	= 1,126 %	$= 14,308E^{-6}$	

estimation of lubrication film for the case when differential equations are difficult to solve. In this connection it is necessary to know the indices of cage rhythmicity and removing those roll pairs that make it worse. Such a refined

Table 4. Rotary and step-like cloning of the solution of differential equation (4a) for  $\alpha=0.03$  rad

Tablica 4. Rotacijsko i stupnjevito cloniranje riješenja diferencijalne jednadžbe (4a) za  $\alpha=0.03$  rad

Techno-	$\varepsilon_0 / (E^{-6} m)$	$\varepsilon_{\rm e}/(\rm E^{-6} m)$	$\epsilon_0/(E^{-6}m)$	$\epsilon_a/(E^{-6}m)$
logical	for	when	when	calculated
para-	un-	2 <sup>nd</sup> and 3 <sup>rd</sup>	2 <sup>nd</sup> and 3 <sup>rd</sup>	according
meter	changed	roll pairs	roll pairs	to the
/ m <sup>-1</sup>	cage	are	are	method
	(rotary)	removed	removed	Monte
		(rotary)	(step-like)	- Carlo
$A_1 = 1858812$	-	-	-	7,831
$A_2 = 1752113$	8,088	-	_	8,238
$A_3 = 1645414$	8,561	-	-	8,694
$A_4 = 1538714$	9,092	9,206	9,207	9,209
$A_5 = 1432015$	9,693	9,786	9,788	9,795
$A_6 = 1325316$	10,381	10,449	10,453	10,469
$A_7 = 1218617$	11,176	11,215	11,219	11,469
$A_{\rm g} = 1111917$	12,104	212,115	12,122	12,172
$A_9 = 1005218$	13,210	13,179	13,188	13,271
$A_{10}$ = 898519	14,548	14,468	14,479	14,607

cage may be a good base to find out approximate solutions for the areas of other engaged angles. In Table 4. such an evaluation (cloning) for differential equation (4a) whose solutions are presented in Figure 2. is given.

Removing  $2^{nd}$  and  $3^{rd}$  roller cage pair principally enhances cloning although only the  $3^{rd}$  roller cage could be removed. Here we can mention the characteristics of this cloning. According to the Table 3. it is necessary to know three successive technological parameters  $A_0$ ,  $A_1$ ,  $A_2$  with

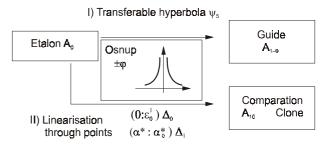


Figure 4. The tool for determining index of cage rhythmicity Slika 4. Alat za određivanje indeksa ritmičnosti kaveza

all initial solutions except solutions for  $\alpha=0.03$  rad from  $A_2$  (9.088E–6). It is cloned on the base of known values for  $\alpha=0.03$  rad from  $A_0$  and  $A_1$ . Then the technological parameter  $A_1$  turns to  $A_0$ , and  $A_0$  turns to  $A_1$ , however  $A_3$  is

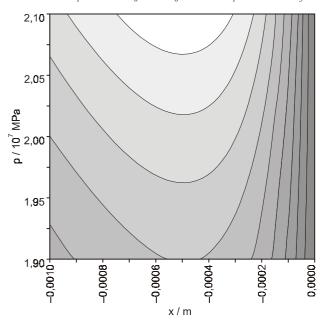


Figure 5. Pictures aimed at the pressure gradient of differential equation (3a) in seeking its maximum for  $A_0$ : R=0.2 m,  $\alpha=0.011058$  rad.  $\epsilon_a=0.001$  m; x=(-0.001m down to 0.0001m), p=19 to 21 Mpa

Slika 5. Ciljani snimci na gradijent tlaka diferencijalne jednadžbe (3a) u traženju njegovog maksimuma za  $A_0$ : R=0,2 m,  $\alpha=0,011058$  rad.  $\varepsilon_a=0,001$  m; x=(-0,001 m do 0,0001m), p=19 do 21 Mpa

Table 5. Possible methods of cold rolling optimization
Tablica 5. Moguće metode optimiziranja hladnoga valjanja

Area	Varia-	Unchange-	Remarks	
per $A_{\scriptscriptstyle  m L}$	bility	ableness	Remarks	
Within the	$\varepsilon_0, v_R,$	A = const	Computation according to	
area A	$\mu_0, \gamma$	$\alpha = const$	hyperbola $v_{\mathbf{R}} \cdot \mu_{0} = \text{const.}$	
	$\mathbf{p}, v_0$	R = const	Solutions are on 3D-surface	
			of expression 8	
Outside the	$\varepsilon_{\rm o}, v_{\rm R},$	A = const	Calculation according to	
area $A_2$	$\mu_0, \gamma$		differential equations	
Figure 3.	$p, v_0,$			
	$\alpha, R$			
Outside the	$\varepsilon_0, v_R,$	_	Iterative method of calcula-	
area A2	$\mu_0, \gamma$		tion according to differential	
	$p, v_0,  $		equations (3a), (4a), and (5)	
	α, R,		with the use of Osnup in the	
	$\varepsilon_{a}, \boldsymbol{R}_{z},$		Figure 4.	
	t			

Table 6. Possible methods of cloning of differential equation
Tablica 6. Moguće metode cloniranja diferencijalnih jednadžbi

Rotationary cloning		To be calculated	Remark	
Calibration	$\overline{A_{\scriptscriptstyle 0}}$	$\alpha^*, \epsilon_a^*$ , $\epsilon_o^{\scriptscriptstyle 1}$	$arepsilon_{0}$ from $oldsymbol{A}_{0+2}$	
Guide	$A_{0+1}$	$\epsilon_{o}$ for cloning angle	follows as the result of	
Clone	$A_{0+2}$	from $A_0$ , $A_{0+10}$	cloning	
Calibration	$A_{o+1}$	$\alpha^*, \epsilon_0^*, \epsilon_0^1$	Now being	
Guide	$A_{0+2}$	$\varepsilon_0$ from $A_{0+1}$ is calculated	cloned $\epsilon_{\scriptscriptstyle{0}}$ from $A_{\scriptscriptstyle{0+3}}$	
Clone	$A_{0+3}$	$\varepsilon_0$ from $A_{0+2}$ is cloned	chosen angle α	
Step-like cl	oning	To be calculated	Remark	
Calibration	$A_{0}$	$\alpha^*, \epsilon_0^*$ , $\epsilon_0^{\scriptscriptstyle 1}$	Two methods of	
Guide	$A_{0+1}$	$\varepsilon_0$ for cloning angle	Osnup application are possible and	
Clone	$A_{\scriptscriptstyle 0+10}$	from $A_{\mathfrak{d}}$ and $A_{\mathfrak{d}+10}$	with: 1. statistic $\phi_0$ 2. dynamic $\phi$	
Calibration	$A_{\scriptscriptstyle 0}$	$A_{0+1}$ is cloned	Recommendation is	
Guide	$A_{0+2}$	$A_{\scriptscriptstyle{0+2}}$ is being cloned	to recheck the clo-	
Clone	$A_{\scriptscriptstyle 0+10}$		ning, when φ on Osnap change its direction of rotation over rhythmicity index	

Calibration and clone are fixed while the guide climbs along the steps till it reaches  $A_{0+10}$ 

a new technological parameter for cloning with known solution in a singular point. As shown in Table 4. this algorithm based on uninterrupted rotation is very useful for the evaluation of the height lubrication film in the areas of engaged angles larger than  $\alpha^*$ .

The  $2^{\rm nd}$  cloning type can develop step-like so that  $A_0$  is always a fixed calibration while guides  $A_{1-10}$  continually climb along a step until they reach  $A_{10}$ . In the Table 4. it is visible that step-like cloning has a insignificant advantage over the rotary cloning. Optimization calculations are accompanied also by changes in pressure gradient within the lubrication film. such a change for  $A_0$  is presented in the Figure 5. It indicates that maximal pressure gradient is at about SE-4 m in front of entry cross section of metal deformation zone.

### **CONCLUSION**

The refining of the region  $A_L$  is a good preliminary work for this area of technological parameters to be used for transferring similarity criterion (cloning) of differential equations solutions on larger or smaller engaged angles. Index of rhythmicity showing these features are defined by functions (11). Optimization and cloning could be presented in the Table 5. and Table 6.

Calibration, Guide, and Clone are continuously in a dynamic motion. A good cloning for the angle  $a_1$  requires cutting off roll pairs - at the initial angle a within the cage, which could disturb the accuracy of cloning at a larger engaged angle  $\alpha_1$  (over the index of rhythmicity).

However the process of cloning may also develop backwards - from higher engaged angles down to lower ones, but it can also check the solutions of differential equations when  $\alpha \to 0$  rad.

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