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EFFECTIVE FLEXURAL MODULUS OF BARS CONTAINING CELLULAR-MATERIAL LAYER, AND THE USE OF THIS MODULUS FOR DETERMINING THE EFFECTIVE ELASTIC MODULUS OF A CELLULAR MATERIAL

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Rectangular bar-shaped sandwich-like samples consisting of a cellular-material core between two standard material layers were prepared. The cellular materials were made of both ferrous and bronze hollow spheres. Using these specimens and testing method based on flexural vibrations, expected differences were observed in effective modulus values determined by means of vibrations parallel and perpendicular to layers. Information on distribution of material throughout the bar cross section was obtained through metallographic studies. The thickness of particular layers, measured values of "parallel" and "perpendicular" effective flexural modulus, and expressions derived for flexural rigidity of quasi-layered bars were used for evaluating the values of effective Young's modulus of material of particular layers constituting the sample. Reasonable agreement was found among results obtained for various samples.

Key words: *cellular material, Young's modulus, dynamic resonant method*

Stvarni modul savijanja motki koje imaju sloj od staničnog materijala i uporaba tog modula za određivanje stvarnog modula elastičnosti staničnog materijala. Pripremljeni su pravokutni sendvič-uzorci motki koji se sastoje od jezgre sa staničnim materijalom između dva sloja standardnog materijala. Stanični materijali su napravljeni od željeznih i brončanih šupljih kugli. Uporabom tih uzoraka i metoda testiranja zasnovane na vibracijama savijanja, očekivane razlike su prijećene u postojećim vrijednostima modula određenih pomoću vibracija paralelnih i okomitih na slojeve. Informacija o rasporedu materijala u presjeku motke dobivena je na osnovu proučavanja metalografije. Debljina pojedinih slojeva izmjerenih vrijednosti "paralelnih" i "okomitih" stvarnog modula savijanja i izraza izvedenih iz savitljive krutosti poluslojevnih motki koje se koriste za procjenjivanje vrijednosti stvarnog efektnog Youngovog modula materijala pojedinih slojeva koji sačinjavaju uzorak. Razumno suglasje je pronađeno među rezultatima dobivenim za razne uzorke.

Ključne riječi: *stanični materijal, Youngov modul, metoda dinamičke rezonancije*

INTRODUCTION

Cellular solids [1] (an assembly of cells with solid edges and/or faces, packed together so that they fill the space) represent ones of materials which are developed, improved and investigated within the framework of the search for new lightweight structural materials. Regarding the prospective application, an acquaintance with elastic characteristics of cellular solids is desirable, even when nowadays their primary use is not mechanical (filters, thermal insulation, energy absorption, etc.). Direct measurements of elastic properties on quasi-homogeneous samples made of cellular material alone are usually complicated or even impossible, as the detectable response of the specimen is often accompanied by the plastic deformation of the material itself. To avoid similar complications more sophisticated methods and approaches are needed.

In this contribution the attention is given on the determining the values of effective elastic modulus by means of flexural vibrations of layered specimens. When the rod undergoes flexural vibration, maximum stress values occur near the surfaces which are perpendicular to the bending plane of the rod. Near the rod center the stress is much smaller [2, 3]. Hence, when the cellular material would be

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situated only near the rod center, it could be protected against the plastic deformation. Furthermore, in such geometry the vibration suppressing effects of another unsuitable properties of cellular solids (high damping, low inertial properties, etc.) can be diminished too.

EXPERIMENTAL

Fe - based samples preparation

As a precursor, hollow particles consisting of Fe oxides were used. Precursor particles are produced as a waste material during the manufacturing of wires. These oxides precursor particles were reduced to metallic iron under following conditions: temperature 1120 °C - 1130 °C for 30 minutes, atmosphere - cracked ammonia, pilot walking beam furnace (FHD).

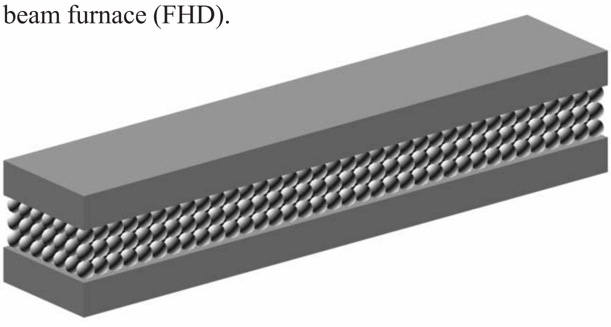

Figure 1. **Schematic sketch of the sandwich rod**
Slika 1. **Skica sendvič-gredice**

We have prepared bar-shaped symmetric three-layer samples (Figure 1.). Specimens were formed under pressure 100 MPa and sintered for 60 minutes at 1200 °C in liquid hydrogen. The central (inner) layer was made of cellular material consisting of sintered ferrous hollow spheres (Figure 2.a). Two fractions of hollow spheres were used: with diameters 0.8 - 0.9 mm and 1- 1.5 mm. Two surface (outer) layers were prepared from iron powder.

Bronze - based samples preparation

The bronze hollow spheres were prepared as follows: As starting material spherical iron powders were produced from iron oxide by reduction in liquid hydrogen for some hrs at 800 °C. This powder was screened and the fraction between 700 and 500 μm was used. Deposition of copper on the spherical particles by cementation was done, even complete replacement of Fe by Cu took place. Since Cu is deposited at the surface while Fe is dissolved in the solution, hollow particles were obtained. These particles were very weak and difficult to handle, for this reason they were strengthened annealing at 800 °C in H_2 atmosphere. This caused the reduction of the oxides and some sintering within the shell, resulting in mechanically reliable particles that could be better handled.

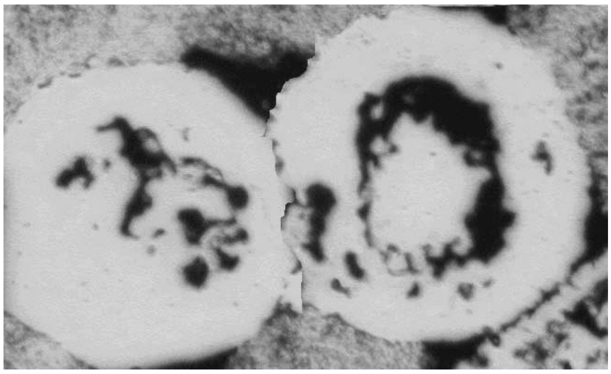

Figure 2.a **Light-microscope micrograph of the section across the iron hollow particles**
Slika 2.a **Mikroskopski snimak presjeka šuplje čestice željeza**

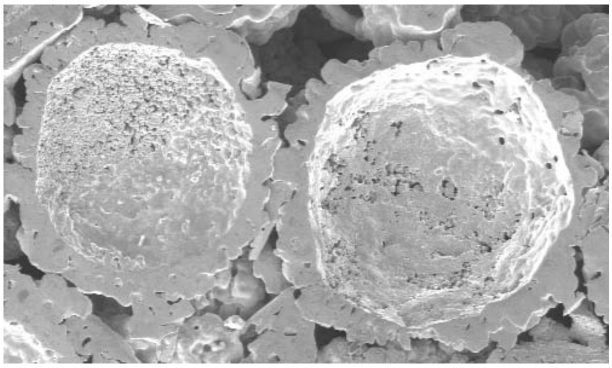

Figure 2.b **SCAN micrograph of the section across the bronze hollow particles**
Slika 2.b **SCAN mikroskopski snimak presjeka šupljih čestica bronce**

The heat treated Cu hollow spheres were tin-coated in aqueous solution. In a single step process, maximum Sn contents of 5 mass % can be introduced.

If more Sn is to be added, an intermediate anneal at 700 °C in H_2 has to be carried out; afterwards a second deposition run is possible by which a Sn content of 10 mass % can be attained (Figure 2.b).

Then the sandwich bar-shaped specimen was prepared (Figure 3.): the outer layers - two bars made of bronze powder (Eckart 90/10) - were prepared separately. Bronze hollow particles were placed between completed outer lay-

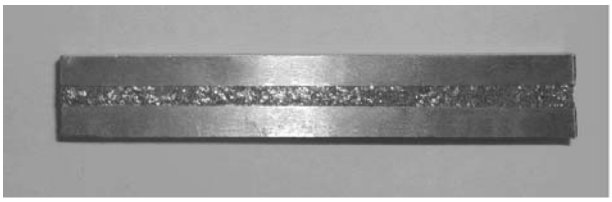

Figure 3. **Three layer rod prepared from the bronze powder (surface layers) and bronze hollow spheres (central layer)**
Slika 3. **Trodijelna gredica pripremljena iz brončanog praha (površinski slojevi) i šupljih brončanih kugli (središnji sloj)**

ers. These 3 layers were tapped into ceramic moulds and gravity sintered at the temperature of 800 °C for 3 hours in hydrogen atmosphere.

The dynamical resonant method [4] was used for determining the effective flexural modulus of such layered specimens. Geometric characteristics of particular layers were determined by metallographic methods.

THEORY

The standard „dynamic-resonant-method” evaluating formula:

$$E^{fl} = 0.94642 \frac{M}{WHL} L^4 \frac{f^2}{t^2} \tag{1}$$

was used for determining the effective flexural modulus values E^{fl}. Expr. (1) stands for a rectangular bar undergoing free flexural vibration with free ends.

- *M* is the mass of the bar;
- *H*, *W* and *L* are height, width and length of the bar, respectively;
- *t* represents the specimen cross-sectional dimension in the direction of vibration, that is, $t = H$ if the bending plane is parallel to *HL*-plane, or $t = W$ if the bending plane is parallel to *WL*-plane;
- *f* represents the resonance frequency of the fundamental flexural mode.

To be able to obtain an information on properties of material of interest, it is necessary to have the relation that connects the value of “overall” effective modulus to values of effective modulus of materials constituting the layered sample. Corresponding theoretical expressions are published in earlier papers [5, 6]. Here we only briefly sketch the method of derivation and state the resultant relations. A rectangular bar of height *H*, width *W* and length *L* was considered. The properties of material of the bar were assumed to vary only in the direction of the bar height. The vibration frequencies are obtained by solving the corresponding equation of motion. The equation of motion is derived by means of the Hamilton’s principle of minimal action. Lagrange’s function, occurring in the expression for action, consists of the kinetic and potential (elastic) energies of a deformed bar. The required elastic energy is determined by means of the strain and stress tensor fields derived for a bent quasi-layered bar under consideration. The geometry of deformation of material fibres and bar planar cross sections in a bent quasi-layered bar is similar to the geometry of deformation in a homogeneous bar. Therefore, the strain tensor is qualitatively similar to the strain tensor in a homogeneous bar. The stress tensor is determined on the basis of the Hooke’s law by means of the above mentioned strain tensor and the inhomogeneous distribution of the material Young’s modulus values along the cross section. The resultant elastic energy of a bent quasi-layered bar, if expressed by means of the neutral-fibre curvature, differs from the elastic energy of a homogeneous bar only by a pre-factor - the flexural rigidity of the bar. The kinetic energy of a quasi-layered bar, if expressed by means of the velocity of the neutral-fibre transverse motion, differs from that of a homogeneous bar by the pre-factor - mass of bar per unit length. These differences lead to the analogous changes of corresponding quantities in the equation of motion and consequently in the relation for frequency of a homogeneous bar when they are rederived for the quasi-layered bar.

Substituting theoretical frequencies into the formula (1), used for the evaluation of the „experimental” modulus, the following relations for the “overall” effective modulus are obtained:

$$E_{\perp}^{fl} = \frac{12}{H^3} \left[\int_0^H E(x) x^2 dx - \left(\int_0^H E(x) x dx \right)^2 \left(\int_0^H E(x) dx \right)^{-1} \right] \tag{2}$$

for vibration in the *HL* plane, i. e. bending occurring perpendicular to quasi-layers, and

$$E_{II}^{fl} = \frac{1}{H} \int_0^H E(x) dx \tag{3}$$

for vibration in the *WL* plane, i. e. bending occurring parallel to quasi-layers. $E(x)$ represents the materials’ Young’s modulus that can vary along the height of the bar.

For symmetric three-layer bars the expressions (2) and (3) acquire the form [5]:

$$E_{II} = E_s + (E_c - E_s) w_c \tag{4a}$$

$$E_{\perp} = E_s + (E_c - E_s) w_c^3 \tag{4b}$$

- E_s and E_c are values of Young’s modulus of material of surface and central layers, respectively;
- w_c represents the volume fraction of the central layer. So, when the E_{II} and $E_{\perp}$ are measured and eqs. (4a), (4b) inverted, the modulus of central layer, E_c can be evaluated as:

$$E_c = E_{II} + \frac{E_{II} - E_{\perp}}{w_c (1 + w_c)} \tag{5a}$$

when the volume fraction of central layer, w_c, is known, or as

$$E_c = E_s + \mathrm{sign}(E_{II} - E_{\perp}) \frac{E_{II} - E_s}{E_{\perp} - E_s} \sqrt{(E_{II} - E_s)(E_{\perp} - E_s)} \quad (5b)$$

when the Young's modulus of surface layer, E_s, is known.

RESULTS AND CONCLUSION

In accordance with the theory [5], the measured values of effective modulus $E_{\perp}$ were higher than E_{II} for all three-layer samples with a cellular core.

These modulus values were measured on samples with various thickness of the central layer, that is, with various volume fraction of the central layer w_c. Subsequently, the modulus of central cellular layer, E_c, was evaluated by means of eq. (5a). We obtained the following values for the iron-based materials:

$E_c = (23.58 \pm 0.68)$ GPa for a material with hollow spheres of diameters 0.8 - 0.9 mm, and

$E_c = (18.59 \pm 0.23)$ GPa for a material with hollow spheres of diameters 1.0 - 1.5 mm.

Measurement the bronze - based rod, was carried out only on one sample. We obtained the following mean value for effective modulus of bronze cellular layer

$E_c = 0.738$ GPa

In this contribution we demonstrated that the sandwich-like sample with a cellular-material core between two standard-material layers can be used for indirect measuring of the effective elasticity modulus of cellular material, otherwise hardly measurable. Using such samples, we obtained the values of elastic modulus for our cellular materials made of both ferrous and bronze hollow spheres. The investigation of effect of various micro structural parameters on this modulus continues.

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