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# THREE - DIMENSIONAL MATHEMATICAL MODEL OF FUSION OF STEEL SCRAP IN THE CONVERTER MELT

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*Preliminary Note - Prethodno priopćenje*

Three - dimensional mathematical model of fusion of cubical steel scrap in the converter melt was developed. The model was solved using modified implicit Brian's method. The obtained algorithm was programmed in ASCII FORTRAN for the computer SPERRY 1106. In the model temperature dependent thermophysical properties of material were incorporated. That gives to the model a nonlinearity. On the basis of the model it was concluded that addition of 1% of low - carbon steel scrap decreases the temperature of a middle carbon steel melt for cca 23 °C. This is in good agreement with experimental data from literature. At the same time a new formula for the fusion time of steel scrap was developed, which enables to calculate the fusion time on the basis of the volume of the steel scrap in the converter melt.

***Key words:*** *mathematical model, fusion, steel scrap, converter*

**Trodimenzijski matematički model taljenja čeličnog otpatka u konvertorskoj talini.** Razvijen je trodimenzijski matematički model taljenja kockastog čeličnog otpatka u konvertorskoj talini. Model je riješen korištenjem modificirane implicitne Brianove metode. Dobiveni algoritam programiran je u ASCII FORTRAN za računalo SPERRY 1106. U model su ugrađena temperaturno ovisna toplofizička svojstva materijala, što modelu daje nelinearnost. Na temelju modela zaključeno je da 1 % niskougljičnog čeličnog otpatka smanji temperaturu srednje ugljične taline za cca 23 °C, što se dobro slaže s eksperimentalnim podacima iz literature. Istodobno, dobivena je nova formula za vrijeme taljenja čeličnog otpatka, koja omogućuje računanje vremena taljenja u ovisnosti o volumenu čeličnog otpatka u konvertorskoj talini.

***Ključne riječi:*** *matematički model, taljenje, čelični otpadak, konvertor*

## INTRODUCTION

From the increase of the share of metallic scrap in the charge, an increased economy of steel manufacturing in the converter processes is expected. The share of steel scrap should approach that in the openhearth process. Steel scrap, usually as low carbon steel refuse, is very economical means of melt cooling. Data in ref. 1 show that 1 % of steel scrap decreases the bath temperature for 12 to 15 °C, while 1 % iron ore decreases the temperature for 30 to 40 °C. However, production expirience shows that steel scrap is better than iron ore.

For example, by using steel the quantity of metal ejected from the converter is diminished, the resistance of refractory lining is increased and better utilization of excess heat in the bath obtained. The regime of scrap fusion depends in significant degree on scrap size, and affects the bath temperature, the slag forming processes, as well as oxidation of carbon and metal desulfurization. For instance, small sized scrap melts faster and cools quickly the bath. This decreases the rate of slag forming, carbon oxidation, desulfurization, and lower the quantity of blown oxigen. Very large pieces of scrap don't melt completly during the processing in the oxigen converter. A mathematical model of low carbon steel scrap melting in the middle carbon steel melt was developed and tested with aim to determined the dependence between the fusion time and volume of cubical steel scrap. Since the bath temperature is above 1550 °C the so - cooled diffusion melting is not considered. The investigated system consists of volume element of melt in which cubical steel scrap is immersed.

## MATHEMATICAL MODEL

For the start of the melting of cubical scrap piece in a volume element of melt the Fourier's partial differential equation of heat conduction has the form [2]:

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$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \quad (1)$$

where $a = k / \rho c_p$ is thermal conductivity.

The basic assumption for the validity of the differential equation (1) is that scrap piece is immersed is physicaly realistic since the difference in density of steel scrap (7860 kg / m$^3$) and melt (7507 kg / m$^3$) is very small. Considering the system it can be concluded that mathematical model is three - dimensional. If the time $t = 0$ the temperature of the melt is $T_L$, and that of the scrap piece is $T_s$.

The initial temperature at the steel scrap / melt boundary interface is obtained by solving the Fourier's differential equation for heat flow through the contact area of two semiinfinite medias [3]:

$$T_i = T_s + \frac{T_L - T_s}{1 + \frac{k_s}{k_m}\sqrt{\frac{a_m}{a_s}}} \quad (2)$$

On the contact steel scrap / melt area a continuous heat flow occurs with boundary condition of the fourth kind [4]:

$$k_m \frac{\partial T_m}{\partial n} = k_s \frac{\partial T_s}{\partial n} \quad (3)$$

In developing the model it was asummed that thermal properties of low carbon steel scrap (0.2 %C) and middle carbon steel melt (0.6 %C) are temperature dependent [5].

## BRIAN'S IMPLICIT METHOD

The three - dimensional differential equation of heat conduction, with initial and boundary conditions, can be solved by Brian's finite difference implicit method [6]. The method is the modification of Douglas - Rachford's method [7], and, according to Brian, it is the most efficient method of numerical integration of three - dimensional heat conduction equation. It can be proved that Brian's method can be written in the form [8]:

$$\delta_x^2 T^*_{i,j,k} + \delta_y^2 T^n_{i,j,k} + \delta_z^2 T^n_{i,j,k} = \frac{1}{a_{i,j,k,n}} \frac{T^*_{i,j,k} - T^n_{i,j,k}}{\Delta t / 2} \quad (4)$$

$$\delta_x^2 T^*_{i,j,k} + \delta_y^2 T^{**}_{i,j,k} + \delta_z^2 T^n_{i,j,k} = \frac{1}{a_{i,j,k,n}} \frac{T^{**}_{i,j,k} - T^n_{i,j,k}}{\Delta t / 2} \quad (5)$$

$$\delta_x^2 T^*_{i,j,k} + \delta_y^2 T^{**}_{i,j,k} + \delta_z^2 T^{n+1}_{i,j,k} = \frac{1}{a_{i,j,k,n}} \frac{T^{n+1}_{i,j,k} - T^{**}_{i,j,k}}{\Delta t / 2} \quad (6)$$

The Brian's method is unconditionally stabne and converges with discretization error, 0 $[(\Delta t)^2 + (\Delta x)^2]$. The solutions of Equations (4) - (6) for general nodes ($i$, $j$, $k$) in scrap and melt, and nodes of scrap - melt interface, are given in Appendix A. On the base of the presented algorithm of fusion a computer program was written in ASCII FORTRAN and solved on SPERRY 1106 computer.

## DISCUSSION

The simulation of fusion of a low carbon steel scrap in the middle carbon steel melt is carried out by space step $\Delta x = \Delta y = \Delta z = 1$ cm and time step $\Delta t = 5$ s till $t_{max} = 1440$ s. The initial melt temperatures of 1700 °C for the melt, 25 °C for the scrap piece and 883 °C for the boundary surface were assumed. On the basis of succesive temperatures prints out for particular net points the fusion time was obtained for low carbon steel cubes of size 3x3x3 (cm) to 7x7x7 (cm), which is presented in Table 1.

Table 1. **Fusion time for low carbon steel cubes**

Tablica 1. **Vrijeme taljeljenja niskougljičnih čeličnih kocki**

| $V$ [ cm$^3$ ] | $t$ [ s ] |
|---|---|
| 27 | 146.5 |
| 64 | 363.6 |
| 125 | 514.5 |
| 216 | 841.0 |
| 343 | 1028.2 |

On the basis of simulation the best approximation shows the curve given by the equation

$$t = 13.1921 \cdot V^{0.76176} \quad (7)$$

The coefficient of correlation is 0.99056, and equation (7) was obtained using Hewlett - Packard 48G computer. Also, it can be concluded that 1 % of steel scrap decrease the temperature of steel melt for cca 23 °C, a value in good agreement with published experimental data [1].

## CONCLUSION

Three - dimensional mathematical model of fusion of low carbon steel scrap piece in middle carbon steel melt in oxigen converter was developed and investigated. The model was solved using modified implicit Brian's method, and the algorithm obtained was programmed in ASCII FORTRAN for the computer SPERRY 1106. The model was checked in the base of experience data. It has been established that the addition of 1 % of steel scrap decreases the melt temperature for cca 23 °C, a value in good agreement with experimental data. Also a new formula for the fusion time of cubical steel scrap was developed, which enables to calculate the fusion time on the basis of volume of the steel scrap in the converter melt.

***Abbreviations used:***

$a$ - temperature conductivity
$c_p$ - specific heat at constant pressure
$k$ - thermal conductivity
$n$ - vertical direction
$t$ - time
$T$ - temperature
$V$ - volume
$x, y, z$ - space coordinate

**APPENDIX A**

**Tridiagonal Coefficients**

1. Point $(i, j, k)$ in the model or metal.

$a_i = c_i = -1$
$b_i = 2(z + 1)$

$$d_i = T^n_{i,j-1,k} + T^n_{i,j+1,k} + T^n_{i,j,k-1} + T^n_{i,j,k+1} + 2(z-2)T^n_{i,j,k} \quad \text{(A-1)}$$

$a_j = c_j = -1$
$b_j = 2(z + 1)$

$$d_j = T^*_{i-1,j,k} - 2T^*_{i,j,k} + T^*_{i+1,j,k} + T^n_{i,j,k-1} + T^n_{i,j,k+1} + 2(z-1)T^n_{i,j,k} \quad \text{(A-2)}$$

$a_k = c_k = -1$
$b_k = 2(z + 1)$

$$d_k = T^*_{i-1,j,k} - 2T^*_{i,j,k} + T^*_{i+1,j,k} + T^{**}_{i,j-1,k} + 2(z-1)T^{**}_{i,j,k} + T^{**}_{i,j+1,k} \quad \text{(A-3)}$$

where:

$$z = \frac{(\Delta x)^2}{a_{i,j,k,n}\Delta t} \quad \text{(A-4)}$$

2. Point $(i, j, k)$ on vertical boundary surface yz between sand (left) and metal (right).

$$a_i = -\frac{2k_s}{k_s + k_m}$$

$b_i = 2(z + 1)$

$$c_i = -\frac{2k_m}{k_s + k_m}$$

$$d_i = T^n_{i,j-1,k} + T^n_{i,j+1,k} + T^n_{i,j,k-1} + T^n_{i,j,k+1} + 2(z-2)T^n_{i,j,k} \quad \text{(A-5)}$$

$a_j = c_j = -1$
$b_j = 2(z + 1)$

$$d_j = \frac{2k_s}{k_s + k_m} T^*_{i-1.j,k} - 2T^*_{i,j,k} + \frac{2k_m}{k_s + k_m} T^*_{i+1,j,k} + T^n_{i,j,k-1} + T^*_{i,j,k+1} + 2(z-1)T^n_{i,j,k} \quad \text{(A-6)}$$

$a_k = c_k = -1$
$b_k = 2(z + 1)$

$$d_k = \frac{2k_s}{k_s + k_m} T^*_{i-1,j,k} - 2T^*_{i,j,k} + \frac{2k_m}{k_s + k_m} T^*_{i+1,j,k} + T^{**}_{i,j-1,k} + T^{**}_{i,j+1,k} + 2(z-1)T^{**}_{i,j,k} \quad \text{(A-7)}$$

where:

$$z = \frac{(\Delta x)^2}{(k_s + k_m)\Delta t}\left(\frac{k_s}{a_s} + \frac{k_m}{a_m}\right) \quad \text{(A-8)}$$

3. Point $(i, j, k)$ on vertical boundary surface xz between sand (left) and metal (right).

$a_i = c_i = -1$
$b_i = 2(z + 1)$

$$d_i = \frac{2k_s}{k_s + k_m} T^n_{i.j-1,k} + \frac{2k_m}{k_s + k_m} T^n_{i,j+1,k} + 2(z-2)T^n_{i,j,k} + T^n_{i,j,k-1} + T^n_{i,j,k+1} \quad \text{(A-9)}$$

$$a_j = -\frac{2k_s}{k_s + k_m}$$

$b_j = 2(z + 1)$

$$c_j = -\frac{2k_m}{k_s + k_m}$$

$$d_j = T^*_{i-1,j,k} - 2T^*_{i,j,k} + T^*_{i+1,j,k} + T^n_{i,j,k-1} + 2(z-1)T^n_{i,j,k} + T^n_{i,j,k+1} \quad \text{(A-10)}$$

$a_k = c_k = -1$
$b_k = 2(z + 1)$

$$d_k = T^{*}_{i-1,j,k} - 2T^{*}_{i,j,k} + T^{*}_{i+1,j,k} + \\ + \frac{2k_s}{k_s + k_m} T^{**}_{i,j-1,k} + 2(z-1)T^{**}_{i,j,k} + \frac{2k_m}{k_s + k_m} T^{**}_{i,j+1,k} \quad \text{(A-11)}$$

4. Point $(i, j, k)$ on horizontal bondary surface xy between sand and metal (sand down, metal up).

$a_i = c_i = -1$
$b_i = 2(z + 1)$

$$d_i = T^{n}_{i,j-1,k} + 2(z-2)T^{n}_{i,j,k} + \\ + T^{n}_{i,j+1,k} + \frac{2k_s}{k_s + k_m} T^{n}_{i.j,k-1} + \frac{2k_m}{k_s + k_m} T^{n}_{i,j,k+1} \quad \text{(A-12)}$$

$a_j = c_j = -1$
$b_j = 2(z + 1)$

$$d_j = T^{*}_{i-1,j,k} - 2T^{*}_{i,j,k} + T^{n}_{i+1,j,k} + \\ + 2(z-1)T^{n}_{i,j,k} + \frac{2k_s}{k_s + k_m} T^{n}_{i.j,k-1} + \frac{2k_m}{k_s + k_m} T^{n}_{i,j,k+1} \quad \text{(A-13)}$$

$$a_k = -\frac{2k_s}{k_s + k_m}$$

$b_k = 2(z + 1)$

$$c_k = -\frac{2k_m}{k_s + k_m}$$

$$d_k = T^{*}_{i-1,j,k} - 2T^{*}_{i,j,k} + T^{n}_{i+1,j,k} + \\ + T^{**}_{i,j-1,k} + T^{**}_{i,j+1,k} + 2(z-1)T^{**}_{i,j,k} \quad \text{(A-14)}$$

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