

## STUDY

### OF THE RELATION BETWEEN THE STATIC AND DYNAMIC MODULI OF ROCKS

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*Preliminary Note - Prethodno priopćenje*

Non-destructive techniques based on the propagation of ultrasonic wave through the elastic medium were used for determination of dynamic modulus of selected samples of rocks in laboratory. The calculated values of dynamic modulus were compared with values of static modulus determined in laboratory on samples tested by applied loading. A method of estimation of the static modulus in situ conditions, where the measurements cannot be carried out, is discussed.

**Key words:** *ultrasonic waves, static and dynamic modulus, rocks*

**Proučavanje odnosa između statičnog i dinamičnog modula stijena.** Za laboratorijsko određivanje elastičnog i dinamičnog modula odabranih uzoraka stijena upotrijebljene su nerazarajuće tehnike temeljene na širenju ultrazvučnih valova kroz elastični medij. Izračunate vrijednosti dinamičkog modula su uspoređene s vrijednostima statičnog modula određenog u laboratoriju na uzorcima testiranim primjenom opterećivanja. Raspravljena je i metoda utvrđivanja statičnog modula u uvjetima "in situ" gdje mjerenja ne mogu biti provedena.

**Ključne riječi:** *ultrazvučni valovi, statički i dinamički modul, stijene*

## INTRODUCTION

Determination of the physical and mechanical properties of rocks as well as the behaviour of the rock massive after the deformation are the most interesting problems, which are still not sufficiently and definitely solved. Solving some stability problems referring not only the mining, but also to other branches of industry, it seems to be very important to make an assessment of the elastic properties of the media. They are mostly determined under laboratory conditions on the collected samples and used directly or modified to the solution of problem. In mining conditions it is often meeting a problem, when the rock is investigated as the rock-massive with a very complicated structure. For the simplicity the elastic properties are determined on the base of measurements in laboratory.

The common method used in laboratory is the static testing, where the mechanic properties of rocks are determined on the samples of rocks. Bearing in mind the assumption of homogeneity of the investigated media, the elastic properties are characterized by constant - static

modulus  $E_s$  which specifies the relation between the applied stress and strain. The other group of techniques involve the application of non-destructive seismic or acoustic waves [1, 2].

The main advantage of these methods is that there does not come to the destruction of the specimen. The basis of this method is the theory of elasticity. From the values of velocities of propagation of waves, which reflect the properties of the elastic, isotropic media, it is possible to determine the dynamic modulus  $E_D$ . The method enabling the determination of the dynamic modulus is relatively simple, inexpensive and suitable for application also in situ.

It is known from various sources [3, 4], that the correlation between two moduli does exist; the aim of this paper is, basing on the values of dynamic elastic modulus, to find an analytical expression that allows the determination of modulus  $E_s$ . Basing on measurements of dynamic modulus measured in situ conditions, it will be possible to estimate the correspondent static modulus, the measurement of which cannot be carried out in operational conditions. It seems to be very important, as the evaluation of the elastic properties of rock should correspond to requirements, which refer to long [5, 6], (stability problems at building works) or short (e.g. rock disintegration at blasting works) termed loading.

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## EXPERIMENTAL

The measurements of elastic moduli were carried out in laboratory conditions of Technical University and Institute of Geotechnics of Slovak Academy of Sciences both in Košice on different rock samples of andesite, amphibolite, diabase, dolomite, granite, limestone, norite, porphyroide, sandstone, siderite and slate, selected from various quarries in Slovakia and Czech republic, which were macroscopically undisturbed.

### The static method used in laboratory

The uniaxial compression tests were performed on several kinds of rocks with different physical properties. For these experiments the specimens with the ratio of height to width equalling (1) were prepared.

$$v_p = \sqrt{\frac{E_D(1-\mu)}{\rho(1+\mu)(1-2\mu)}} \quad (1)$$

The extent of deformation was recorded by tensometer located in a half of the specimen's height. The recorded values of the uniaxial compression tests and loads of press have been treated by computer. From the assumption about the homogeneity of the investigated materials, the static modulus  $E_s$  was determined according to the Hook's law as the ratio between the strength  $s$  and elastic relative strain  $\varepsilon - \varepsilon_i$ :

$$E_s = \frac{\sigma}{\varepsilon - \varepsilon_i} \quad (2)$$

The calculated values of static modulus  $E_s$  for selected specimens of rocks exposed to uniaxial compression tests are introduced in Table 1.

### The dynamic method used in laboratory

The experiments were carried out with the ultrasonic pulsing method using the ultrasonic instrument Material tester (Model 543). According to the sizes of specimens, the frequency of 1 MHz was used for exciting of ultrasonic waves. The rock specimens were mounted between the transmitter and receiver transducer holders. The velocity of the longitudinal propagation  $v_p$  of ultrasonic waves covering the certain distance of the specimens can be calculated measuring the time between the sending and receiving waves. From the equation (1) it is possible to determine the dynamic modulus  $E_D$ , if the values of specific densities  $\rho$  as well as the Poisson's numbers  $\mu$  are known. The latter can be calculated from the equation (3), into which the velocities of longitudinal  $v_p$  as well as shear waves  $v_s$  are included.

$$\mu = \frac{\frac{1}{2} \left( \frac{v_p}{v_s} \right)^2 - 1}{\left( \frac{v_p}{v_s} \right)^2 - 1} \quad (3)$$

The calculated values of dynamic moduli are also introduced in Table 1.

Table 1. The values of static and dynamic moduli for various rocks

Tablica 1. Vrijednosti statičkih i dinamičkih modula za razne stijene

Rock	Deposit	Static modulus $E_s$ [MPa]	Dynamic modulus $E_D$ [MPa]
Andesite	Ruskov	71478	80741
Amphibolite	Mútnik	77941	92102
Dolomite	Mútnik	41373	116745
Granite	Ťahanovce	35102	43000
Granite	Zlatá Idka	47451	52196
Limestone	Nižná Slaná	56471	72394
Siderite	Nižná Slaná	80686	113406
Sandstone	Vířaz	25600	26300
Sandstone	Tvarožec	26667	28003
Magnezite	Miková	116827	136680
Magnezite	Košice	82006	109068
Marble	Silická Brezová	56471	78225
Sandstone	Příbram	77810	92310
Norite	Příbram	83610	88210
Diabase	Příbram	85410	95810
Diabase	Čermel'	99510	113135
Slate	Branisko	67700	87400

## RESULTS AND DISCUSSION

The differences between the values of both moduli are influenced with different conditions under which the static and dynamic tests are performed. The values of applied strength in static experiments are of several tents of MPa, while the strength at dynamic methods does not exceed the value of 100 Pa. The loading at uniaxial tests can cause the closing of microcraks, that leads to the growth of deformation and consequently to the decreasing of the elastic constant. In comparison, the non-destructive dynamic tests do not change the structure of the material, what is the biggest advantage of their use. The time of applying stress is also different; at static tests it lasts several minutes while at dynamic tests it lasts only several microseconds.

The manner of the using of the methods for determination of the static as well as the dynamic modulus lies in demands of the practise. If the requirement of determination of the elastic modulus is formulated from the long-termed loading point of view - as the problem of stability at building works, mining works etc., the elastic properties are derived from the static modulus. In opposite side, if the loading is short-termed, as for example the rock disintegration at blasting works, earthquakes, etc., the determination of dynamic modulus is required. The measurements of static modulus in operational conditions cannot be carried out, that is why an alternative method of its determination should be useful.

There were some attempts to describe the differences between the moduli by mathematical relation [7, 8]. Savich (1984) suggested an expression in form of a power law. When looking for correlation of laboratory obtained experimental data on elastic moduli of rocks from Slovak and Czech queries, there is a possibility to predict the static modulus values from dynamic values laboratory determined according to an empirical function. The most appropriate function relating to the high correlation coefficient 0.9639 is the linear function (Figure 1.):

$$E_S = a E_D + b \quad (4)$$

where the coefficients  $a$  and  $b$  are found as follows:  $a = 0.77075$ ;  $b = 5854.109$ .

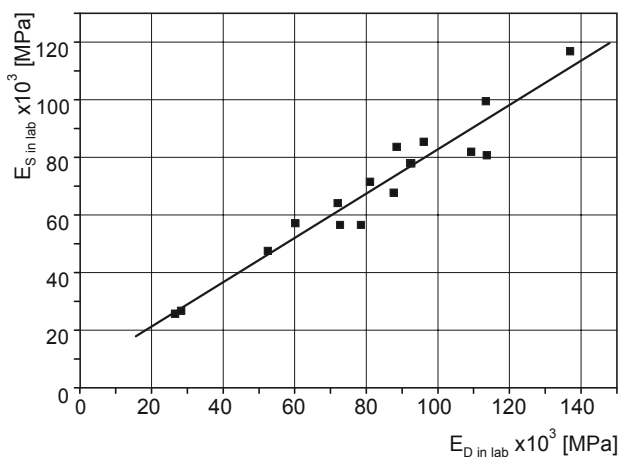


Figure 1. Dependence between static  $E_S$  and dynamic  $E_D$  elastic modulus

Slika 1. Ovisnost između statičnog  $E_S$  i dinamičnog  $E_D$  elastičnog modula

The measurements and calculation of elastic static modulus have been done on the assumption of the homogeneous

isotropic media (non-disturbed or quasi non-disturbed rocks), where the Hook's law is valid; but rocks in general do not fulfil these conditions. The investigation of the space occupied by the objects - samples of rocks - from the fractal point of view, where the value of fractal dimension can reflect the degree of material's disordering, can give an assessment of the real structure of rocks. The using of the derived empirical formula for estimation of the static modulus from the dynamic modulus, measured in situ condition, requires making another correlation between the values of dynamic modulus measured in lab and in situ. The result in [6] shows that some correlation could be determined, it is necessary to continue in this research.

## CONCLUSION

The aim of this paper was to show that the using of non-destructive dynamic tests has its importance for solving stability problems of engineering projects. It was shown, that basing on the experimentally obtained dynamic Young moduli in laboratory there could be found a correlation expressed in form of analytical function helping to determine the corresponding static modulus. The results have been achieved under simplified assumptions, as the relation for Young's modulus calculations is valid only for homogeneous and isotropic media, while the rock in general does not fulfil this condition.

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