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SIGNAL DE-NOISING METHOD BASED ON PARTICLE SWARM ALGORITHM AND WAVELET TRANSFORM

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Original scientific paper
Wavelet analysis is a new time-frequency analysis tool developed on the basis of Fourier analysis with good time-frequency localization property and multi-resolution characteristics, which is in a wide range of applications in the field of signal processing. This paper studies the application of wavelet transform in signal filtering, by using an improved particle swarm optimization, proposes an intelligent signal de-noising method based on wavelet analysis. The method uses a Center Based Particle Swarm Algorithm (CBPSO) to select the optimal threshold for each sub-band in different scales, learning the type of noise from the signal itself intelligently, which does not require any prior knowledge of the noise. The improved particle swarm algorithm is used to enhance the optimal choice of the different scales of the wavelet domain threshold, which realized the signal De-noising under different types of noise background, and improved the speed of wavelet transform and wavelet construction, and has greater flexibility. The experimental results showed that CBPSO algorithm can get better De-noising effect.

Keywords: *wavelet analysis, particle swarm algorithm, signal De-noising*

Metoda uklanjanja šuma iz signala temeljena na algoritmu roja čestica i Wavelet transformu

Izvorni znanstveni članak
Wavelet analiza je novi alat za analizu odnosa vrijeme-frekvencija, razvijen na temelju Fourierove analize s dobrim svojstvom lokaliziranja vremena i frekvencije i mogućnosti donošenja višestrukih rješenja. Koristi se u cijelom nizu primjena u području obrade signala. U ovom se radu analizira primjena wavelet transforma u filtriranju signala korištenjem poboljšane optimalizacije roja čestica i predlaže inteligentna metoda uklanjanja šuma iz signala zasnovana na wavelet analizi. Metoda koristi Center Based Particle Swarm Algorithm (CBPSO) za izbor optimalnog praga za svaki pod-pojas u različitim mjerilima, inteligentno razaznavajući vrstu šuma iz samog signala, što ne zahtijeva nikakvo prethodno poznavanje šuma. Poboljšani algoritam roja čestica koristi se da potakne optimalni izbor različitih mjerila praga wavelet domena, što je dovelo do uklanjanja šuma iz signala kod različitih tipova pozadinskog šuma, i povećane brzine wavelet transforma i wavelet konstrukcije te ima veću fleksibilnost. Eksperimentalni rezultati su pokazali da se CBPSO algoritmom može postići bolji učinak uklanjanja šuma.

Ključne riječi: *wavelet analiza, algoritam roja čestica, uklanjanje šuma iz signala*

1 Introduction

Failures occur from time to time in a realistic environment, industrial equipment, military equipment or in a building structure. To ensure the safe and reliable functioning of the large structure and the important equipment, automatic, continuous and regular collection and analysis of the monitored signal are necessary. Large structures under load will release a mechanical wave (vibration, acoustic development signal), due to local instability (such as fracture, instantaneous deformation, etc.). When and in what pattern the mechanical wave appears as well as the waveform characteristic and other information all indirectly demonstrate the structural state. Extraction of the signals or of the characteristics of these from strong background noise has an important significance on the structural condition monitoring. In practical applications, it has always been a widely discussed issue in the field of signal processing to find the best approach to reduce the noise for signals of various natures. According to the characteristics of the analyzed signals and the main objective, it matters a great deal to select the most appropriate noise reduction.

Traditional signal analysis is built on the basis of Fourier transformation (Fourier). But it can only tell us the range of the signal scales, but not give us the structure of the signal or the cascade process of different scales it contains. That is to say in the space-time domain the Fourier transformation has no resolution. Moreover, Fourier analysis is a global transform, either completely in the time domain, or completely in the frequency domain. But the singularity position of the signal cannot be given, nor can the time-frequency, localization properties of signal be expressed. However, it is this very nature of the non-stationary signal that is most fundamental and critical in the practical application process. The Short-time Fourier Transform introduced in order to deal with non-stationary signals does overcome the defects of the standard Fourier transform in local analysis to some extent, but there also exist own insurmountable defects i.e., when the short-time window function is determined, the shape of the window is determined. Therefore, it is a method of signal analysis of a single resolution. To change the resolution, you must re-select the window function.

Wavelet analysis can solve these problems. The idea of Wavelet Theory stems from dilation and translation of signal analysis, first introduced by Morlet, 1980. As a development of Fourier analysis, wavelet analysis method is a time-frequency localization analysis method where window size is fixed, yet the shape of the window variable time window and frequency window is changeable, which means a higher frequency resolution and a lower time resolution in the low-frequency portion and a higher time resolution and a lower frequency resolution in the high-frequency portion. It is suitable for detecting a transient anomaly in the normal signal and demonstrating its ingredients. In practical engineering applications, the signal analyzed may contain many spikes or mutation part, and the noise is not smooth white noise. The traditional Fourier transform analysis is powerless in de-noising processing of this signal, because it cannot illustrate how a signal changes at a certain point of time. Wavelet analysis is a time-frequency analysis which can simultaneously analyze the signal in both the time and frequency domains to effectively distinguish the mutant signal and noise in order to achieve the de-noising of the signal. Wavelet analysis in signal de-noising process is an important application of wavelet analysis.

2 Wavelet De-noising research trends

Mallat is one of the first researchers in the wavelet signal processing. He proposed the classic signal de-noising method using the wavelet transform modulus maxima principle. The basic principle is to remove modulus maxima points of noise and retain only the modulus maxima points of real signal in the wavelet transform domain. However, great errors will occur in signal reconstruction using only these limited modulus maxima points. Therefore, based on the modulus maxima principle of signal de-noising, there exists a problem concerning wavelet. Mallat's alternating projection method solved this problem. However, it entails a lot of calculation, including an iterative, and sometimes the result is not stable [1, 2].

The academic community devoted to the signal de-noising in Stanford University led by Donoho, made a great deal of achievements. Donoho and Johnstone et al. proposed soft threshold and hard threshold signal de-noising method (WaveShrink), and proved the optimality of WaveShrink from the progressive sense. Coifman and Donoho proposed shift-invariant wavelet de-noising method [3].

Johnstone et al. proposed a noise removal wavelet threshold estimator. Nowak et al. proposed the wavelet transform domain filtering algorithm for photon imaging systems where noise is a Poisson noise. Nowak proposed PRESS-the optimal nonlinear wavelet filtering method, adjusting the PRESS-the optimal filter according to the size of the local area of the image, to make it match the variance of the noise level with the Poisson. In fact, PRESS-the optimal nonlinear wavelet filtering method is in between a soft threshold and a hard threshold [4].

Chang et al. combined adaptive closed-valued and translation invariant de-noising to propose an airspace adaptive wavelet threshold method for image. The threshold selected adaptively changes with the statistical characteristics of the image. Zhang et al. proposed image de-noising algorithm based on neural network [5].

In short, the field of wavelet de-noising is hot with new methods coming forth one after another. Results are particularly good in removal of Gaussian noise.

3 Analysis of wavelet De-noising principle

3.1 Wavelet De-noising principle

Additive white Gaussian noise is the most common noise model [6], the observed signal for additive white Gaussian noise "pollution" can be expressed as:

$$y_i = f_i + \sigma z_i, i = 1,..., n, \tag{1}$$

in the formula above, y_i represents noised signal, f_i "pure" sampling signal, z_i an independent identical distribution of Gaussian white noise expressed as $z_i \sim^{iid} N(0,1)$, σ noise level, and n signal length. In order to restore the true signal f_i from the noised signal y_i, we can process the wavelet coefficients to separate signal and noise, given the different characteristics of the signal and noise shown in wavelet transform.

In applications in the practical engineering, the useful signal usually demonstrates characteristics such as low-frequency or sometimes being more stable, while the noise signal manifests itself as the high-frequency signal, thus enabling us to decompose the noised signal with wavelet [7] (such as the Three layer decomposition):

$$\begin{aligned} S &= CA_1 + CD_1 = CA_2 + CD_2 + CD_1 = \\ &= CA_3 + CD_1 + CD_2 + CD_3. \end{aligned} \tag{2}$$

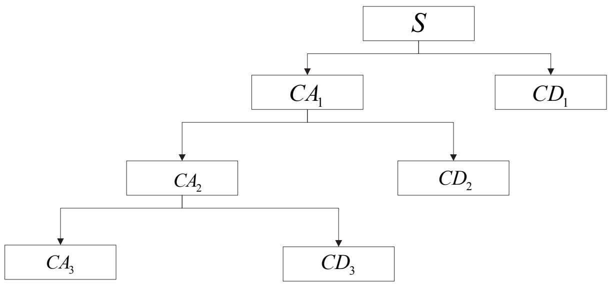

Figure 1 Three layer wavelet decomposition

In Fig. 1, cA_i is the approximate section of the decomposition, and cD_i is the detailed part, i = 1, 2, 3. The noise portion is usually contained in cD_1, cD_2, cD_3. To process the wavelet coefficients with the threshold value, we can get reconstructed signals, and thus the purpose of de-noising is achieved.

Conducting the signal de-noising by lifting wavelet decomposition of the signal, achieves the purpose of de-noising through threshold quantization to the high-frequency coefficients of wavelet decomposition, as Figs. 2 and 3 below.

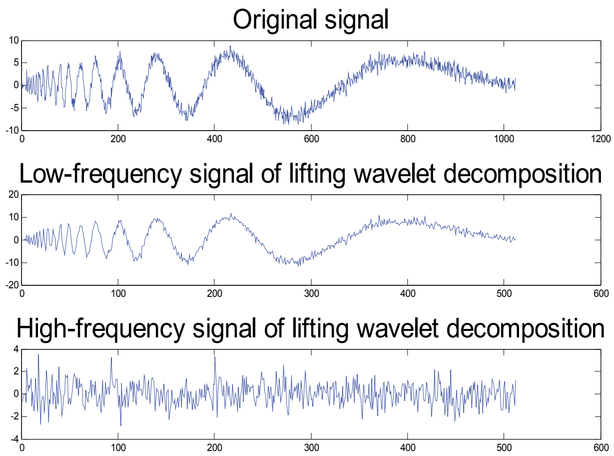

Figure 2 Monolayer lifting wavelet

We can use the wavelet analysis to extract the pure sinusoidal signal frequency from the synthesized signal. Because under the wavelet decomposition, different scale has different time and frequency resolution, and thus wavelet decomposition could separate the different frequency components of the signal.

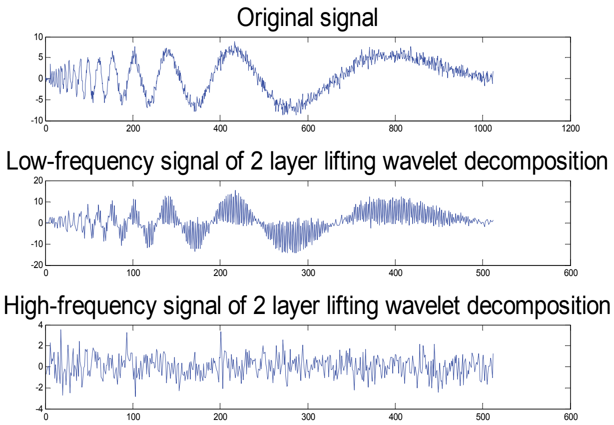

Figure 3 Layer lifting wavelet

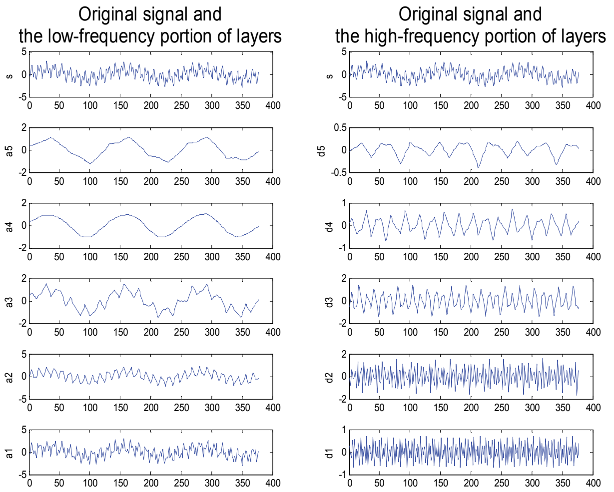

Figure 4 Analyze the different frequencies of the signal through wavelet analysis

3.2 Wavelet De-noising step

The de-noising process can be divided into the following three steps:

1) Wavelet decomposition for the observed data:

$$w_0 y = w_0 f + \sigma w_0 z, \tag{3}$$

wherein y represents the observation data vector, denoted as y_1, y_2,...., y_n, f is the true signal vector, denoted as f_1, f_2,..., f_n, and z is a Gaussian random vector, denoted as z_1, z_2,..., z_n, which uses the fact that a wavelet transform is linear in nature.

2) To process wavelet coefficients w_0 with the threshold value (using the soft threshold processing or hard threshold processing depends on the context and moreover, there are various forms of threshold). For instance, we select the most famous form of threshold:

$$t_n = \sigma \sqrt{2 \cdot \text{lb}\, n}. \tag{4}$$

Threshold processing can be expressed as η_{tn}. It can be proved that in the threshold formula (4), when n tends to infinity, the noise in the observation data can be almost completely removed if wavelet coefficients are processed with soft threshold.

3) Reconstruct signal through inverse transformation w_0^{-1} of the processed wavelet coefficients:

$$f^* = w_0^{-1} \eta_{tn} w_0 d. \tag{5}$$

The de-noised signal is thus obtained from the polluted sampling signal.

3.3 Threshold value selection and quantization

The key step of Donoho-Johnstone wavelet shrinkage de-noising method is the selection of the threshold value and the threshold processing, which will be discussed in detail.

3.3.1 Soft threshold and hard threshold

In the threshold processing of the wavelet coefficients, you can use the soft threshold or hard threshold method. The hard threshold retains only the larger wavelet coefficients and sets those smaller wavelet coefficients to zero:

$$\eta_H(w,t) = \begin{cases} w, |w| \geq t \\ 0, |w| < t \end{cases}. \tag{6}$$

Soft threshold sets those smaller wavelet coefficients to zero and also shrinks larger wavelet coefficients to zero:

$$\eta_S(w,t) = \begin{cases} w-t, w \geq t \\ 0, |w| < t \\ w+t, w \leq t \end{cases}. \tag{7}$$

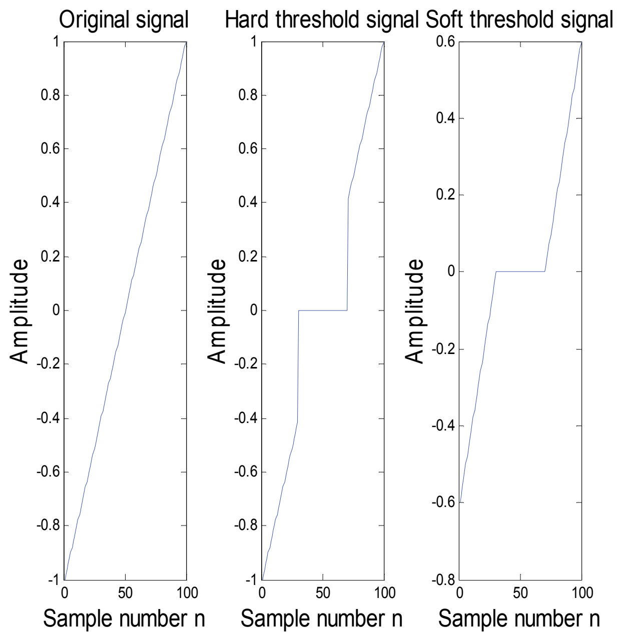

Figure 5 Hard threshold and soft threshold

The intuitive form is shown in Fig. 5 (take t = 1). From the figure we can see that the soft threshold is a smoother form, which produces smoother results after de-noising, and the hard threshold better reserves the true signal peak characteristic. Soft threshold essentially shrinks the wavelet coefficients, therefore Donoho-Johnstone named this de-noising technique as wavelet shrinkage.

3.3.2 Several forms of threshold

There are many ways of the threshold selection, and the basic rules are based on the noised signal model Eq. (1), where the signal level is 1. For the unknown or non-white noise, threshold value can be achieved in the re-adjustment of the de-noising.

At present, there are four common forms of threshold function to choose:

(1) Using a fixed threshold value, such as the formula (4), since this form of threshold can obtain good de-noising effect in the soft threshold in an intuitive sense.

(2) Using the Minimax Principle to select threshold which is also a fixed value as (1). It produces a minimum mean square error of the extreme value. The formula is expressed as:

$$t = \begin{cases} 0, n \leq 32 \\ 0{,}3936 + 0{,}1829 \cdot \mathrm{lb}\, n\,. \end{cases} \tag{8}$$

(3) Using Unbiased Risk Estimate for threshold selection. Obtain risk estimates of a given threshold, and select the minimum risk threshold as the final choice.

(4) The heuristics threshold. It is a combination of (1) and (3), when the SNR is very small, it is estimated that there is a lot of noise, then use the method (3); when the SNR is large, it is estimated that there is a small amount of noise, then use the method (1).

3.3.3 Threshold selection

The key issue of the threshold value processing is to select a suitable threshold value. If the threshold value (threshold) is too small, the de-noised signal still contains noise; conversely, if too large, the important signal characteristics will be filtered out, causing deviation. Intuitively, for a given wavelet coefficient, the greater the noise is, the larger the value of threshold is. Most threshold selection process is to calculate a threshold value for a set of wavelet coefficients, i.e. to gain a value that suits well the statistical characteristics of this group of wavelet coefficients.

Based on a typical threshold selection method proposed by Donoho and others, threshold value and the variance of the noise is presented and proved to be inverse ratio theoretically. It is expressed as formula (4):

This threshold value mainly depends on the data length of the sample n. When it is too small, the result still contains a lot of noise. If it is too large, the important signal will be deleted.

Therefore, the choice of threshold directly affects the results of the de-noising. To overcome this drawback, the researchers did a lot of research. Some scholars have proposed an adaptive dual lifting wavelet chaotic signal noise reduction method, using a gradient descent. The basic idea of this method is to add a threshold filter to the detail coefficient calculating step. Adjust the threshold value of the filter parameters according to the gradient descent method, until the detailed coefficients achieve the smallest RMSE. This method further improves the SNR of the system, and reduces the reconstruction error. However, this method is only applicable to specific types of noise and requires careful adjustment of the threshold value of the parameters of the filter, thus reducing the flexibility in practical applications. Therefore, it is significant in practical applications to develop a simple intelligent de-noising method that is capable of removing different types of noise. Bearing that in mind, we propose a de-noising method based on Information Center-Based Particle Swarm Optimization (CBPSO) to eliminate different types of noise. In this method, CBPSO is used in the lifting wavelet framework of selecting the optimal threshold value.

4 The improvement based particle swarm optimization and the intelligent nonlinear De-noising of the lifting wavelet

The wavelet transform has been widely used in noise reduction. It provides a compromise between the time and frequency resolution, and the flexibility to develop the signal between the different resolution levels. The most popular de-noising method based on wavelet transform is the wavelet threshold (shrinkage). This is a simple and effective method in most engineering applications. In this method, the noise signal is decomposed into sub-band, the noise filtering was mainly obtained by eliminating some coefficients that were smaller than a set of given threshold. In practical applications, the choice of threshold is essential to the de-noising results. When selected threshold is too large, the result will be unsatisfactory; conversely, if the threshold is too small, it will lead to large errors. These defects reduced its performance in practical applications. To overcome these drawbacks, many scholars in the study on the ability to adapt to the threshold have done a lot of works, more of the same noise reduction method based on wavelet transform has been proposed. However, through careful analysis, we can find that the above method only considers the influence of the additive white noise on signals. However, in practical applications, the signal is often polluted by different types of noise, such as white noise and colored noise. Therefore, an ideal de-noising method should learn intelligently the type of noise from signals themselves, which does not require any priory knowledge of the noise. In view of these considerations, we proposed a new de-noising method, which was de-noising in different types of noise background , which uses a Center Based Particle Swarm Algorithm (CBPSO) to select the optimal threshold in each sub-band of different scale. This method uses the second generation wavelets. Compared with the classical wavelet, the second generation wavelet implements wavelet transform and wavelet construction faster and provides greater flexibility.

4.1 The De-noising principle based on wavelet lifting

The noise reduction based on wavelet transform has received extensive attention in the estimation of signal. More and more of such attention was due to the following factors: 1) the simplicity of the method, 2) a lower computational complexity of relevant algorithms, 3) the ability to analyze different signals in different frequency bands. Currently the most popular de-noising methods based on wavelet transform are wavelet threshold or shrinkage method. This method resolves the observation signals to generate different types of sub-band, and reduces noise through filtering coefficients less than a given threshold.

However, engineering experience has shown that different types of wavelet functions have different time-frequency distributions. It is usually very difficult to choose one of the best wavelet functions for a given signal from which you want to extract the required information. In addition, inappropriate wavelet function will reduce the accuracy of the signal detection. Therefore, it is necessary to develop a new adaptive wavelet function to overcome these limitations. The appearance of the lifting wavelet transform has made up for the deficiencies of the traditional wavelet transform. Lifting wavelet transform is a new spatial domain method. Compared with the traditional wavelet transform, lifting wavelet transform is a faster wavelet transform. Lifting wavelet transform generally includes three basic steps: split, predict and update [8, 9].

Split: the original data set $x[n]$ was divided into two parts: the even index point $x_e[n]=x[2n]$ and the odd index point $x_o[n]=x[2n+1] (n=1,2,3,\dots,N)$.

Forecast: the neighboring even coefficient $\{x[2n]\}$, the prediction odd coefficient $\{x[2n+1]\}$, and the defined prediction deviation $d[n]$ are detail signals.

$$d[n]=x_o[n]-P(x_e[n]). \tag{9}$$

In which P is the prediction operator, to each $x_o[n]$, P is a linear combination of adjacent even coefficients, as defined in (10):

$$P(x_e[n])=\sum_{i=1}^{N}P_i x_e[n+i] \tag{10}$$

in which, N means how many data points will participate in the weighted prediction, P_i is the weighting factor of a series of wavelet coefficients (filter factors).

Update: scaling factor (approximate coefficients) $a[n]$ is produced by combining $x_e[n]$ and $U(d[n])$.

$$a[n]=x_e[n]+U(d[n]). \tag{11}$$

$x_e[n]$ is an even index point, $U(d[n])$ is a linear combination, it is generated by applying the update operator U to the detail coefficients $d[n]$, as defined in (12):

$$U(d[n])=\sum_{i=1}^{\bar{N}}u_i d[n+i]. \tag{12}$$

In which $\bar{N}$ means the number of wavelet coefficient points to participate in weight update, u_i is called boosting factor from where we can see the $U(d[n])$ determines the nature of the original wavelet and double scaling function. The following (13) and (14) are the structure of the Deslauriers-Dubuc wavelet:

$$d[n]=x_o[n]-(x_e[n-1]+9x_e[n]+ 9x_e[n+1]+x_e[n+2])/16. \tag{13}$$

$$a[n]=x_e[n]+(d[n-2]+9d[n-1]+ 9d[n]+d[n+1])/32. \tag{14}$$

The above prediction and update steps are equal to the implementation of the single-stage (4,4) Deslauriers-Dubuc wavelet transform. The forecast steps canceled the cubic polynomial and left residue to the high-pass signal $d[n]$. The update procedure resulted in that one of the $x[n]$'s low-pass and sub-sampled version was on the $a[n]$. Fig. 6 shows the forward lifting scheme.

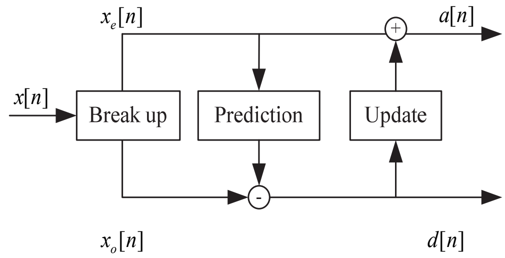

Figure 6 Forward lifting scheme

Unlike Fourier transform, lifting scheme can easily achieve the inverse lifting transform. Repeating the three steps in the approximate output, we can get different levels of detail signal and approximation signal. Inverse transformation could be directly obtained by a simple inverse operation and symbols conversion of equations (13) and (14). Inverse lifting transform step: the update undo, undo forecasting and consolidation, can be written as (15), (16) and (17):

$$Undo-update:\ x_e[n]=a[n]-U(d[n]). \tag{15}$$

$$Undo-predict:x_0[n]=d[n]+P(x_e[n]). \tag{16}$$

$$Merge:\ x[n]= merge(x_e[n],x_0[n])=d[n]- U(d[n])-U(d[n])+d[n]+P(x_e[n]). \tag{17}$$

The merge operation is the combination of the update operation withdraw and forecast operation withdraw. As for the forward transform and inverse transform, it is assumed that we choose the same P and U, and we ensure perfect reconstruction for any P and U lifting wavelet construction. Fig. 7 shows the inverse lifting transform.

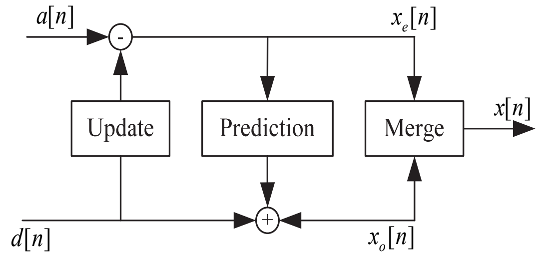

Figure 7 Inverse lifting scheme

From the above description, we can see that the lifting scheme can be used to extract the desired characteristics of a given data set, which can be used for signals' noise reduction. Since the noise energy is heavily concentrated in the detail coefficients of the high-band, the signal energy is concentrated in the low-band approximation coefficients. Using the threshold technology to process detail coefficient, we can eliminate the noise from the signals observed. At present, there are two options to shrink wavelet coefficients: "keep or kill" hard threshold program, as well as "shrink or kill" soft threshold program. The hard threshold scheme reserved wavelet coefficients greater than the threshold value and set coefficients less or equal to the threshold value to zero. In the soft threshold scheme, if the wavelet coefficient is greater than the threshold value, then conduct threshold reduction, if the wavelet coefficient is less than the threshold, then set it to zero. After careful analysis, we can see that the hard threshold is unstable or more sensitive to small changes and shows some discontinuities, and the soft threshold is stable, and avoids discontinuity.

Therefore, the choice of threshold function or threshold, is the key of the threshold method based on wavelet transform. Donoho has made an important contribution to the threshold selection research, and proposed the global threshold

$$Thr = \sqrt{2 \cdot \text{lb}\, N}. \tag{18}$$

This threshold value depends mainly on the data sample length N, when N is too small, the result still contains a lot of noise. If it is too large, the important signal will be deleted. Therefore, the choice of threshold directly affects the results of the de-noising. In this paper we proposed CBPSO (Center-Based Particle Swarm Algorithm) de-noising method, to eliminate different types of noise. In this method, CBPSO is used in the the lifting wavelet framework of selecting the optimal threshold value [10,11] .

4.2 Threshold selection of improvement based particle swarm algorithm

First, conduct three lifting wavelet transforms to the signal. Every level includes four frequency sub-bands of low-low (LL), low-high (LH), high-low (HL), and high-high (HH) . In this case, three frequency sub-bands need to be shrunk, so that each particle is a 9-dimensional real-valued vector, $Thr_j = \{Thr_j^1, Thr_j^2, \ldots, Thr_j^9\}$ in which j is the j^{th} instance of the P population, for each particle, each position represents a threshold value. The threshold values are allocated in the l hierarchical's b sub-band, in which $1 \leqslant b \leqslant 3, 1 \leqslant l \leqslant 3$, and l is the decomposition level of the lifting wavelet, b is the sub-band of the level l. Fig. 8 is the chromosome coding scheme of the lifting wavelet threshold .

$$\underbrace{Thr^{[1,1]} \quad Thr^{[1,2]} \quad Thr^{[1,3]}}_{Threshold\ for\ HL\ subband}$$

$$\underbrace{Thr^{[2,1]} \quad Thr^{[2,2]} \quad Thr^{[2,3]}}_{Threshold\ for\ LH\ subband}$$

$$\underbrace{Thr^{[3,1]} \quad Thr^{[3,2]} \quad Thr^{[3,3]}}_{Threshold\ for\ HH\ subband}$$

$Thr^{[1,1]}$ – Threshold for scale - 1

$Thr^{[1,2]}$ – Threshold for scale - 2

$Thr^{[1,3]}$ – Threshold for scale - 3

Figure 8 Chromosome encoding for the CBPSO-based lifting wavelet threshold

4.2.1 Fitness function and selection operations

Design suitability of an objective function evaluation solution. In this study, the objective function is defined as follows:

$$SNR = 10 \times \lg \left[\frac{\sum_{n=1}^{N} \bar{f}^2}{\sum_{n=1}^{N} (f - \bar{f})^2} \right]. \tag{19}$$

$$RMSE = \sqrt{\frac{1}{2N} \sum_{N-1}^{N} (\bar{f} - f)^2}. \tag{20}$$

In which, $\bar{f}^{[b,l]} = W^{-1}(f_{lwd}^{[b,l]})$, $f_{lwd}^{[b,l]} = T_{soft}(f_{lw}^{[b,l]}, Thr^{[b,l]})$, $f_{lw}^{[b,l]} = W(f)$, W and W^{-1} enhance the wavelet transform and inverse lifting wavelet transform; f and $\bar{f}$ represent input noisy signal and de-noising signal respectively, $f_{lw}^{[b,l]}$ and $f_{lwd}^{[b,l]}$ denote the wavelet transform and the threshold coefficient on the hierarchical l and the sub-band b ; $T_{soft}(f_{lw}^{[b,l]}, Thr^{[b,l]})$ expresses the soft threshold operation conducted on specific level l and sub-band b by using threshold $Thr^{[b,l]}$ to the lifting wavelet coefficients.

4.2.2 Particle search operation

Particle Swarm Optimization (PSO) is a population based stochastic optimization technique developed by Dr. Eberhart and Dr. Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling. PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA). The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. In PSO, the potential solutions, called particles, fly through the problem space by following the current optimum particles. Each particle keeps track of its coordinates in the problem space which are associated with the best solution (fitness) it has achieved so far. The fitness value is also stored. This value is called pbest. Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the neighbors of the particle. This location is called lbest. When a particle takes all the population as its topological neighbors, the best value is a global best and is called gbest [12,13].

In order to avoid falling into local optimum, we should design a central particle search based on the central population information. Search implementation of the central information-based particle is as follows:

Step 1: Initializing set of the CBPSO algorithm parameters: inertia weight, learning factor.

Step 2: Generate a 9-dimensional random vector c_1 ,in which each element belongs to the interval [0,1], and then, through iteration

$$c_k = 4c_{k-1}(1-c_{k-1}), k = 2,\dots,CL . \tag{21}$$

generate new vectors, in which $CL = [M_r \times L \times T]$ is the length.

Step 3: Mapping vector to the search space (22) for individuals

$$Thrc_k'' = TC_g + \xi(c_k - 0.5), k = 1,\dots,CL . \tag{22}$$

wherein $Thrc_k''$ a unit, ξ is a factor controlling range, c_k is a vector, TC_g is a standardized population information center, which is defined as (23)

$$TC_g = \frac{2}{T(T+1)}\sum_{j=1}^{T} Thr_j' \cdot fit(Thr_j') . \tag{23}$$

wherein T is the population size, Thr_j' is a standardized unit. We can find from equation (23) that the population center is weighted center of all individual positions, we use the fitness value as the weighted center of the weight. Obviously, this center may vary in different generations, it is mainly used in the process of guiding the particle flight.

Step 4: Assess individual, then determine the best historical position that individual winged and the best historical position that groups winged.

We use the maximum number of generations as a stopping rule, and record the best location of each generation of particle swarm.

4.3 Improvement based particle swarm algorithm and intelligent de-noising of the lifting wavelet

The main steps of the signal de-noising method based on CBPSO is summarized as following:

Step 1: Initialize the particle position and velocity.

Step 2: Calculate each particle's fitness value (objective function value), determine the best point of each particle and the best point of all particles (evaluate particles pros and cons).

Step 3: If the termination condition is satisfied, then stop, otherwise, go to step 2.

Step 4: Update the position and velocity of each particle.

Step 5: Apply the lifting wavelet signal to obtain the corresponding sub-band.

Step 6: Use the optimized threshold shrink all sub-bands except the LL sub-band.

Step 7: Apply the inverse lifting wavelet to transform signals after reconstruction and noise reduction.

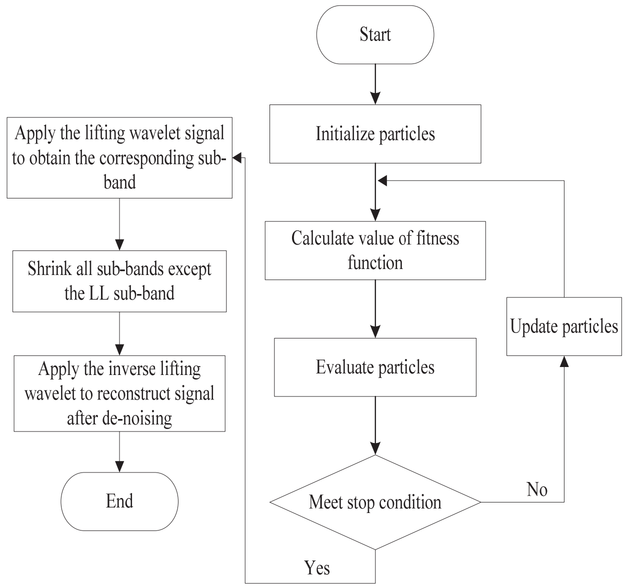

Figure 9 Flow chart of the CBPSO-based De-noising method

5 Experimental results and analysis

To illustrate the effectiveness and superiority of the CBPSO in signal de-noising, this paper conducted a large number of experiments by using the traditional hard, soft threshold function and the CBPSO algorithm respectively. Fig.10 shows the de-noising experiment results of the four selected noisy signal of the Blocks, Bumps, Heavy sine and Doppler. The signal length is 1024, and the experiment used the db3 wavelet.

6 Conclusion

This paper studies the problem of signal de-noising based on wavelet threshold, proposed a new signal de-noising method based on the center particle swarm optimization (CBPSO). In threshold selection of this method, based on the deference of change law of signal-to-noise wavelet coefficients in multi-scale, we consider to select different threshold values on deferent scales, and enhance local self-adaptability and flexibility of the threshold value, to obtain the optimal threshold value adaptively. The numerical results showed the effectiveness and the feasibility of this algorithm. This method has better de-noising effect, compared with the common threshold method.

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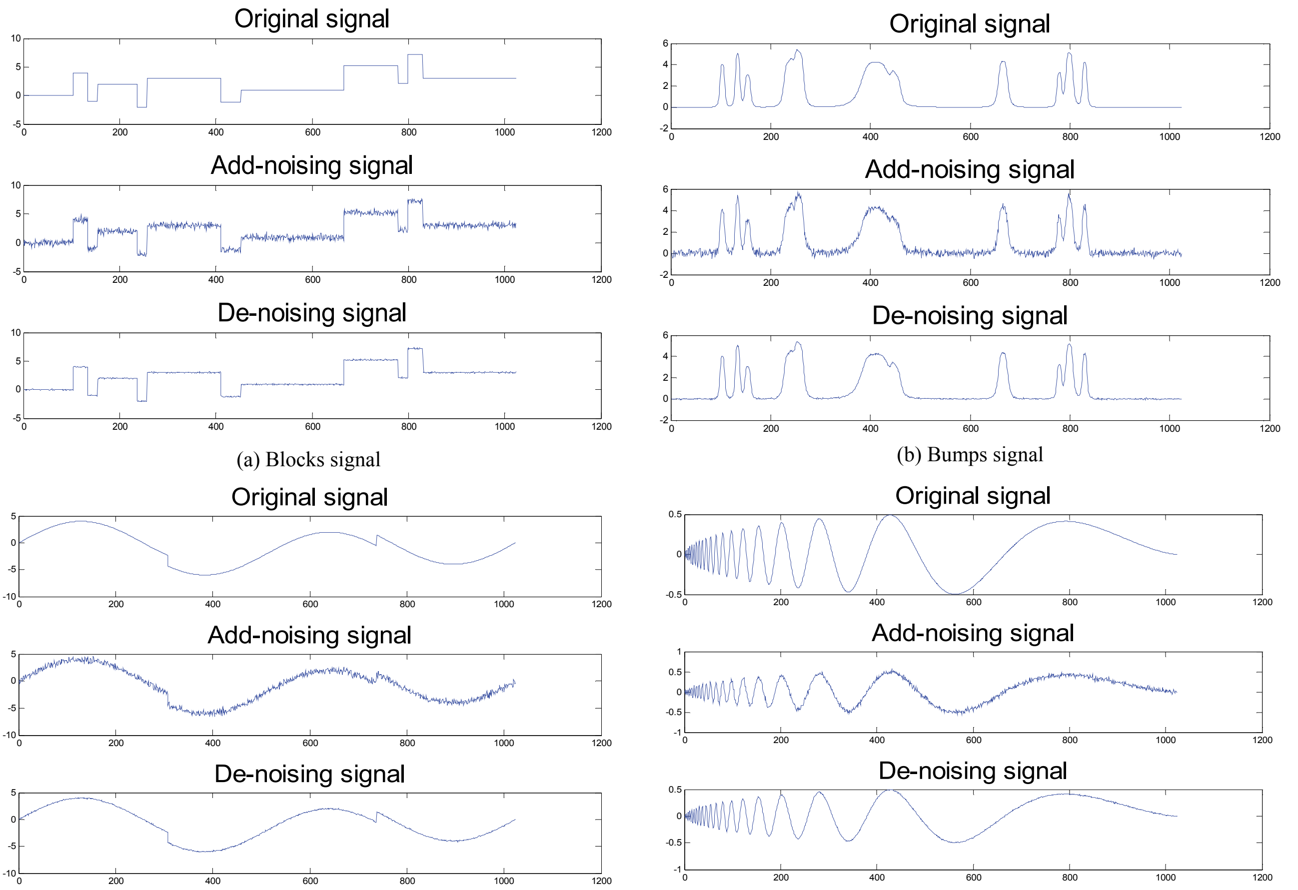

(a) Blocks signal (b) Bumps signal

(c) Heavy Sine Signal (d) Doppler signal

Figure 10 The de-noising results of the four noisy signals

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