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ANALYTICAL AND NUMERICAL METHODS OF DETERMINING THE DISTRIBUTION OF TEMPERATURE OF AIR-COOLED STRIP

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The paper analyzes the distribution of temperature of 60S2A steel strip cooled in air. Both analytical and numerical methods were applied in investigation. From among the known analytical methods of temperature distribution determination, Tselikov's [1], and the Fourier's [2] methods were used. The numerical examination of temperature distribution presented in the paper involved carrying out a computer simulation of strip air cooling. For this purpose, the Forge 2D software [3] was employed, which relies on the finite-element method in performing computations. Comparison of results obtained by analytical methods with numerical computation results and for the real technological process has also been made within the study.

Key words: *strip cooling, analytical methods, numerical analysis, FEM*

Analitičko i numeričko utvrđivanje prostiranja temperature na traci hlađenoj zrakom. U radu se analizira prostiranje temperature u čeličnoj traci 60S2A hlađenoj zrakom. Tijekom istraživanja primjenjivala se i jedna i druga metoda. Između poznatih analitičkih metoda utvrđivanja prostiranja temperature koristile su se Tselikova [1] i Fourierova [2] metoda. Numeričko ispitivanje temperature koje je prikazano u radu uključuje i provođenje kompjuterske simulacije hlađenja trake zrakom. U tu je svrhu primijenjen Forgeov software 2D [3] koji se u računanju oslanja na metodu konačnih elemenata. Unutar studije je također provedena i usporedba rezultata dobivenih analitičkim metodama s numeričkim izračunima.

Ključne riječi: *hlađenje trake, analitičke metode, numerička analiza, metoda konačnih elemenata*

INTRODUCTION

Literature [1, 2] describes numerous analytical methods of calculating strip temperature distribution. From among them, three have been used in this study for calculating the distribution of temperature of air cooled strip.

The first of the methods applied relies on A. I. Tselikov's model [1]. Change of the temperature of strip during its passage from the heating furnace to the first stand of a rolling train can be calculated from formula (1). The increment in strip temperature, caused by deformation in the preceding rolling stand, will be equal to zero ($\Delta T_d = 0$) in this case:

$$\Delta T = T_0 - \\ - \frac{1000}{\sqrt{\frac{0,0255 \cdot Perim \cdot t}{S} + \left(\frac{1000}{T_0 + \Delta T_d + 273} \right)^3}} + 273 \qquad (1)$$

where:

T_0 - stock heating temperature / °C,
$Perim$ - strip perimeter / mm,
S - strip cross-section surface area / mm^2,
t - time of strip cooling during passage from the furnace to the first stand / s,
ΔT_d - increment in strip temperature caused by deformation in the preceding rolling stand / °C.

The second method is the model of air cooling of thin stock. The classification of stock into thin and thick stock is based on Biot's number, Bi [2]:

$$Bi = \frac{\alpha \cdot L}{\lambda} \qquad (2)$$

where:

α - coefficient of heat exchange between the strip and the environment / $\frac{\mathrm{W}}{\mathrm{m^2K}}$,

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L - thickness of the flat layer (half of the thickness of symmetrically cooled strip) / m,

λ - thermal conductivity / $\frac{\mathrm{W}}{\mathrm{mK}}$.

Assuming the strip as being thin, its temperature in time t is calculated from relationship [2]:

$$T = \left(T_0 - T_a\right)\exp\left(-\frac{k\cdot\alpha}{L\cdot\rho\cdot C_p}\cdot t\right) + T_a \tag{3}$$

The coefficient of heat exchange between the strip and the environment is calculated using Newton's equation [2, 4 - 6]. The density of the heat flux, q, taken over by the environment is determined based on the Stefan-Boltzmann law [2, 6, 7], whereas the stock emissivity, ε_s, is determined from relationships proposed in work [6].

To calculate the first approximation of the heat flux density q and stock emissivity, ε_s, the strip surface temperature should be taken as equal to the average stock temperature:

$T_{\mathrm{surf}} = T_{\mathrm{av}}$.

The difference of stock surface and ambient temperatures, [°C], is calculated from the formula:

$$\Delta T = T_{\mathrm{surf}} - T_a \tag{4}$$

where:

T_0 - initial strip temperature / °C,
T - strip temperature / °C,
T_{surf} - strip (stock) surface temperature / °C,
T_{av} - average stock temperature / °C,
T_a - ambient temperature / °C,
k - shape factor,
α - strip and environment heat exchange coefficient / $\frac{\mathrm{W}}{\mathrm{m^2K}}$,
L - thickness of the flat layer (half of the thickness of symmetrically cooled strip) / m,
ρ - stock density / $\frac{\mathrm{kg}}{\mathrm{m^3}}$,
C_p - stock specific heat / $\frac{\mathrm{J}}{\mathrm{kgK}}$,
t - stock cooling time / s.

The thermophysical properties of the rolled material should be determined for the average stock temperature, T_{av}.

The third method is the model of strip air cooling, which relies on Fourier's equation. For the determination of variations in strip temperature due to taking over of stock heat by the environment, the criterial solution of Fourier's equation for a constant heat flux density is used in the model [2].

The input parameters in this model are:

h - strip thickness / mm,
T_a - ambient temperature /°C,
T_{av} - average stock temperature / °C,
t - stock (strip) cooling time / s.

The parameters to be determined are:

T - stock temperature / °C,
T_{surf} - stock surface temperature / °C,
T_{axis} - stock mid-thickness (axis) temperature / °C,
ΔT_s - temperature difference between the axis and surface of the stock / °C,
T_{av} - average stock temperature / °C.

The strip temperature is determined from formula [2], as written in the form below:

$$T = T_{\mathrm{av}} - \frac{q\cdot L}{2\cdot\lambda}\cdot\left[\frac{2\cdot a\cdot t}{L^2} + \left(\frac{x}{L}\right)^2 - \frac{1}{3} + \sum_{p=1}^{\infty}\frac{4\cdot(-1)^{p+1}}{\varepsilon_p^{\ 2}}\cdot\cos\left(\varepsilon_p\cdot\frac{x}{L}\right)\exp\left(-\varepsilon_p^2\cdot\frac{a\cdot t}{L^2}\right)\right] \tag{5}$$

where:

$\varepsilon_p = \pi\cdot p$, for $p = 1, 2, 3, \ldots$,
$L = h / 2000$,

a - temperature conductivity [2] / $\frac{\mathrm{m^2}}{\mathrm{s}}$.

To determine the stock surface temperature, it is assumed that $\left(\frac{x}{L} = 1\right)$, whereas for the determination of temperature in the stock axis, it is assumed to be $\left(\frac{x}{L} = 0\right)$. On this basis, the strip surface temperature and strip axis temperature can be determined from the following formulae:

$$T_{\mathrm{surf}} = T_{\mathrm{av}} - \frac{q\cdot L}{2\cdot\lambda}\cdot\left[\frac{2\cdot a\cdot t}{L^2} + \frac{2}{3} + \sum_{p=1}^{\infty}\frac{4\cdot(-1)^{p+1}}{\varepsilon_p^{\ 2}}\cdot\cos\left(\varepsilon_p\cdot\frac{x}{L}\right)\exp\left(-\varepsilon_p^2\cdot\frac{a\cdot t}{L^2}\right)\right] \tag{6}$$

$$T_{\text{axis}} = T_{\text{av}} - \frac{q \cdot L}{2 \cdot \lambda} \cdot \left[\frac{2 \cdot a \cdot t}{L^2} - \frac{1}{3} + \right.$$
$$\left. + \sum_{p=1}^{\infty} \frac{4 \cdot (-1)^{p+1}}{\varepsilon_p{}^2} \cdot \cos\left(\varepsilon_p \cdot \frac{x}{L} \right) \exp\left(-\varepsilon_p^2 \cdot \frac{a \cdot t}{L^2} \right) \right] \tag{7}$$

The density of heat flux, q, taken over by the environment should be determined similarly as for the use of Model II, from the Stefan-Boltzmann law.

To determine the first approximation of the heat flux density, q, and stock emissivity, ε_S, from the equations proposed in references [2, 6, 7], the stock surface temperature, T_{surf}, should be assumed to be equal to the average stock temperature:

$$T_{\text{surf}} = T_{\text{av}}.$$

After determining the stock surface temperature from formula (6), the temperature difference between the stock surface and axis should be determined from relationship [2], as written in the following form:

$$\Delta T_s = T_{\text{axis}} - T_{\text{surf}} = \frac{q \cdot L}{2 \cdot \lambda} \tag{8}$$

The stock mid-thickness temperature, [°C], can be determined either from formula (7), or from the formula below:

$$T_{\text{axis}} = T_{\text{surf}} + \Delta T_s \tag{9}$$

The average stock temperature can be determined from equation [2], as written in the form:

$$T_{\text{av}} = T_{\text{surf}} + \frac{2}{3} \cdot \Delta T_s \tag{10}$$

The thermophysical properties of the stock should be determined for the average stock temperature, T_{av}.

Among the numerical methods enabling computer simulations of the strip air cooling process to be carried out, the finite-element method is particularly noteworthy. For this reason, the Forge 2D program [3] was used for numerical examination.

PURPOSE AND SCOPE OF THE STUDY

An analysis of the distribution of temperature of air-cooled strip when conveyed from the furnace exit the to the first rolling stand was carried out in the study.

The purpose of the analysis was to determine the correctness of results obtained both from numerical computations and from computations by analytical methods, compared to the real conditions.

Examination results obtained by using analytical methods were compared with computer simulation results and experimental results. The experimental part involved the measurement of temperature performed using an optical pyrometer during the process of rolling flat bars from a 130 × 130 mm square in the conditions of the Rolling Mill Department of a Steelworks. Computer simulation was carried out for a similar strip cooling process, while using the input data given in Table 1.

Table 1. **Main parameters of the air cooling process and the thermophysical properties of 60S2A steel strip**

Tablica 1. **Glavni parametri procesa hlađenja zrakom i parametri termofizičkih svojstava čelika 60S2A**

Main process parameters		Thermophysical properties of 60S2A steel strip		
Dimensions of strip cross-section	130×130 mm	Thermal conductivity, λ_{1100}	27,16 W/mK	[12, 13]
Strip temperature, T at the furnace exit	1100 °C	Specific heat, C_{p1100}	675 J/kgK	[2, 13]
Air cooling time, t_0	30 s	Density (specific mass), ρ_{1100}	7630,67 kg/m^3	[13]
Ambient temperature, T_a	20 °C	Strip and environment heat exch. coef., α_{1100}	90,57 W/m^{2}K	[2, 4 - 6]

The material used for tests was the 60S2A spring steel [8 - 11].

Because of the symmetry of the cross-section, the simulation of the air cooling process was performed for a quarter of the strip.

ANALYSIS OF INVESTIGATION RESULTS

Table 2. gives both testing and computation results concerning the distribution of temperature of air-cooled

Table 2. **Distribution of strip temperature, measured and calculated using analytical and numerical methods**

Tablica 2. **Raspodjela temperature trake, izmjerene i izračunate analitičkim i numeričkim metodama**

<table>
<tr><td>Strip temperature, T measured during the process</td><td>Strip temperature, T calculated using Model I</td><td>Strip temperature, T calculated using Model II</td></tr>
<tr><td>1070 °C</td><td>1073,20 °C</td><td>1085,20 °C</td></tr>
<tr><td colspan="3">Strip temperature, T calculated using Model III</td></tr>
<tr><td>T_{surf} / °C</td><td>T_{axis} / °C</td><td>T_{av} / °C</td></tr>
<tr><td>1048,89</td><td>1099,99</td><td>1082,96</td></tr>
<tr><td colspan="3">Strip temperature, T obtained from numerical computation (simulation)</td></tr>
<tr><td>T_{surf} / °C</td><td>T_{axis} / °C</td><td>T_{av} / °C</td></tr>
<tr><td>1031,00</td><td>1100,00</td><td>1065,50</td></tr>
</table>

strip. Also, an analysis of comparison of experimental results with numerical computation and analytical results is presented below.

The results of examination of the temperature distribution of air-cooled strip while being conveyed from the furnace exit to the first rolling stand, obtained from computer simulation are shown in Figure 1.

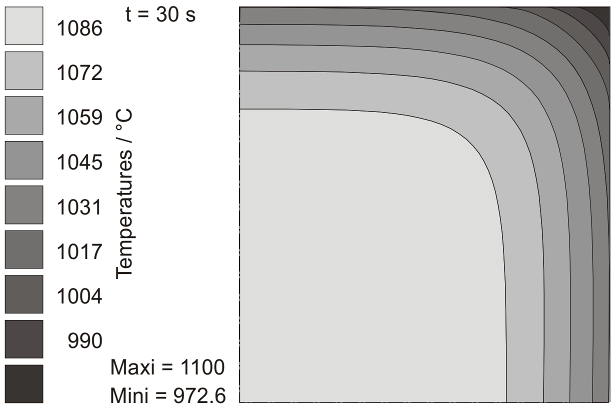

Figure 1. **Temperature distribution over the strip cross-section obtained from computer simulation**

Slika 1. **Raspored temperature po presjeku trake prema kompjutorskoj simulaciji**

It can be found from the investigation carried out that the results obtained by using mathematical models correspond, within some permissible limits, to the experimental testing results (Table 2.).

The first two mathematical models of those discussed in Section 2 assume a uniform value of temperature over the strip cross-section, which is a certain limitation and prevents these models from being used in the event, where it is necessary to know the temperature distribution for at least a few points of the strip examined. Moreover, the first of the models does not directly take account of the shape of the material and its thermophysical properties. This has a substantial effect on obtained calculation results. These models can, however, be utilized with a fairly high accuracy in cases, where a particularly high accuracy is not a requirement, and where it is not necessary to know the distribution of temperature over the strip cross-section, e.g. when calculating temperature distribution in a rolling process [14].

During calculation of temperature by these methods, the error of examination results in relation to real conditions was 0,29 % for the first model and 1,42 % for the second model.

The third of the mathematical models discussed enables the calculation of cooled strip surface temperature, and it allows the calculation of temperature in the strip axis, as well as the average temperature. Also, similarly as model II, this model takes into account thermophysical properties, which has a significant influence on the obtained results.

In the case of using the third analytical model, the error related to real conditions was: 1,97 % for surface temperature, 2,80 % for strip axis temperature, and 1,20 % for the average temperature.

More accurate than the presented analytical methods are numerical methods, as they enable the determination of temperature over the whole area of the stock cross-section. When analyzing the results of performed computer simulation, a distribution of temperature on the cross-section of the air cooled strip can be observed (Figure 1.). The boundary conditions taken for the numerical analysis of the cooling process were similar as for the experimental conditions. Computer simulation results correspond with a satisfactory accuracy to the temperature distribution results measured during the real process, as well as to the results obtained from mathematical models. Owing to their computation speed and accuracy, numerical methods enjoy an increasing popularity, in contrast to analytical methods. Furthermore, they enable the prediction of the temperature distribution of strip supplied to the rolling stand prior to rolling. This gives process engineers a capability of designing correctly rolling processes for new products in a particular rolling mill.

In the case of using numerical methods, the error related to real conditions was: 3,64% for surface temperature, 2,80 % for strip axis temperature, and 0,42 % for the average temperature.

The distribution of temperature of air cooled strip depends both on the parameters of the process itself, including material heating temperature and cooling time, and on the ambient temperature. The temperature distribution depends also on the thermophysical properties of cooled material, such as thermal conductivity λ, specific heat C_p, and density ρ.

CONCLUSIONS

Upon carrying out the investigation of the distribution of temperature of air cooled 60S2A steel strip and comparing the results of this investigation with the real values of temperature distribution, as measured during the process of rolling flat bars in the bar rolling mill by using an optical pyrometer, the following conclusions can be drawn:

- results obtained by using both analytical and numerical methods agree, with a high accuracy, with the experimental results (Table 2.);
- owing to their computation speed and ability to completely and accurately reflect temperature distributions over the cross-section of cooled strip, numerical methods are more advantageous when designing new technological processes (Figure 1.);
- temperature distribution results obtained from performed computer simulation are close both to the results calcu-

lated using mathematical models and the results measured during the process;

- differences in temperatures calculated, respectively, by using mathematical and numerical methods are caused by numerous factors, including the assumptions taken and the initial conditions.

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