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ESTIMATION OF ACTIVATION ENERGY FOR CALCULATING THE HOT WORKABILITY PROPERTIES OF METALS

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The hot workability of metals is most commonly described by the hyperbolic-sine constitutive equation, which relates stress with strain rate and temperature. In the present paper a simple method for calculating the parameters of this equation is introduced. Hot compression experiments of an AA6082 aluminium alloy were carried out to check the reliability of the proposed method. Based on analysis of the experimental results, a simple semi-empirical equation to describe the flow stress as a function of strain, strain rate and temperature was formulated. A good agreement between the predicted and the experimental data was achieved.

Key words: *hot working, activation energy, hot compression test, flow curves, aluminium alloys*

Određivanje aktivacijske energije za proračun pri toploj preradi svojstava metala. Prerađivačka svojstva metala u toplom stanju obično opisuju sa hiperboličko-sinusnim jednadžbama, gdje je naprezanje funkcija brzine deformacije i temperature. U članku je predložena jednostavna metoda za izračunavanje tih parametara. Metoda je provjerena na temelju tlačne probe tople deformacije aluminijske slitine AA6082. Na osnovu analize eksperimentalnih rezultata bila je formulirana jednostavna polu-empirijska ovisnost za opis naprezanja tečenja kao funkcije deformacije, brzine deformacije i temperature. Analiza iskazuje dobru skladnost između eksperimentalnih i prognoziranih vrijednosti.

Ključne riječi: *topla prerada, aktivacijska energija, topla tlačna proba, krivulje tečenja, Al slitine*

INTRODUCTION

Recent rapid advances in hot working, such as controlled hot rolling, extrusion or forging, demand a knowledge of precise and accurate process variables as to maximize the production efficiency. Analysis of metal forming operations relies on an accurate knowledge of the flow behaviour of the workpiece under a variety of operating conditions. This knowledge is contained in constitutive equations, which relate plastic stress to variables such as strain, strain rate and temperature. Such data are usually determined by thermo-mechanical testing.

For the purposes of analytical or computer modelling of hot working operations, it is necessary to express the relationship between flow stress, temperature, strain and strain rate mathematically [1 - 3].

In the present work we first propose and explain a method for calculating material parameters and activation energy from experimentally obtained flow curves, which we are convinced gives more accurate results and is much faster than widely used graphical or nonlinear regression methods [4 - 6].

Furthermore, a mathematical expression based on the theory of Knocks and Mecking [2, 7] is given for description of the flow curves for those aluminium alloys under hot working conditions for which dynamic recovery is the only softening mechanism. Hot compression experiments on AA6082 aluminium alloy were carried out to check the reliability of the proposed method.

MODELLING

It is now well accepted that the influence of strain rate and temperature on steady state flow stress during hot working operations could be satisfactorily expressed with the equation proposed by Sellars and Tegart [8 - 10]:

$$Z = \dot{\varepsilon} \exp\left(\frac{Q}{RT}\right) = A(\sinh \alpha\sigma)^n \tag{1}$$

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where Z is the temperature compensated strain rate known as the Zenner-Hollomon parameter and a the inverse of flow stress, which indicates when the equation becomes a power or an exponential function. At low values of stress (for as $\alpha\sigma < 0,8$) equation (1) reduces to a power relationship of the form:

$$Z = \dot{\varepsilon} \exp\left(\frac{Q}{RT}\right) = A(\alpha\sigma)^n = A'\sigma^n \tag{2}$$

and at high stress values (for $\alpha\sigma < 1,2$) it simplifies to:

$$Z = \dot{\varepsilon} \exp\left(\frac{Q}{RT}\right) = A2^{-n} \exp(n\alpha\sigma) = A'' \exp(\beta\sigma) \tag{3}$$

Constant n is the inversely temperature compensated strain rate sensitivity. At high T it can be described physically as a structural factor, which describes how many activation sites in the material will be activated during the deformation process [11]. Q is activation energy and R is the gas constant.

We can find all the parameters of the Sellars equation by fitting experimental data to this model. For this, equation must be firstly expressed logarithmically and properly arranged. Thus

$$\ln(\sinh \alpha\sigma) = \frac{\ln \dot{\varepsilon}}{n} + \frac{10^4 Q}{10^4 RnT} - \frac{\ln A}{n} \tag{4}$$

Next we define function χ^2 as

$$\chi^2 = \sum_{i=1}^{N} \frac{(z_i - a_1 x_i - a_2 y_i - a_3)^2}{e_i^2} \tag{5}$$

where N is the number of measurements, $z_i = \ln(\sinh \alpha\sigma_i)$, $x_i = \ln \dot{\varepsilon}_i$ in $y_i = 10^4\, T^{-1}$. Parameter $a_1 = n^{-1}$, $a_2 = 10^{-4} Q n^{-1} R^{-1}$ and $a_3 = n^{-1} \ln A$. For the error calculation we take into account only measurement errors of the parameter z_i, given by $e_i = \alpha e_i^\sigma \coth \alpha\sigma_i$, where e_i^σ are the measurement errors of the flow stress. We explain this assumption later in the text.

Minimization of the equation (5) is in general quite complicated because of its non-linearity. Such functions usually have a lot of local minima where also the best algorithms could be easily captured. To avoid these situations we must have very good first guess values, but even then we cannot be sure if the minimum is a global or a local one. For this reason we rather try to find a different approach.

If we know the parameter α, then after, $k = 1, 2, 3$ we obtain a system of three linear equations with three unknowns with the solution:

$$a_1 = \frac{(SS_{yy} - S_y^2)(SS_{xz} - S_x S_z) - (SS_{xy} - S_x S_y)(SS_{yz} - S_y S_z)}{(SS_{xx} - S_x^2)(SS_{yy} - S_y^2) - (SS_{xy} - S_x S_y)^2}$$

$$a_2 = \frac{(SS_{xx} - S_x^2)(SS_{yz} - S_y S_z) - (SS_{xy} - S_x S_y)(SS_{xz} - S_x S_z)}{(SS_{xx} - S_x^2)(SS_{yy} - S_y^2) - (SS_{xy} - S_x S_y)^2}$$

$$a_3 = \frac{(S_z - a_1 S_x - a_2 S_y)}{S} \tag{6}$$

where we defined the following sums

$$S = \sum_{i=1}^{N} \frac{1}{e_i^2}\ ,\ S_x = \sum_{i=1}^{N} \frac{x_i}{e_i^2}\ ,\ S_y = \sum_{i=1}^{N} \frac{y_i}{e_i^2}\ ,\ S_z = \sum_{i=1}^{N} \frac{z_i}{e_i^2}$$

and

$$S_{xx} = \sum_{i=1}^{N} \frac{x_i^2}{e_i^2}\ ,\ S_{yy} = \sum_{i=1}^{N} \frac{y_i^2}{e_i^2}\ ,$$

$$S_{xy} = \sum_{i=1}^{N} \frac{x_i y_i}{e_i^2}\ ,\ S_{zz} = \sum_{i=1}^{N} \frac{x_i z_i}{e_i^2}\ ,\ S_{zy} = \sum_{i=1}^{N} \frac{z_i y_i}{e_i^2}$$

The calculated parameters of equation (6) should yield the minimum of function (5). Now, the idea is to take α as a known parameter and try to find solution as

$$Q = Q(\alpha),\ \ n = n(\alpha),\ \ A = A(\alpha) \tag{7}$$

which for each prescribed α minimizes equation (5). With this procedure instead of solving a system of four nonlinear equations with four unknowns, we seek for the minimum of the χ^2 function, which is now a function of one parameter only:

$$\chi^2 = \chi^2(\alpha) \tag{8}$$

This could be done by "Golden Section Search" or by any other method for finding the minimum of a function of one variable.

As we mentioned earlier, we take into account only measurement errors of the flow stresses. More rigorously, measurements errors should be written as

$$e_i^2 = (\alpha\sigma_i \coth\alpha\sigma_i)^2 (r_i^\sigma)^2 + a_1^2 (r_i^{\dot{\varepsilon}})^2 + \left(\frac{10^4 a_2}{T_i}\right)^2 (r_i^T)^2 \quad (9)$$

where $r_i^\sigma = e_i^\sigma / \sigma_i$, $r_i^{\dot{\varepsilon}} = e_i^{\dot{\varepsilon}} / \dot{\varepsilon}_i$ in $r_i^T = e_i^T / T_i$ means relative errors. The assumption is justified if

$$(\alpha\sigma_i \coth\alpha\sigma_i)^2 (r_i^\sigma)^2 >> a_1^2 (r_i^{\dot{\varepsilon}})^2 + \left(\frac{10^4 a_2}{T_i}\right)^2 (r_i^T)^2$$

A rough calculation shows that this is true if the relative errors of temperatures and strain rates are an order of magnitude smaller than the relative errors of flow stresses. If this is not true, then each case must be analyzed separately. In the case that we must use equation (9) for errors, than a different procedure must be found.

Once we have found the parameters, we need a function which describes how flow stress changes with applied strain for a given strain rate and temperature. The flow curves for materials that undergo dynamic recovery under loading conditions of constant strain rate and temperature are characterized by an initial increase of flow stress with deformation to a steady state value where a steady state sub-structure is developed. It is known that the dislocation density during hot working depends on two competing processes: work hardening and dynamic recovery (softening). Kocks and Mecking [2, 7] have followed a phenomenological approach (KM model) to predict the variation of dislocation density with strain. The model is based on the assumption that the kinetics of plastic flow are determined by a single internal variable - dislocation density ρ. In this model the dislocation storage rate is proportional to $\rho^{1/2}$, and the annihilation of dislocations is proportional to ρ. Thus we can write:

$$\frac{d\rho}{d\varepsilon} = k_1\sqrt{\rho} - k_2\rho \quad (10)$$

where k_1 is a constant which represents hardening, and k_2 is the softening parameter which represents recovery of dislocations and is temperature and strain rate dependent. It has been found for steels and for aluminium that the high temperature flow stress is related to the density of dislocations within the subgrains by the relationship [2]:

$$\sigma = \alpha G b\sqrt{\rho} \quad (11)$$

where α is a dislocation interaction term which is between 0,5 to 1,0 for most metals, G is the shear modulus and b is Burger's vector. Differentiation of equation (11) with respect to ε and combining it with equation (10), after some rearrangements, yields:

$$\frac{d\sigma}{d\varepsilon} = \theta_o\left(1 - \frac{\sigma}{\sigma_s}\right) \quad (12)$$

where instead of $\alpha Gbk_1/2$ we write θ_o and instead of $\alpha Gbk_1/k_2$ we write σ_s. The value of θ_o can be obtained from the slope of experimental flow curves and the saturated stress σ_s for the specific thermo-mechanical conditions ($\dot{\varepsilon}$ and T) could be calculated from equation (1) once we have found the parameters α, n, A and Q (e.g. with the proposed method). Equation (12) can be simply integrated yielding:

$$\sigma(\varepsilon) = \sigma_s\left(1 - \exp\left(-\frac{\theta_o\varepsilon}{2\sigma_s}\right)\right) \quad (13)$$

and by substituting σ_s from equation (1) we finally have an explicitly written constitutive formula

$$\sigma(\varepsilon,\ \dot{\varepsilon},\ T) = \alpha^{-1} A\sinh\left(A^{-1}\dot{\varepsilon}\exp\left(\frac{Q}{RT}\right)\right)^{\frac{1}{n}} \cdot$$
$$\cdot\left(1 - \exp\left(\frac{\alpha\theta_o\varepsilon}{2A\sinh\left(A^{-1}\dot{\varepsilon}\exp\left(\frac{Q}{RT}\right)\right)^{\frac{1}{n}}}\right)\right) \quad (14)$$

EXPERIMENTAL

Cylindrical specimens of 6082 alloy, produced in Impol, Slovenia, as alloy AC30, with the typical chemical composition shown in Table 1. were machined from cast and homogenized bar.

Table 1. **Typical chemical composition of 6082 Al-alloy**
Tablica 1. **Prosječni kemijski sastav Al slitine 6082**

Element	Si	Fe	Cu	Mn
weight / %	0,7 ... 1,3	< 0,50	< 0,10	0,4 ... 1,0
Element	Mg	Cr	Zn	Ti
weight / %	0,6 ... 1,2	< 0,25	< 0,20	< 0,10

The size of the cylinder was 15 mm in length and 10 mm in diameter, and the accuracy of machining was ± 0.05 mm. As the lubricant for reduction of friction between the alloy specimen and the tools graphite was used. Hot compression deformation tests were performed on a Gleeble 1500 thermomechanical simulator at three strain rates, i.e. 0.01 s^{-1} 0.1 s^{-1} and 1.0 s^{-1}, and at the various temperatures of 300 °C, 350 °C, 400 °C, 450 °C, 500 °C and 540 °C.

Maximal deformation (strain) of the experiments was 0,4. Because of the low oxidation of the specimens no special protective gas (N or Ar atmosphere) was needed.

The course of temperature changes during the deformation is shown in Figure 1., i.e. the specimen was heated to the 540 °C in 3 minutes at 3 °C/s (1), held for 30 s (2), then cooled at a rate of 5 °C/s to the deformation temperature (3) and after 15 s held at the deformation temperature (4) and deformed at constant temperature (5). Immediately after

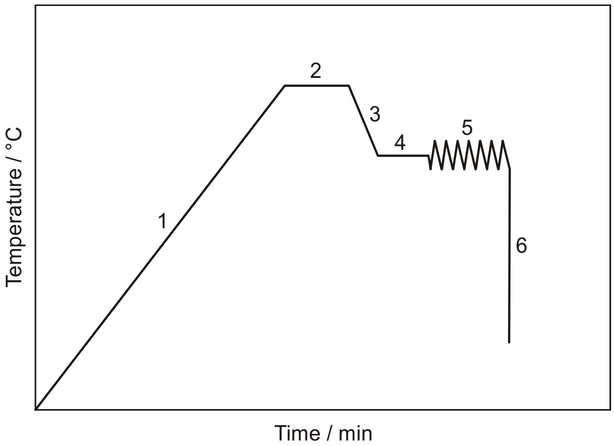

Figure 1. **Schematic representation of temperature change during a hot cylindrical compression experiment**

Slika 1. **Shematski prikaz promjene tempreture u toku tople tlačne cilindrične probe**

deformation was finished the specimen was quenched (6) to preserve the deformed microstructure. The force-length reduction curves were recorded automatically and from these curves the true strain-stress curves were calculated.

RESULTS AND DISCUSION

All the parameters obtained by minimization procedure explained above are collected in Table 2. It must be noted here that sums in equation (6) are very susceptible to round of errors, so special care is needed when summing over a lot of experimental data. For minimization of

Table 2. **Values of material parameters of 6082 Al-alloy calculated from experimental data**

Tablica 2. **Vrijednosti materialnih parametra Al slitine 6082 računatih na osnovu eksperimentalnih podataka**

Q / (kJ/mol)	n	A / s^{-1}	α / MPa^{-1}
158,4 (1±0,01)	14 (1±0,01)	$2{,}85\cdot10^{18}(1\pm0{,}04)$	0,0043 (1±0,02)

equation (8) we used the "Golden Section Search" algorithm and we had no problems in finding the minimum. As can be seen from Figure 2. where the dependence of χ^2 function on parameter a is shown, the function has a very sharp minimum at $\alpha = 0{,}0043$ MPa^{-1} and there are no other local minima in its vicinity.

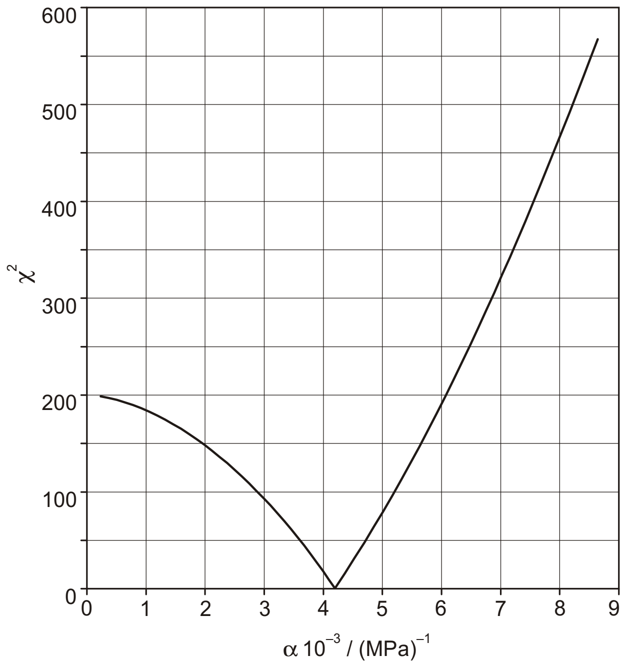

Figure 2. **Dependence of the χ^2 function on parameter α for AA6082 Al-alloy. A very sharp global minimum at $\alpha = 0{,}0043$ MPa^{-1} is clearly seen**

Slika 2. **Funkcija χ^2 u ovisnosti od parametra α za slitinu AC30. Karakterizirana je šiljatim globalnim minimumom za $\alpha = 0{,}0043$ MPa^{-1}**

The calculated activation energy is 158,4 kJ/mol and is in the range of activation energies for aluminium alloys [4 - 6, 11]. The good agreement between calculated and experimental results can be seen on Figure 3a. where the dependence of steady state stress on temperature at different strain rates is given.

It was recognized that equation (1) is an appropriate one because it very accurately describes experimental data over the whole range of temperatures and strain rates. The stress - strain curves obtained by the compression test are presented in Figures 3b-c.

The flow curves for all tested temperatures and strain rates initially exhibit a rapid work hardening until dynamic recovery balances the hardening, and a steady state is reached. As expected the flow stress decreases with decreasing strain rate and increasing temperature, which is typical behavior of metals deformed under hot working conditions [2-11]. Figures (3b-c) also show the calculated flow curves by means of equation (14) where for parameter θ_o we found a value of 4×10^9 Pa to be the most suitable.

Very good agreement between calculated and experimental flow curves was obtained. The small discrepancies between them, which can be seen on Figures 3c and 3d, can be attributed to errors in temperature and strain rate control. Another reason could also be friction between the specimen holder and the specimen. A further could also be material inhomogeneity, which can vary from sample to sample.

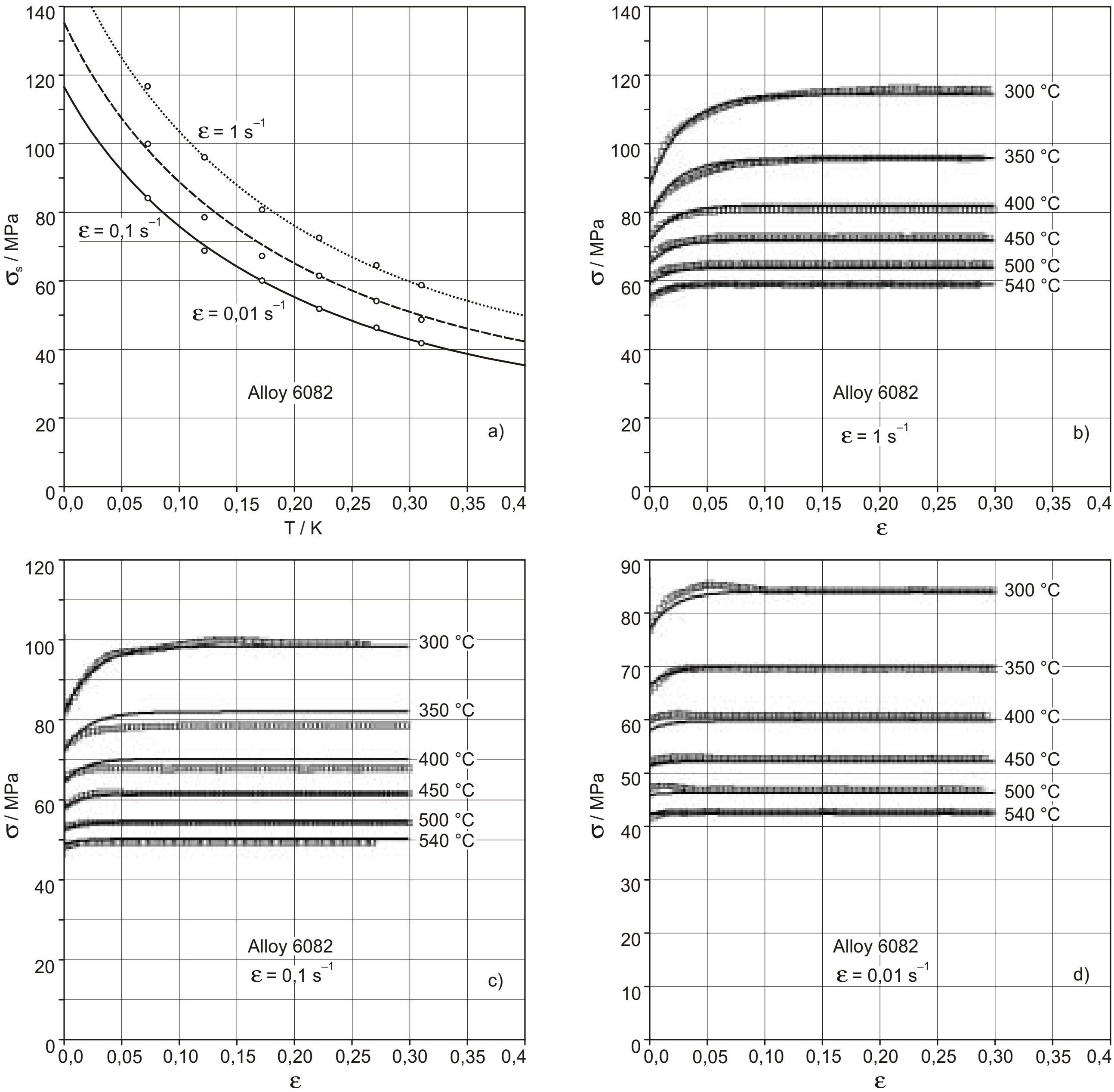

Figure 3. **Comparison between the calculated (solid lines) and experimentally (points) obtained influence of temperature on steady state flow stress at different strain rates (a) for AC30 alloy. Comparison of the calculated (solid lines) and experimental (points) flow curves for strain rates of 1 s^{-1} (b), of 0,1 s^{-1} (c) and of 0,01 s^{-1} (d) at different temperatures for AA6082 alloy**

Slika 3. **Usporedba između izračunatog (puna crta) i eksperimentalno dobivenog utjecaja temeprature na stalan napon tečenja za različite brzine deformacije (a) za slitinu AC30. Uspoređenje izračunatih i eksperimentalnih krivulja tečenja za brzine deformacija 1 s^{-1} (b), 0,1 s^{-1} (c) i 0,01 s^{-1} (d) kod različitih temperatura (slitina AA6082)**

CONCLUSION

Compression tests were carried out on AA6082 aluminium alloy in the temperature range 300 to 540 °C and for three different strain rates (0,01, 0,1, and 1 s^{-1}). It was found that for all tested temperatures and strain rates the material exhibited strain hardening and dynamic recovery. The entire set of experimental data can be well represented using the Sellars equation with activation energy of 158,4 $kJmol^{-1}$. If this equation is combined with function (13), the constitutive equation, which connects strain, strain rate temperature and flow stress can be obtained. This type of constitutive equation can be easily used for interpolation to other temperatures and strain rates.

The proposed constitutive equation and material constants (Q, α, n, A) identification method were been applied successfully to the AA6082 aluminium alloy and also appear to be valid for other materials that are characterized by dynamic recovery as the only softening mechanism under hot working conditions.

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