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THEORETICAL BOUNDS ON THE ELECTRICAL CONDUCTIVITY OF SINTERED MATERIALS AND THEIR RELATION TO BOUNDS ON THE YOUNG'S MODULUS

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The constraints to the effective electrical conductivity were derived as function of distribution of material electrical conductivity throughout the sample. If the sample consists only of homogeneous isotropic matrix and pores, the constraints to effective electrical conductivity take the form very similar to the constraints derived previously for the effective Young's modulus of porous materials. The differences occurring in expressions are due to the fact that elastic properties of an isotropic matrix material are characterized by two quantities, e.g. Young's modulus and Poisson's ratio, while the conductivity is characterized only by one quantity. But regardless of the small differences, in the both cases the key role in determining the properties is played by the minimum load-bearing cross section of the sample considered.

Key words: *electrical conductivity, Young's modulus, porous materials*

Teoretske granice električne provodnosti sinterovanih materijala i njihov odnos prema Youngovom modulu. Zapreke učinkovitoj električnoj provodnosti izvedene su kao funkcije distribucije električne provodnosti i poprimaju oblik vrlo sličan onim smetnjama koje su ranije izvedeni za učinkovit Youngov modul poroznog materijala. Razlike koje se pojavljuju u izrazima nastale su zbog činjenice da su svojstva elastičnosti nekog izotropnog matričnog materijala okarakterizirane dvjema količinama, npr. Youngovim modulom i Poissonovim odnosom, dok je provodnost okarakterizirana samo jednom količinom, ali bez obzira na male razlike, u obadva ta slučaja ključnu ulogu u određivanju svojstva igra primjer uzorka pod minimalnim naponom.

Ključne riječi: *električka provodnost, Youngov modul, porozni materijali*

INTRODUCTION

In addition to many other merits, powder metallurgy (PM) represents a technology well suited for producing multiphase materials. PM products can consist even of phases that are usually incompatible.

Moreover, powder metallurgy enables formation of parts with microstructures that are inaccessible to other production processes [1].

As a consequence of inherent microheterogeneity, practically all properties of sintered parts are closely related to their microstructure. Total porosity level, pore shape, pore connectivity as well as material composition, microstructural and macrostructural inhomogenity have a significant influence on parameters which are important for structural applications, such as stiffness, strength, toughness, etc.

In that context, quality control of semi- and final PM products requires detailed information on the material status at the microstructure level [2]. The standard methods for evaluating the microstructural properties (such as metallography, fractography, etc.) belong to destructive test types, they are time-consuming and laborious. In addition, tests are predominantly realized by research institutes and only in a limited range also by producers and users of materials.

For practical reasons, it would be therefore helpful to evaluate the microstructure by means of another one, experimentally simple, reliable, non-destructive technique that might be applied for a quality control. Such a simple method, which could be immediately used by a producer, enabling him an effective control of the technological procedure during the material production necessary changes in the production process for obtaining the materials of required properties.

Among others, the attention is concentrated on measuring the electrical conductivity as a prospective non-destructive microstructure-evaluating method. As regards the sin-

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tered materials with a homogeneous matrix microstructure and quasi-homogeneous sample macrostructure, electric conductivity measurements revealed to be a very attractive and prospective method and its applicability for quality control was proven in principle [3].

But in the case when microgradients occur both in matrix composition and properties, and/or the sintered sample is macroscopically inhomogeneous, less information is available on the electrical conductivity relationship - microstructure - macroscopic mechanical properties. Therefore, every piece of knowledge concerning at least a part of this chain is helpful.

In this contribution, preliminary results concerning the theoretical constraints in the effective electrical conductivity of (in general heterogeneous) sintered materials are presented.

The tightest currently known bounds to the effective properties of multiphase materials depend on knowledge of the properties of constituents, relative volume fractions and one or more parameters characterizing the microgeometry. The microgeometry of the complex sample may be (and very often is) characterized by quite complicated parameters introduced, for example, through the multi-point spatial correlation functions of constituent distribution [4], etc.

However, it is turning out (e.g. [3]) that a fairly simple parameter - the load-bearing cross-sectional area - is just the relevant feature of microgeometry that significantly affects the macroscopic properties of sintered samples. Therefore, the load-bearing cross-sectional area as a function of position is chosen as the microgeometry characterizing parameter entering to the theory throughout this article.

Using the obtained constraints, and the constraints derived previously for the effective Young's modulus [5] (unfortunately, only for a porous material with a homogeneous matrix), possible common and different features of electrical conductivity and Young's modulus are partly discussed.

Theoretical results are compared with experimental ones, which were obtained for specimens made of sintered powder iron.

THEORY

Effective electrical conductivity is an "integral" parameter that is in general influenced by all microstructural features of material.

Hence, to be able to evaluate the effective conductivity exactly, it is necessary to find the electrical field and current density in the real complex body, which appears to be an impossible task.

Therefore, a more attractive approach consists in the use of varied principles in order to bound the effective conductivity. For determining the constraints on electrical conductivity values, the principle of minimum production of entropy [6] was used and various trial distributions of electrical field and current density within the sample were considered.

To simplify the theoretical analysis, a prismatic sample (macroscopic cross-sectional area A, length L) was considered (Figure 1.). The matrix material was isotropic and characterized by a local electrical conductivity $\sigma(x, y, z) = \rho^{-1}(x, y, z)$ at each point of the sample. $\rho(x, y, z)$ represents the resistivity of material. If the pores are present in the sample, $\sigma(x, y, z) = 0$ for the pores.

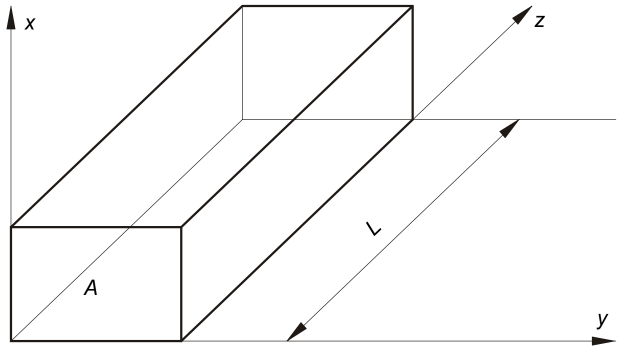


Figure 1. **Schematic sketch of the shape of the sample considered**
Slika 1. **Skica shema oblika razmatranih uzoraka**

Using a variety of trial functions and taking into account that the microgeometry of porous samples should be characterized by the load-bearing cross-sectional area, the following constraints for an effective electrical conductivity σ^*_{zz} were obtained as the most suitable till now:

$$\frac{1}{A}\iint_A \frac{\mathrm{d}x\mathrm{d}y}{\frac{1}{L}\int_L \rho(x,y,z)\mathrm{d}z} \le \sigma^*_{zz} \le$$

$$\le \left[\frac{1}{L}\int_L \frac{\mathrm{d}z}{\frac{1}{A}\iint_A \sigma(x,y,z)\mathrm{d}x\mathrm{d}y}\right]^{-1} \tag{1}$$

z axis is oriented along the length of the rod-shaped sample.

The constraints (1) can be considered as preliminary ones. The search for much tighter bounds still continues.

In the case when the sample consists only of homogeneous matrix (conductivity σ_0) and pores, the constraints (1) obtain the form:

$$J_1 \le \frac{\sigma^*_{zz}}{\sigma_0} \le \frac{1}{J_2} \tag{2}$$

where:

$$J_1 \equiv \frac{1}{A}\iint_A \frac{\mathrm{d}x\mathrm{d}y}{\frac{1}{L}\int_L \frac{\mathrm{d}z}{\Delta(x,y,z)}}, \quad J_2 \equiv \frac{1}{L}\int_L \frac{\mathrm{d}z}{\frac{1}{A}\iint_A \Delta(x,y,z)\mathrm{d}x\mathrm{d}y}.$$

$\Delta(x, y, z) = 1$ if the point (x, y, z) belongs to the matrix and $\Delta(x, y, z) = 0$ if the point (x, y, z) belongs to a pore. The quantity $A(z) = \iint_A \sigma(x,y,z)\mathrm{d}x\mathrm{d}y$ represents the load-bearing cross-sectional area at point z. The quantity J_1 is more or less connected with the minimum load-bearing cross-sectional area within the sample.

Before [5] it was derived that the constraints for an effective Young's modulus E^* of the material consisting of a homogeneous isotropic matrix (Young's modulus E_0, Poisson's ratio ν_0) and pores are of the form:

$$J_1 \le \frac{E^*}{E_0} \le \frac{1}{J_2}\left[1 + \frac{2\nu_0^2}{(1+\nu_0)(1-2\nu_0)}\frac{J_2 J_3 - 1}{J_2 J_3}\right] \quad (3)$$

where:

$$J_3 \equiv \frac{1}{LA}\iiint_{LA} \Delta(x,y,z)\mathrm{d}x\mathrm{d}y\mathrm{d}z.$$

ILLUSTRATIVE MODEL EXAMPLE

For purpose of illustration, the bounds on effective electrical conductivity values and effective Young's modulus values are evaluated for a model structure simulating a sintered material. The structure consists of uniform elementary cells arranged to the simple cubic lattice. The elementary cell represents the conjunction of a sphere of radius r and a cube of edge $2a$ $(a \le r \le a\sqrt{3})$ (Figure 2.). The

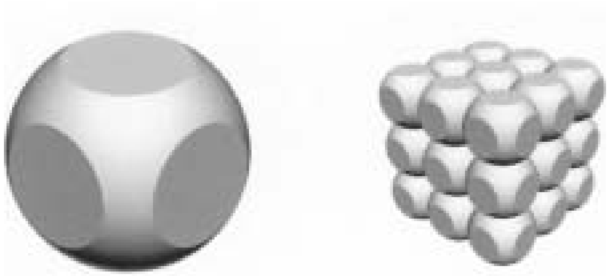
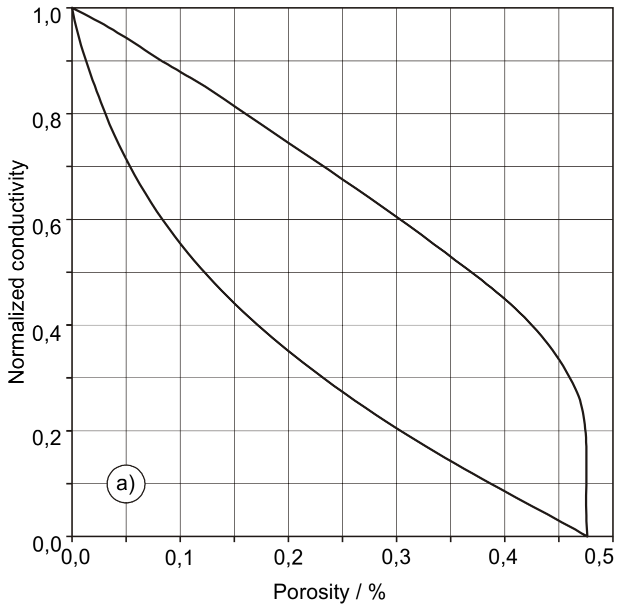


Figure 2. **Elementary cell of the structure simulating a "sintered" material (left), and a block of 27 (3×3×3) elementary cells (right)**

Slika 2. **Elementarna stanicaa strukture koja simulira "sinterovan" materijal (lijevo) i blok od 27 (3×3×3) elementarnih stanica (desno)**

change of entire porosity is caused by the change of ratio r/a. Theoretical bounds on corresponding effective parameters of the model structure are presented in Figure 3.



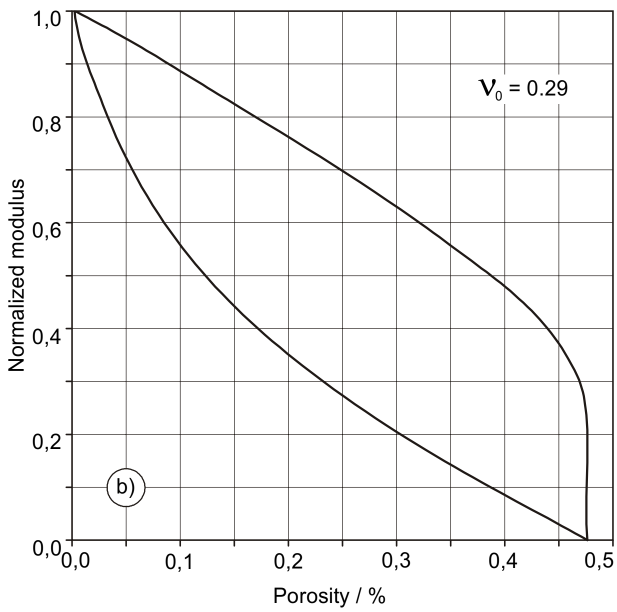
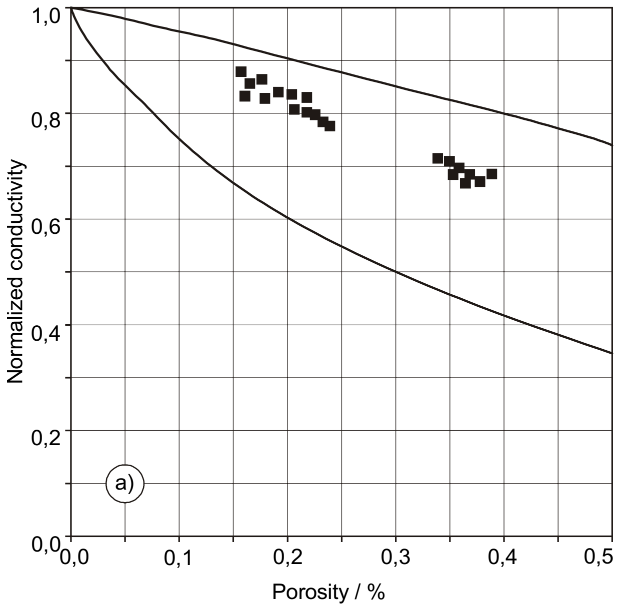


Figure 3. **Lines represent the lower and upper bounds for the effective electrical conductivity (a) and the effective Young's modulus (b) calculated for the model structure under consideration**

Slika 3. **Crte predstavljaju donje i gornje veze za učinkovitu električnu provodljivost (a) i učinkovit Youngov modul (b) izračunat za model strukture koja se promatra**

For comparison's sake, some experimental data on properties of interest obtained for the real sintered samples are also presented (Figure 4.). The specimens were made of the water atomised iron powder (ASC 100.29). The compaction pressures 400, 600 and 700 MPa were used and compacts were sintered for 30 minutes at temperatures 875, 1120 and 1200 °C in the atmosphere of cracked amonia . The electrical conductivity of specimens was examined by using the Thomson bridge of high sensitivity. Resonance frequency tester ERUDITE was used for measurement of the effective Young's modulus [7].

It can be seen that even when a rather crude and artificial theoretical model of PM material was used, the values obtained for the real sintered samples lie within the bounds derived for the theoretical model.



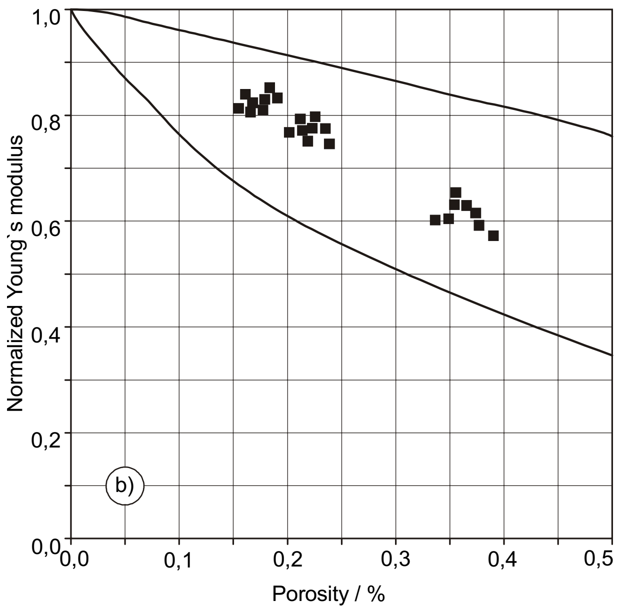


Figure 4. **Dependence of normalized effective electric conductivity (a) and normalized effective Young's modulus (b) on the porosity of the sample. Lines are for the theoretical bounds calculated for the model structure and the black marks represent experimental values obtained for the specimens from iron powder**

Slika 4. **Ovisnost normalizirane učinkovitosti električne provodljivosti (a) i normaliziranog učinkovitog Youngovog modula (b) na poroznost uzorka. Crte su teoretske veze izračunate za strukturu modela a crne oznake predstavljaju eksperimentalne vrijednosti dobivene za uzorke od željeznog praha**

DISCUSSION AND CONCLUSION

The aim of the article was to contribute theoretically to the investigation of the question whether, how and to a which extent the measurement of the electrical conductivity can provide a piece of information on mechanical properties (namely on the Young's modulus) of sintered samples.

Due to the mathematical forms of procedure, the bounds on the effective electrical conductivity of a heterogeneous sample were derived instead of a search for the exact expression for this quantity. The bounds obtained were compared with the bounds derived previously for the effective Young's modulus.

For the samples consisting of a (quasi)homogeneous matrix and pores, the constraints on effective conductivity (2) and effective Young's modulus (3) are very similar. Slight differences result from the fact that electrical properties of isotropic material are characterized only by one quantity (conductivity σ_0), while elastic properties are characterized by two quantities (e.g. Young's modulus E_0, Poisson's ratio ν_0). This fact has to reveal itself in expressions for particular constraints. But in situations when the constraints are useful, that is, they are tight enough, the differences are negligible and the effective conductivity and Young's modulus are correlated. This is due to fact that in the case of homogeneous matrix practically all properties of a porous sample are essentially affected by the distribution of load-bearing cross sections along the sample.

In the case of inhomogeneous matrix the relationship between effective electrical conductivity and effective Young's modulus is not so conclusive and it needs further investigation.

The fact that the experimental data for real PM samples lie within the bounds derived for the theoretical model indicates that the model used and bounds derived represent a good starting point for the further improvement in both model and bounds.

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