

## FRACTAL GEOMETRY AND IMPULSE DYNAMIC METHOD APPLIED AT EVALUATION OF DISTURBANCE OF ROCKS

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*Preliminary Note - Prethodno priopćenje*

In this paper the fractal geometry, interpreting the size effect was used for evaluation of disturbance of samples of rocks. Fractal geometry offers a possibility to describe the non-regular structure of natural objects by fractal dimension, from which it is possible e.g. to estimate the measure of rock disturbance. The micro-disturbance of rock samples was compared with the results of ultrasonic pulsing method, where the disturbance indexes IQ were calculated from the velocities of exciting ultrasonic waves spreading through the rock medium.

**Key words:** *rocks, size effect, fractal geometry, impulse dynamic method*

**Fraktalna geometrija i primjena impulsne dinamičke metode u procjenjivanju poremećaja u stijenama.** U radu je korištena fraktalna geometrija za tumačenje efekta veličine radi procjene poremećaja u uzorcima stijena. Fraktalna geometrija nudi mogućnost da se frakタルним dimenzijama opiše neregularna struktura prirodnih objekata iz čega je moguće, npr. utvrditi dimenzije poremećaja u stijenama. Mikropromjene u uzorcima stijena uspoređene su s rezultatima metode ultrazvučnog titranja gdje se indeksi promjena IQ računaju iz brzine širenja pobuđenih ultrazvučnih valova kroz medij stijena.

**Ključne riječi:** *stijene, efekt veličine, fraktalna geometrija, metoda impulsne dinamike*

### INTRODUCTION

The investigations of physical-mechanical parameters of disturbed rocks are showing very clear size effect, when measuring the dependence of strength on the size of rock specimens. To describe this dependence, more theories were elaborated [1, 2], where the decrease of the strength with respect to specimen size is expressed by convergence integral tending asymptotically to a constant value. Carpinteri used another approach in his works [3, 4], where he have a new explanation for size effect basing on the fractal geometry, that enables to characterize the complex geometry of some natural objects by non-integer parameter - fractal dimension [5]. Contrary to regular well-defined Euclidean shapes like circle, square or ball, the irregular objects occurring in the nature are fractal referring their boundary, surface or volume. The fractal dimension can define the degree of their irregularity, giving an information of how these objects fill the one, two or three-dimensional space. For example the determination of fractal

dimension of fractal Koch curve has revealed that the value of it ( $D = 1,262$ ) is as much higher than 1, as the Koch curve is more winding than a line.

This paper deals with evaluation and comparison of disturbance of selected rock samples measuring the strength at uniaxial loading, utilizing the fractal geometry and from the impulse dynamic method utilizing the spreading of the acoustic waves through the rock medium.

### DETERMINATION OF FRACTAL DIMENSION

The basic idea of the model [4] consists in an assumption, that the micro disturbance of the material is described by the fractal model - the cross-section of disordered rock including all the microcracks, defects and pores has the fractal dimension smaller than 2, what is the case of ideal two-dimensional area. The property of self-similarity is valid for the whole defect population. Studying the size effect on geometrically similar bodies with linear dimension  $L$ , the values of strength  $s$  can be calculated as the ratio of applied force to the square area of cross section. When considering the reaction section as a fractal area it corresponds to more realistic picture of reality. By the help of renormalization group theory, which is useful for de-

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scribing physical phenomena that show the same behavior on different scales, it is possible to define the fractal stress  $\sigma^*$  with physical dimension (Force)/(Length) $^{2-\delta}$ , as the ratio of applied force  $F$  and the fractal area, which has the fractal dimension  $D = 2 - \delta$ . For determining the fractal dimension  $D$  it is necessary to find the decrement  $\delta$ . Applying the force  $F$  to the cross section it is possible, after simple generalization, to obtain the following equation:

$$\ln \sigma = \ln \sigma_1 - \delta \ln L \quad (1)$$

where:

$\sigma_1$  is the strength value measured on the unit specimen.

The changes of strength  $\sigma$  on sizes of specimen  $L$  give a possibility to determine the decrement  $\delta$  from graphic interpretation  $\ln \sigma$  on  $\ln L$ . The value of calculated fractal dimension  $D = 2 - \delta$ , with  $0 \leq \delta < 1$ , is expressing the measure of disturbance of cross-section of rock sample; values closed to 2, are expressing about a weak disturbance, while values tending to 1,5 are valid for the very disturbed materials [6].

## EXPERIMENTAL WORK

### Determination of disturbance by impulse dynamic method

Before the uniaxial loading tests were applied, the disturbance of rock specimens was determined utilizing the impulse dynamic method. The experiments were carried out on the rocks samples of granite from locality Hnilec and the esite from Ruskov, using the ultrasonic apparatus Material tester, 543. The frequency of 1MHz was used for geometrically symmetrical specimens prepared in a prismatic shape with square base of length  $L$  ranging from 1-7 cm and with slightness ratio 2:1. The rock specimens were mounted between the transmitter and receiver transducer holders. The velocity of longitudinal propagation of ultrasonic waves covering the certain distance of the specimens was calculated measuring the time between sending and receiving waves. The used rock material did not show high level of anisotropy, as the values of velocities of longitudinal waves measured in three perpendicular directions were differing only in some tens of m/s.

To test the disturbance of rocks, the quality index IQ suggested by Formaintraux [7] was used, where the disturbance is evaluated from the volume density of pores and surface discontinuities between the crystals of minerals forming the rock material. The influence of the pore density and surface discontinuities is expressed by the changes of velocities of longitudinal waves. Comparing the velocities measured in specimens [8, 9] with those calculated for monocrystals forming rocks IQ (%) can be defined:

$$IQ = \frac{v_{LM}}{v_{LC}} 100 \quad (2)$$

where:

$v_{LM}$  /km·s<sup>-1</sup> is velocity of propagation of longitudinal waves measured in rock massive,  
 $v_{LC}$  /km·s<sup>-1</sup> is velocity of propagation of longitudinal waves, which can be calculated using the correlation between the velocity of elastic waves and volume density:

$$v_{LC} = v_{p0} e^{\sqrt{\rho_0 - 2.6}} \quad (3)$$

where:

$$v_{p0} = 5,45 \pm 0,30 \quad (4)$$

This kind of correlation is applicable if specific density  $\rho_0$  of rock-forming minerals is varying from  $\langle 3,5 \dots 4,5 \rangle$ . If the density is higher  $\rho_0 > 4,0$ , it is possible to use another correlation:

$$v_{LC} = \frac{1}{\rho_0 - 1} \quad (5)$$

The static measurements on unconfined compressive strength have been realized on the mentioned samples of rock. The values of strength  $R_m$  (MPa) measured at uniaxial loading were calculated as the ratio of normal force applied at the moment of specimen failure to the size of cross section and are introduced in Table 1.

Table 1. The characteristics of rocks at uniaxial loading:  
\* the mean value found from 60 measurements on specimens of andesite and granite [10]  
\*\* the value is arithmetic mean from 2 measurements  
Tablica 1. Svostva stijena u jednosmernom opterećenju:  
\* srednja vrijednost utvrđena na osnovu 60 mjerena provedenih na uzorcima andesita i granita [10]  
\*\* vrijednost je aritmetička sredina iz 2 mjeranja

Type of Rock	Volume density $\rho$ / g cm <sup>-3</sup>	Size of specimen $L$ / cm	Mean value of Strength $\sigma$ / MPa	Standard deviation $s$ / MPa
Granite	2,68	1	153,07 *	39,72
	2,68	3	134,02 *	35,29
	2,68	5	115,13 *	29,50
	2,68	7	98 **	-
Andesite	2,67	1	264,45 *	61,72
	2,67	3	238,04 *	56,35
	2,67	5	207,77 *	47,33
	2,67	7	195 **	-

## RESULTS AND DISCUSSION

The equation (1) was applied at elaboration of results of unconfined compressive strength on sizes measured for a narrow class of specimens sizes. The experimental data from the uniaxial loading tests were drawn to the bi-logarithmic diagram (Figure 1.) and the values of fractal di-

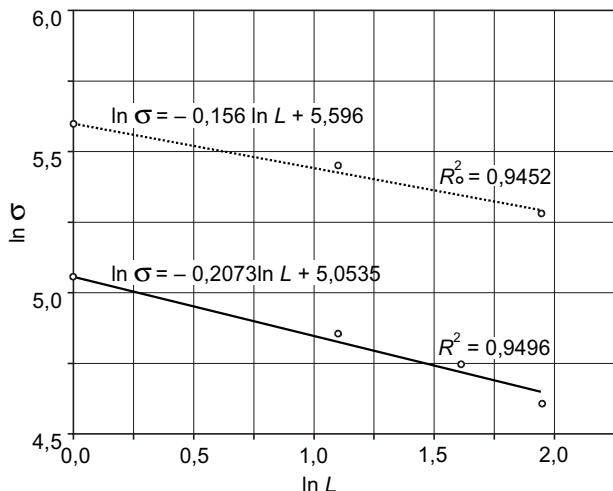


Figure 1. Size effect on strength of andesite and granite samples  
Slika 1. Efekt veličine na čvrstoću uzorka andesita i granita

mension of cross sections of rock samples were determined according to the regression line equation:

Andesite  $D = 1,84$ , Granite  $D = 1,79$

These values were used for evaluation of contact areas disturbance. Basing on the above described theory, the measure of disturbance of specimens with size ranging from 1 to 7 cm was estimated as follows: the andesite has less disturbed contact area, while the disturbance at granite was a little higher.

Table 2. Categories of rock disturbance by Fourmaintraux.  
Tablica 2. Kategorije promjena na stijenama prema Fourmaintrauxu

Index of disturbance IQ / %	Degree of rock disturbance	Classification
90 ... 100	Undisturbed up to moderately disturbed rocks	1
75 ... 90	Moderately up to medium disturbed rocks	2
50 ... 75	Medium disturbed up to strongly disturbed rocks	3
25 ... 50	Strongly up to very strongly disturbed rocks	4
0 ... 25	Extremely disturbed rocks	5

The values of velocities of longitudinal ultrasonic waves measured on samples of rocks have been used at determination of the index of disordering IQ, expressing the volume disturbance. For the andesite specimen with specific density  $\rho = 2,67 \text{ g} \cdot \text{cm}^{-3}$  was determined the index disturbance  $\text{IQ} = 78,56\%$  from the equation (2). According to the Table 2., this value corresponds moderate-to medium disturbed rocks. For granite specimen with density  $\rho = 2,68 \text{ g} \cdot \text{cm}^{-3}$  is  $\text{IQ} = 64,42\%$ , that value is valid for medium to strongly disturbed rocks.

## CONCLUSION

In this paper, the evaluation of disturbance of rocks was discussed from two points of view. The first attempt allowed the estimation of disturbance basing on fractal geometry applied when interpreting the size effect. The determined fractal dimension was used for estimation of disturbance of cross section of rock specimen. In the second approach the impulse dynamic method was used for determination the index of rock disturbance.

The measure of disturbance of contact areas of rocks determined from fractal dimension was compared with the index of disturbance IQ and from the comparison follows the same sequence in rock disturbance: the more disturbed is the sample of granite and less disturbed is the sample of andesite. In spite of fact, that the index IQ is relating to the volume disturbance can be stated, that results confirm those coming from application of fractal geometry.

The method basing on fractal geometry requires measuring the dependence of strength on wider group of specimen sizes, what should make results relating to the rock disturbance more reliable.

## REFERENCES

- [1] Z. T. Bieniawski, Int. J. of Rock Mechanics, 5 (1968), 325 - 335.
- [2] E. Hoek, E. T. Brown: Underground excavation in rock, The Institute of Mining Metallurgy, London, 1980.
- [3] A. Carpinteri, Mechanics of Materials, 18 (1994) 2, 89 - 101.
- [4] A. Carpinteri, Int. Journal of Solids and Structures, 31 (1994) 3, 291 - 302.
- [5] B. Mandelbrot: The Fractal Geometry of Nature, W. H. Freeman and Company, New York, 1982.
- [6] J. Feder: Fractals, Plenum Press, New York, 1988.
- [7] D. Fourmaintraux: Méthodes et applications. Bulletin Liaison Labo P. et Ch., (1975) 1 - 2, 69 - 76.
- [8] B. Pandula: Určovanie porušenosti hornín seismickými a seismoakustickými metódami a ich aplikácia pri rozpojovaní hornín výbuchom. Thesis, BERG Faculty, Technical University, Košice, 1995.
- [9] I. Múrová, M. Lovás, Š. Jakabský, J. Briančin, E. Boldižárová, Acta Montanistica, 7 (2002) 1, 19 - 22.
- [10] M. Labaš, F. Krepelka, Acta Montanistica, 7 (2002) 1, 15 - 18.