V. A. GRYNKEVYCH, I. MAMUZIĆ, V. M. DANCHENKO

ISSN 0543-5846
METABK 43 (3) 187-192 (2004)
UDC - UDK 621.98:004.94=111

ON DEVELOPMENT OF METHODS OF COMPUTER SIMULATION FOR PROCESSES OF METAL FORMING

Received - Primljeno: 2003-10-31
Accepted - Prihvaćeno: 2004-03-20
Review Paper - Pregledni rad

The comparative analysis of the main modern methods of the solution of metal forming boundary value problems is executed. The brief retrospective review of development of methods of the solution of metal forming boundary value problems (including methods of computer simulation) is made. The new method of computer simulation of metal forming processes. It is a method of linearization of nonlinear problems and is based on the variation of a finite element method. The outcomes of computer simulation of the upsetting process of stuffs with different flow characteristics is presented. The offered method and its modifications allow considerably to reduce time of the solution of boundary value problems, providing sufficient veracity of outcomes.

Key words: *metal forming, comparative analysis, computer simulation*

O razvoju metoda kompjuterske simulacije procesa oblikovanja metala. Dana je usporedna analiza glavnih modernih metoda rješavanja problema graničnih vrijednosti oblikovanja metala. Napravljen je kratak retrospektivni očevid razvoja metoda rješavanja problema graničnih vrijednosti oblikovanja metala (uključujući metode kompjuterske simulacije). Nova metoda kompjuterske simulacije procesa oblikovanja metala je metoda linearizacije nelinearnih problema a zasniva se na varijaciji metode konačnih elemenata. Prikazani su rezultati kompjuterske simulacije postupaka savijanja materijala različitim karakteristikama protoka. Ponuđena metoda i njene modifikacije omogućuju značajno smanjenje vremena rješavanja problema graničnih vrijednosti, osiguravajući dovoljnu istinitost rezultata.

Ključne riječi: *oblikovanje metala, usporedna analiza, kompjuterska simulacija*

INTRODUCTION

All modern methods of the solving of the boundary-value problems of metal forming (MF) can be divided rather tentatively into the following large groups:
- methods based on some physical law of conservation,
- methods having variation principles as a basis,
- methods relaying on other principles.

To the first group belong approaches framed on the law of conservation of momentum (equilibrium), substance conservation law (condition of incompressibility), energy conservation law (including heat energy), principle of conservation of the moment of momentum etc. Usually such an approach comes to forming and solving the systems of differential or integral equations. As applied to MF problems, these can be: Stokes differential equations (flow of the viscous incompressible fluid), differential equilibrium equations in stresses; integral (including boundary) equations obtained on the basis of the theorem of reciprocity of the works or the method of fictive loads; differential equations of heat conduction, equations of the heat power balance. In combination with powerful modern numerical methods (finite element and boundary-element methods, finite difference method) such an approach allows to formulate and to solve sufficiently broad class of problems of plastic deformation, including 3D problems.

The second group of methods is based on some variation principles of mechanics and thermal physics. In this case the solution of the problem is obtained from condition of stationarity of the functional corresponding to the chosen variation principle. While solving the boundary-value MF problems, the most used formulations are these on the basis of Lagrange's or Markov's (Hermann) functional. As an instrument of discretization, mostly used is the finite-element method, sometimes - variation-difference method.

V. A. Grynkevych, V. M. Danchenko, National Metallurgical Academy of Ukraine, Dnepropetrovsk, Ukraine, I. Mamuzić, Faculty of Metallurgy University of Zagreb, Sisak, Croatia

Since the 50-es of the past century, the variation approach in combination with the finite element method has found a wide use in the solving of boundary value problems of a linear theory of elasticity (heat conduction). This caused a good theoretical development and, as a consequence, sufficiently simple adaptation to MF problems. It may be argued that the given group of methods is today most often applied to solving of the technological MF problems.

Methods based not on physical, but on some mathematical principles may fall into the third group. They may be classified as follows: different formulations of the method of fluidized discrepancies (methods of subregions, dot collocation, Galerkin's method), method of flow functions and method of the least squares (which in a sense may be related both to the variation method and to the method of fluidized discrepancies) etc.

COMPARATIVE ANALYSIS AND THE DEVELOPMENT OF METHODS

Let us conduct a brief comparative analysis. As the experience of many years of using the methods of all three afore-cited groups shows, the main factor influencing the reliability of results is the practical implementation - analytical or numerical. The Table 1. gives comparison of analytical and basic numerical approaches: the finite element method (FEM), boundary element method (BEM), method of grids (finite difference method - FDM). As criteria of comparison: accuracy, simplicity of implementation, speed of obtaining results, requirements to computing resources were chosen. Criterion "required parameters" characterizes applicability of each of approaches, including applicability for the control of MF processes. As obvious from the Table 1., though all the methods of implementation correspond, as a whole, to the claims laid at present, any of them satisfies criteria of optimality.

Table 1. **Comparative analysis of existing methods of solution the boundary problems of MF**
Tablica 1. **Usporedna a naliza postojećih metoda rješavanja graničnih problema oblikovanja metala**

	Analytical methods	FEM	BEM	FDM	Required parameters
Accuracy	High (for solutions obtained)	High at sufficient qualification	High, as a rule	Moderate (other things being equal)	Minimum indispensable for the concrete process
Simplicity of realization	High (for solutions obtained)	Moderate	Low	High (for regions of the simple form)	Moderate
Speed of obtaining solution	Very high	Moderate or low	Moderate	Moderate or high	High or very high
The requirements to computing resources	Low	High	Moderate	Moderate or high	Moderate or low

Historically, the development of methods for the solution of the boundary MF problems was performed step by step. In the Table 2. are presented their approximate time scope, principal problems, which were solved as a whole, and methods of solution too.

As obvious from the Table 2., the main recent tendencies of the development of methods of solution may also be clearly followed:

- deepier account of physics of the process of deforming (density, motion of dislocations and disclinations, reactions between them, presentation of inclusions of different kind and so on),
- presentation of the complex rheological properties of medium (including bimetals, reinforced, hardened by dispersion, powder metals), light and super light metals and alloys (including these on the basis of titanium, magnesium, lithium and so on),
- creation of the fast algorithms and methods suitable for operational control of the MF processes,
- creation of the integrated programm complexes combining not only apparatus for the solution of the boundary problem, but also the expert systems, systems of artificial intelligence allowing to solve a great number of problems on the development of optimum technological processes.

One may suppose that the tendencies mentioned above will remain up to the end of the present decade. It will be favored both by industrial demand and development of the hardware potentiality of computer technique.

It is possible also to predict appearance of the new numerical methods of the solution of the boundary MF problems, alternative as to FEM, which would combine power, adaptability and reliability of results to be obtained and considerably higher speed of obtaining results.

Experience of application of programs based on calculation of the stress - strain state of metal in MF processes has shown that such an approach at designing and development of technology allows to save considerable material resources being spent for the carrying out of the trial rolling, extrusion, die-stamping and manufacture of trial equipment. The complex and deep mathematical simulation allows by means of computing experiments to develop technologies of MF, to specify rational temperature ranges of treatment.

At the same time it is necessary to note that the modern software packages based on the numerical methods

Table 2. **The main stages of development of methods for solution the boundary MF problems**
Tablica 2. **Glavne faze razvoja metoda za rješavanje graničnih problema oblikovanja metala**

Period	Content of the stage	Basic methods of solution
The 20-es - 50-es of the XX century	Determination of some integral characteristics of the process total force, average contact pressure, average broadening, average coefficient of friction (by the way of conversion) and so on	Engineering (in terms of the content reducing the problem to unidimensional one and rather crude assumptions)
The 50-es - 70-es	Determination of fields of the stress tensor and velocity vector for the 2D problems and ideally plastic medium	Method of slip lines; variational method in a classic statement; method of conformal mappings and so on
End of the 70-es - 80-es	Determination of fields of the stress tensor, velocity vector and temperatures for the 2D-problems and medium with deformation and velocity hardening	Variational method in a classic statement; the same on the FEM basis
The 90-es	The same for the 3D problems in view of zone of deformation of unspecified shape, complex conditions of the contact interaction. Creation of the programmatic complexes for industrial application	Mainly on the FEM basis
Beginning of the XXI century	The same, but in view of physics of the process of deforming, complex rheological properties of the medium, including the composite and powder materials, light and super light alloys and so on. Development of programmatic complexes for industrial application	FEM, BEM, method of the neuron networks

(finite element method, boundary element method, finite difference method and so on.), demand large computing resources. It concerns especially three-dimensional problems when the number of unknowns reaches several tens thousands; in these cases even on the modern powerful personal computers the time of problem solution is measured by hours and days.

One may infers that in spite of all their power and depth of analysis the modern numerical methods are unsuitable for the solution of problems of operating control of MF processes - rolling, forging etc. Therefore, in the systems of the control for the actual MF processes are used, as a rule, algorithms, based on processing data of laboratory and industrial experiments. Such algorithms work effectively in the narrow range of parameters of technological processes - change of the latter requires to carry out new experiments and to treat their results. Besides, while carrying out laboratory and industrial experiments, it is impossible (or nearly impossible) to take into account influencing of many important factors of technological processes: temperature field in zone of deformation, state of stress and strain, change in microstructure etc. A presentation of these factors is possible at numerical simulation, but it needs the carrying out of preliminary computing experiments.

Thus, the problem of development of methods for the solution of problems of plastic deforming, which would combine adaptability, depth of analysis, accuracy of the modern numerical methods and speed of the solving suitable to the systems of the control for MF processes, has both scientific and practical importance.

Solution of the boundary MF problems in terms of the content represents the solving of the problems of mathematical physics with essential nonlinearity of physico - mechanical properties of material to be deformed. Presentation of physical nonlinearity of the material being deformed represents one of the fundamental problems. At present the most used methods of linearization of nonlinear problems are:

- method of elastic solutions,
- method of hydrodynamic approximations (MHA),
- method of additional stresses,
- method of additional loads.

The first two methods are nearly identical as to their mathematical content and represent substitution of the solution of the problem for the non-linear medium with the sequence of solutions of elastic (linear-viscous) problems for the linear but inhomogeneous media - on each iteration is performed the tuning of the modulus of elasticity μ or of conventional viscosity G. These algorithms are described in detail in the literature [1, 2]. Note that methods of elastic solutions and hydrodynamic approximations have the substantially higher speed of convergence than methods of additional stresses and additional loads. Usually only several iterations are needed to reach the necessary accuracy.

One more fundamental problem arising at the solving of the boundary MF problems consists in nonlinearity of boundary conditions. The stresses of friction and relative velocity of sliding on the contact surface of metal with tool depend on each other, in this way, the solution of this problem requires some iterative algorithms too.

Each iteration of this kind is rather cumbersome in computing sense: counting of integrals in the volume and surface of the zone of deformation, forming the system of linear algebraic equations and its solving.

It appears to be expedient to develop an alternative approach to the solution of the problems of such kind -

approach, which would considerably reduce the required quantity of iterations or eliminate them at all. Generalization of the theorem on reciprocity of the works was formulated and proved in the paper [3] - integral equations, linear relative to the unknowns of the problem are obtained. After discretization of initial integral equations solving the system of linear algebraic equations is in order. The given approach is applicable to the solution of the boundary elastoplastic problems, in the case, when the change (speed of the change) of the volume is substantial. This approach referred conventionally to as "direct solution method" supposes availability of the so-called "basic" solution of a similar boundary problem for the physically linear medium. Obtaining such a solution represents a rather difficult problem, which itself could also require considerable computing resources. Besides, to the presentation of the nonlinearity of boundary conditions, in this case some quantity of iterations would be in order, too.

An approximated algorithm allowing, on the basis of the solution of the boundary-value problem for the linear - elastic (linear - viscous) medium, to determine components of stress - strain state in the vicinity of selected point of the zone of deformation was suggested in the paper [4] - discrete variable viscosity method (DVVM).

The suggested approach is based on the known in theory of elasticity Betty identity:

$$\sigma'_{ij}\varepsilon''_{ij} = \sigma''_{ij}\varepsilon'_{ij} \tag{1}$$

where: σ_{ij}, ε_{ij} the components with one prime represent the known ("basic") solution and components with two primes - solution to be found.

Apparently, by virtue of the linearity of connection between tensors of stresses and deformations (strain - rate for the linear - viscous medium) equation (1) is to be true in vicinity of any point of the body under deformation. The generalization of expression (1) was obtained for the case of physically nonlinear media to be deformed:

$$\bar{\varepsilon}''^{p}_{ij} = \bar{\varepsilon}''^{e}_{ij} + \frac{2G - 2G''}{2G''}\bar{\varepsilon}''^{e}_{ij} \tag{2}$$

In this case the following assumptions were adopted:

- there is a solution of the boundary problem for the linear - viscous medium satisfying boundary conditions both in stresses and in displacements (velocities),
- boundary conditions are linear,
- the medium to be deformed is subject to the points of the theory of the plastic flow of Saint-Venant - Lévy - Mises (the functional dependence is set between deviators of stresses and strain - rate),
- medium is supposed to be incompressible.

In expression (2) G represents conventional viscosity to be used while obtaining the linear solution, G'' - conventional viscosity being in agreement with solution for the nonlinear medium with given rheological characteristics. As a first approximation let us suppose that the flow stress does not depend on temperature, degree of deformation and is only function of the strain - rate, i.e.:

$$G'' = \frac{T(H)}{H} \tag{3}$$

where:

T - intensity of shearing stresses,
H - intensity of shear strain - rates.

Algorithm of determination of conventional viscosity in selected points of the zone of deformation can be described as follows:

1. Determination of intensity of shear strain - rates H in agreement with solution for the linear - viscous medium.
2. Determination of conventional viscosity G'', expression (3).
3. Determination of components of the tensor of rates of deformations for the medium with a given rheological curve $T(H)$, expression (2).
4. Determination of the value H on the basis of their values.
5. Conversion of conventional viscosity G'' in terms of results of item 4.

As a matter of fact, the given algorithm contains two iterations: the first coincides with MHA, the second is an extrapolation in accord with the given rheological curve. When using the given algorithm, the key problem is obtaining of linear medium solution. As a method of obtaining the basic solution, in the beginning the finite element method was chosen.

For testing the developed algorithm of the numerical modelling was carried out for the process of upsetting cylindrical sample with ratio of initial height to diameter equal to 1. To eliminate the influence of friction between metal and tool, condition of sticking was assumed along the contact surface.

The change of form was simulated by means of the program FORGE developed by one of the authors of presented paper V. A. Grynkevych. In the program, the variational - energy approach (minimization of Lagrange's functional) is used. The condition of incompressibility will be realized by means of the method of penalty functions. The presentation of rheological properties of the metal is provided by the method of hydrodynamic approximations. As an initial approximation, the temperature field was assumed as homogeneous.

Upsetting with degree of deformation in height 30% was simulated as running in six stages at 5 % each. The rheological curve of the medium to be deformed was described by exponential function of the kind $T = H^b$, where the exponent b in calculations was taken as 0,9, 0,7, 0,5, 0,3, 0,2, 0,15, 0,10, 0,05.

The calculation for each rheological curve was carried out in this case both according to the method of hydrodynamic approximations and to the above - cited algorithm. Comparison was carried out in terms of the finite change of the billet form and distortion of initial finite element grids.

It was noted that for the values of the exponent b equal to 0,9, 0,7 and 0,5 coincidences of results were very good what, however, was possible to forecast. For the exponent equal to 0,3 the coincidence in kinematic and deformational parameters remains rather good.. A rather good coincidence in the field of rates of deformation and satisfactory in distribution of the value H is observed for the exponent 0,2. As to even more "nonlinear" rheological curves, the good coincidence is observed only along the vertical component of velocity vector.

Comparison of the finite forming was carried out in terms of distortion of initial finite - element grids calculated by different methods for different values of the exponent b. The superposed finite - element grids calculated according to the MHA and DVVM for the values of exponent b conformably 0,2 and 0,05 are represented in the Figures 1. and 2.

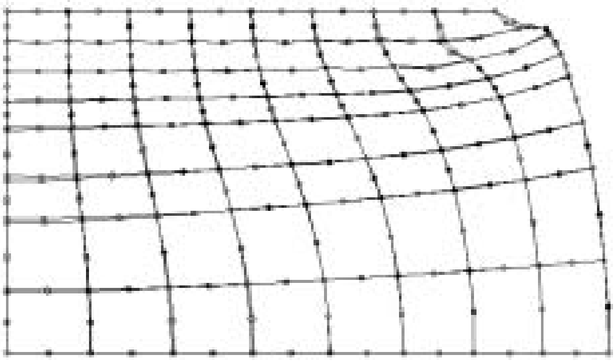

Figure 1. **A final forming of sample after 30 % upsetting at b = 0,2**
Slika 1. **Finalno oblikovanje uzoraka nakon 30-postotnog savijanja za b=0,2**

As follows from the Figure 2., even at rather poor coincidence of fields of speeds and rates of deformations the final forming (for the given problem) gives good convergence.

Thus, it is shown that the given approach can be successfully employed for the nonlinear - viscous media with exponential rheological curves in the range of exponent values 0,9 … 0,2.

Then the solution of the same problem was performed for the media with the work hardening. In particular, for steels 10 and 35CrNi2Mo with corresponding rheological curves [5]:

$$T = 12231\ e^{-0.00250}\ 1.494\ \Lambda^{0.174} \cdot 0.726 H^{0.139} \tag{4}$$

$$T = 15.738\ e^{-0.00275}\ 1.620\ \Lambda^{0.210} \cdot 0.727 H^{0.141} \tag{5}$$

In the cited expressions the temperature was assumed as constant (1000 °C); L is accumulated degree of shear deformation after V. L. Kolmogorov.

It was established that for the given media with the work hardening the coincidence of results in the field of velocities, in the field of strain - rates and in final forming

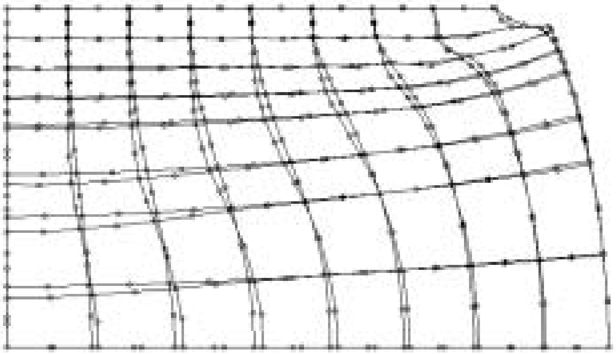

Figure 2. **A final forming of sample after 30 % upsetting at b = 0,05**
Slika 2. **Finalno oblikovanje uzoraka nakon 30-postotnog savijanja za b=0,05**

was rather good. Relative errors in solutions for these steels and for the nonlinear - viscous media with the exponent 0,3 turned to be of the same order. It is explained by the fact that in each stage of upsetting degree of deformation L was determined as multiple product of the value H into the change in time $\Delta\tau$.

Thus, the summary exponent in rheological curve was increased and fell within the range of good coincidence with solution by the method of hydrodynamic approximations. It is possible to suppose that the discrete variable viscosity method is applicable for simulation of plastic deformation of steels and alloys with rheological curves described by exponential functions of the kind (4, 5), i.e. for the solving of problems of MF. Quality of solutions obtained by the developed method and by the method of hydrodynamic approximations is comparable; yet the speed of solving is substantially higher - for performed calculations speed factor is in average 2,43.

Estimation of the proposed approach was carried out and at simulation of the axial symmetric upsetting of the billets from materials with complex rheological characteristics (having if only one maximum on the rheological curve) [6]. The procedure of obtaining such rheological relationships is presented in [7, 8]. It was shown that the discrete variable viscosity method qualitatively and correctly models the effect related to localization of the plastic deformation for media with substancial softening. As to final forming of a sample, here the good quantitative coincidence with method of hydrodynamic approximations has place too. Substantial discordance of results, as well as in [4], is observed in the zone of maximum values of *H*.

The modification of this algorithm, in which at determination of conventional viscosity in the points of the zone of deformation the influence of the neighbour nodal points is taken into account by means of the of weight coefficients method, was proposed too [9]. It was shown that such a presentation allows in a number of cases to decrease substantially the average error of solution.

At implementation of the DVVA at each stage of deforming one has to carry out iterations for revision of the nonlinear boundary conditions on the contact of metal and tool. Apparently, the most “energy - intensive” blocks of the given algorithm are constructing the reference field of velocity and constructing the actual field of velocity after revision of distribution of conventional viscosity.

However, there is a scope able to accelerate still more the obtaining of the solution. In the paper [10] the so-called theorem of the state was formulated and proved - the theorem connecting the components of velocity and stress vectors in a point of boundary of two deformed bodies - one linear - viscous and other with the given rheological dependency created by the action of identical system of external loads. This circumstance opens the chance to determine unknown components of velocity and stress vectors by the known solution of analogous problem for the linear - viscous body, boundary conditions and components of the tensor of the rate of deformation immediately in a point of boundary of a body.

This theorem allows also to refine computing - experimental methods of the solution of the boundary MF problems permitting to determine components of the stress tensor by means of the known from the experiment field of velocity [11].

CONCLUSION

In the case of its successful testing and an available and sufficiently fast algorithms of obtaining the linear solutions, the discrete method of the direct solving can become one of the most effective methods of solving the problems of plastic deforming.

REFERENCES

[1] A. A. Iljušin: Continuum mechanics, MGU, (1971), 248.
[2] G. Ja. Gun: Mathematical modeling of processes of metal forming. Metalurgija, (1983), 352.
[3] V. A. Grinkevič: On the method of the direct solution the boundary elastoplastic problems, Tom 3, Zbirnik naukovih prac, Dnipropetrovsk, Navčalnaja kniga, (1998).
[4] V. A. Grinkevič: The discretic variable viscosity method of solution of the problems of metal plastic deforming, Metalurgič. i gorno-rudn. prom-st. 1 (2001), 64 - 67.
[5] A. Henzel, T. Špitel: Calculation of the energy-force parameters in processes of metal forming, Sprav. izd.: Per. S nem., Metalurgija, (1982), 360.
[6] V. A. Grinkevič: Application of the discrete method of variable viscosity for simulation of plastic deformation of metals with complex rheology, Teorija i praktika metalurgii, 5 (2001), 25 - 29.
[7] Ja. S. Švarcbart: Interconnection of the high-temperature processes of the work hardening, stress relaxation and creep of metals, Izvestija Akademiji nauk, Metali, 4 (1978), 137 - 144.
[8] I. G. Zuev, G. S. Nikitin, Classification of kinds of curves of metal high-temperature work hardening and their analytical description, Izvestija Akademii nauk, Metali, 1 (1984), 138 - 144.
[9] V. A. Grinkevič: Computer simulation of the process of upsetting by means of the modified discretic variable viscosity method, Udoskonalenhaja procesiv ta obladnannja obrobki tiskom u metalurgii ta mašinobuduvanni: Zbornik naukovih prac, Kramatorsk, 2001.
[10] V. A. Grinkevič: On the of boundary conditions of the boundary problems of the theory of plasticity, Sučasni problemi metalurhii, Tom 5, Plastična deformacija metaliv, Dniepropetrovsk: Sistemni tehnologii, (2002), 47 - 51.
[11] V. A. Grinkevič, G. G. Šlomčak, V. N. Dančenko: Experimentally - computational method of solution the boundary MF problems, Sučasni problemi metalurgii, Tom 5, Plastična deformacija metaliv, Dniepropetrovsk: Sistemni tehnologii, (2002), 115 - 118.