Original Paper UDC 53/Cassirer, Einstein
Received May 16th, 2006

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The Role of Invariance in Cassirer's Interpretation of the Theory of Relativity

Abstract

The paper considers Cassirer's account of the philosophical problems raised by the theory of relativity. The main question the paper addresses is how Cassirer, as a Neokantian, responds to the discoveries made by Einstein. The problem here is especially the presupposition of the a priori nature of Euclidean geometry. Cassirer's answer lies in showing that Kant's philosophy is broad enough to include also non-Euclidean geometries in the determination of the physical world. He does this by showing that though Kant conceived space and time as forms of pure intuition he already connected them with certain theoretical factors, with the rules of the understanding. Space as the pure form of coexistence and time as the pure form of succession imply no special relations of measurement and it is thus a mistake to assume the a priori nature of Euclidean geometry. The way different geometries can figure in the determination of the physical world is explained in reference to the Klein approach to geometry, which defines geometrical properties as those that stay invariant according to a certain group of transformations. It is the concept of a group that is the real concept a priori. Group theory plays an even larger role in physical theories as well as Cassirer's epistemology. Namely, with the theory of relativity it becomes evident that physical theories are theories of invariants according to a group of transformations. Cassirer claims that the general doctrine of invariability of certain values must recur in some form in any theory of nature, because it belongs to the logical and epistemological nature of such a theory.

Key words

theory of relativity, Ernst Cassirer, transcendental philosophy of space and time, non-Euclidean geometry, theory of invariance

1. Introduction

Cassirer published his account of the philosophical problems raised by the theory of relativity in 1923 in his text "Einstein's Theory of Relativity Considered from the Epistemological standpoint". His main point is that the theory of relativity presents in the purest form the advance from the copy theory of knowledge to the functional theory. The objects and concepts of physics are no thing-concepts, no copies of particular contents given in perception, but theoretical assumptions and constructions which transform the merely sensible into something measurable.

What will primarily interest us in this paper is how Cassirer, as a Neokantian, responds to the discoveries made by Einstein. Namely, his philosophy of space and time is still based on that of Kant and even though he rejects them as forms of pure intuition they are still the a priori forms that make experience possible. Kant's philosophy resting on Newtonian physics, the crucial question becomes to what extent is the fate of transcendental philosophy entangled with that of Newtonian physics. Cassirer's main concern is to discover whether "the doctrines of the Transcendental Aesthetic offer a foundation, which is broad enough and strong enough to bear, along with the structure of the Newtonian mechanics, also that of modern physics". (Cassirer, 1953: 355)

There are two points of Cassirer's answer that this paper concentrates on, one has to do with Cassirer's accommodation of the transcendental philosophy of space and time in view of modern developments and the other has to do with the role that the theory of invariance plays in this attempt and also in the broader context of Cassirer's epistemology.

Regarding the first point, Cassirer's approach lies in comparing Einstein's methodological and epistemological presuppositions, especially those regarding the measurement of space and time and their function in a physical theory, to Kant's most general doctrine of the two forms of pure intuition regardless of the presuppositions of Euclidean geometry and the like.

There are several points of this comparison we shall consider: the concept of the objectivity of space and time, the determinations of the measurement of space and time according to the laws of nature, the difference in the meaning of the term coordination for a physicist and a philosopher and the choice of non-Euclidean geometry in general relativity.

Cassirer's main concerns here are to show that though Kant conceived space and time as pure forms of intuition he already connected them with certain theoretical factors, with the rules of the understanding, that in their most general meaning, space as the pure form of coexistence and time as the pure form of succession imply no special relations of measurement and it was thus a mistake to assume the a priori nature of Euclidean geometry.

2. The Objectivity of Space and Time

The first point of comparison is Einstein's statement that with his theory the last remainder of physical objectivity is taken from space and time. Cassirer considers the meaning of the phrase "physical objectivity" in this statement. He concludes that this must surely not mean that space and time are not things or objects in the sense of naïve realism, for physics must have left this behind from its very start. Space and time are no thing concepts but pure concepts of measurement as are all other objects of physics. In view of this Einstein's statement must mean something more, namely, that space and time here represent concepts and forms of measurement of an order higher than the first order.

Cassirer is here alluding to the fact that in the theory of relativity the physicist has to hold in mind not only the measured objects but also the conditions of this measurement. Before we can compare measurements made in different systems of reference a universal principle of transformation must be given. Only in this way can we combine them in a unitary result and use them in the determination of the laws of nature. Cassirer considers this reflection on the conditions of measurement a step forward from the epistemological point of view. Namely, he attributes all the conflicts in philosophy and exact science to the lack of such reflection, which allows for every new and fruitful concept of measurement to be transformed into a thing concept.

> "The ultimate constants of physical calculation are not only taken as real, but they are ultimately raised to the rank of that which is alone real." (Cassirer, 1953: 358)

Within the theory of relativity this should therefore no longer be possible.

It is in this sense that Cassirer sees in the theory of relativity only the accomplishment and application of the standpoint of critical idealism within empirical science itself. Namely, for Kant, too, space and time possess no separate existence either in the objective or in a subjective sense. They are not empirical objects that we could investigate but are the very conditions of this investigation, of experience itself. They are the forms of pure intuition and as such cannot be met again as contents of experience. The way "objectivity" is to be understood in this set-up lies in the significance these two forms have in the total structure of empirical knowledge.

"Space and time signify only a fixed law of the mind, a schema of connection by which what is sensuously perceived is set in certain relations of coexistence and sequence." (Cassirer, 1953: 412)

In this sense they possess besides their transcendental reality also empirical reality, but this must always be understood as validity for all experience and not as an independent existence of their own.

3. The Measurement of Space and Time

Furthermore, space and time as types of order can be comprehended only through what is ordered. In the case of the measurement of time, the temporal determinations of empirical events cannot be derived from the relations of these events to absolute time, but the phenomena must make necessary their positions in time for each other. So too, space can be empirically known to us by the community of substances in space, by a whole of physical effects found in experience. So the ordering in space and time can take place only on the basis of empirical knowledge of natural laws. This would suggest that Kant connects the determinations of space and time with certain theoretical factors, certain rules of the understanding.

In the theory of relativity this doctrine that it is the rule of the understanding that forms the pattern of all our temporal and spatial determinations is verified anew. For if in the theory of relativity we want to define time, i.e. determine the methods of its measurement, we first have to define the concept of the simultaneity of two events. In the special theory we base this definition on the principle of the constant velocity of light, and in the general relativity on the doctrine that all Gaussian coordinate systems are of equal value for the formulation of the laws of nature. Cassirer points out here that these principles are not the expression of an empirically observed fact but are norms that the understanding uses hypothetically in the interpretation of experience.

4. The Concept of Coordination

According to Cassirer the crucial point of comparison between Einstein's theory and Kant's philosophy lies in the concept of coincidence to which the theory of relativity reduces the content and the form of all laws of nature.

"The following statements hold generally: Every physical description resolves itself into a number of statements, each of which refers to the space-time coincidence of two events A and B. In terms of Gaussian co-ordinates, every such statement is expressed by the agreement of their four co-ordinates x[1], x[2], x[3], x[4]." (Einstein, Part 2, Chapter 27)

The above passage can be found in the section on the non-Euclidean geometry of the space-time continuum and the impossibility of constructing a Cartesian coordinate system. We surmount this difficulty by referring the four-dimensional space-time continuum in an arbitrary manner to Gauss co-ordinates. We assign to every point of the continuum (event) four numbers, x[1], x[2], x[3], x[4] (co-ordinates), which have not the least direct physical significance, but only serve the purpose of numbering the points of the continuum in a definite but arbitrary manner. Einstein justifies the fact that we assign to an event particular co-ordinates which in themselves have no significance as follows: If we consider a material point with only a momentary existence without duration, then it would be described in space-time by a single system of values x[1], x[2], x[3], x[4]. Thus its permanent existence must be characterised by an infinitely large number of such systems of values, corresponding to the material point, we thus have a (uni-dimensional) line in the four-dimensional continuum. In the same way, any such lines in our continuum correspond to many points in motion. The only statements regarding these points which can claim a physical existence are in reality the statements about their encounters. In our mathematical treatment, such an encounter is expressed in the fact that the two lines which represent the motions of the points in question have a particular system of co-ordinate values, x[1], x[2], x[3], x[4], in common.

"…in reality such encounters constitute the only actual evidence of a time-space nature with which we meet in physical statements". (Einstein, Part 2, Chapter 27)

We reach the construction of physical space and time only in this way; the space-time manifold is nothing but the whole of such coordinations. At this point, Cassirer emphasizes the difference between the philosopher and the physicist, namely for the latter space and time are a concrete measurable manifold gained as a result of the coordination, while for the philosopher they are just this coordination itself. Space is coordination from the standpoint of coexistence and time from the standpoint of succession.

Still, it is exactly on the basis of the concept of coincidence that Cassirer ascribes the Kantian presupposition of pure intuition in the sense of the possibility of relating point to point also to the theory of relativity. Namely, even though we can conceive the world-points and the world-lines so abstractly as to mean nothing but certain mathematical parameters, their coincidence acquires meaning only on the basis of the possibility of succession we call time.

»A coincidence which is not to mean identity, a unification, which is still a separation, since the same point is conceived as belonging to different lines: all this finally demands that synthesis of the manifold, for which the term 'pure intuition' was formulated.« (Cassirer, 1953: 418)

Furthermore, Cassirer insists that it is in this most general meaning that we have to comprehend space and time only as pure forms of coexistence and succession. It is crucial here that we presuppose nothing as to the special relations of their measurement.

"Thus in reality, the description of the time-space continuum by means of Gauss co-ordinates completely replaces the description with the aid of a body of reference, without suffering from the defects of the latter mode of description; it is not tied down to the Euclidean character of the continuum which has to be represented." (Einstein, Part 2, Chapter 27)

It is here that, according to Cassirer, Kant made his mistake, because he did not always grasp the general meaning of the serial form of coexistence and succession with equal sharpness but applied more special meanings to it. This is why he takes Euclidean geometry to be a priori. But with the theory of relativity it not only turns out that the geometry of space-time is not Euclidean but that there is no one geometry we can apply to the whole of reality, since the relations of measurement depend on the gravitational potential which changes from point to point and thus gives rise to different geometrical structures at different places in the manifold.

5. Kant and non-Euclidean Geometries

In order to accommodate these discoveries Cassirer has to modify Kant's theory and he does so by allowing also non-Euclidean geometries as a priori, i.e. allowing the axioms of non-Euclidean geometry to enter into the determination of the understanding in which the empirical world arises for us. This presents no problem for Cassirer since on the one hand he continuously distinguishes between the space of intuition (which still remains Euclidean in a sense) and the space of physics that arises from the relations of measurement based on natural laws, and on the other hand he points out that Kant himself already attributed such construction of physical space to the rules of understanding and not to intuition. There still remains the question of how to explain the unity of space as a form of coexistence and the different geometries. The answer can be found in Cassirer's philosophy of mathematics, which gives another solution to the above problem.

Cassirer's discussion of geometry is based on the work of Felix Klein and his algebraic set-up of geometry wherein the concept of a group plays the decisive role. According to the theory of invariance each geometry depends on the group of transformations, and its properties depend on the choice of this group. Geometrical properties are only those that are preserved by a certain group of transformations. With such an approach Klein was able to construct and systemize different geometries – metric, affine, projective and inversive geometry plus *analysis situs* (topology). These geometries differ in the level of their universality, even more they are hierarchically ordered since the group connected with a more general geometry contains the group connected with a more special geometry. Klein managed to include also non-Euclidean geometries in his account by relating them with projective geometry, namely the group of non-Euclidean transformations forms a subgroup of projective transformations and we can thus view the non-Euclidean geometry as a subgeometry of projective geometry.

The Klein approach to geometry preserves the unity of space as all geometries presuppose and use the general form of space, i.e. the form of possible coexistence. In this all geometries are alike but they differ in that they relate this space to different groups of transformations. Furthermore these geometries form an ordered sequence (hierarchy) where each element includes its predecessor, so that taken all together they are united into a whole.

Having established the space of pure mathematics there arises the question of its application in physics. Cassirer refers here to Poincaré, who argues for the a priori nature of geometry on the basis of its definition in connection with group theory. He claims that the concept of a group pre-exists at least potentially in our mind, and that it is not a form of our sensibility but the form of our understanding. We apply a certain geometry to the empirical manifold by selecting among the various groups such as leads to the simplest and most convenient description of physical phenomena. Cassirer emphasizes here that though it is certain elementary experiences that lead us to construct such a geometry and that there is in experience a principle of our choice of geometry, this does not mean that the geometrical axioms themselves are empirically grounded.

For example, in going from the special theory of relativity to the general theory we choose non-Euclidean geometries over Euclidean. Einstein's argument is as follows: In the special theory of relativity space-time co-ordinates can be regarded as four-dimensional Cartesian co-ordinates. This was possible on the basis of the law of the constancy of the velocity of light. But the general theory of relativity cannot retain this law, according to it the velocity of light must always depend on the co-ordinates when a gravitational field is present. But the presence of a gravitational field invalidates the definition of the coordinates in the special theory of relativity. In view of the results of these considerations we are led to the conviction that, according to the general principle of relativity, the space-time continuum cannot be regarded as a Euclidean one.

Cassirer is right in pointing out that the choice of non-Euclidean geometry in the general theory leads to a greater unity and systematic completeness in the formulation of the laws of nature. The Euclidean expression of the linear element proves to be insufficient in the working out of the fundamental thought of the general theory of relativity since it does not fulfil the fundamental demand of covariance. The relations of measurement within the physical manifold find their simplest expression in the language of non-Euclidean geometry. As Cassirer emphasizes this language is and must remain symbolic, just as the language of Euclidean geometry could alone be. The reality it expresses is not the reality of things but that of laws and relations.

"The structures of geometry, whether Euclidean or non-Euclidean, possess no immediate correlate in the world of *existence*. …The existence, that belongs to them by virtue of their definition, by virtue of a pure logical act of assumption is, in principle, not to be interchanged with any sort of empirical 'reality'." (Cassirer, 1953: 433)

To sum up, the step beyond Kant is in always keeping in mind that space is just a pure form of coexistence, a rule of the understanding, that the determinations of measurement and the choice of geometry depend on the laws of nature and that we have to allow also the non-Euclidean geometry to play a role in the determination of the physical world.

6. Group Theory and Invariance

The theory of groups, especially in reference to the theory of invariants, proved essential in Cassirer's vindication of the transcendental philosophy of space and time. We should now like to consider whether it does not play an even larger role in Cassirer's further consideration of the theory of relativity.

With the theory of relativity it becomes evident that physical theories are theories of invariants according to a group of transformations, of the Lorentz transformations in the case of SRT and in GRT of more general transformations. Furthermore Cassirer claims that the general doctrine of invariability of certain values must recur in some form in any theory of nature, because it belongs to the logical and epistemological nature of such a theory.

Cassirer was right on this, which the modern mathematical interpretation shows. The group-theoretic classification of space-time theories associates Newtonian mechanics to the Galilean group, special relativity to the Lorentz group and general relativity to a group of all admissible one-one sufficiently continuous transformations (differentiable), these are all invariance groups, leaving certain geometrical objects of theory unaffected.

The invariance group of »common sense« Newtonian mechanics with absolute space and time is the group of all spatial translations, spatial rotations and time translations. Adding the Galilean transformations to this group we get Newtonian (Galilean) mechanics leaving only absolute time. Considering classical electrodynamics, this holds true only in connection with the first group, but experimental evidence shows that it holds in every inertial frame. This implies that the Galilean transformations linking two inertial systems are incorrect. The correct transformations are the Lorentz transformations which added to the group of »common sense« Newtonian mechanics produce the special theory of relativity.

Cassirer also sees in the relativization of the spatial and temporal magnitudes the necessary condition through which the new invariants of the theory are discovered and grounded. Thus the new invariants in the special theory of relativity are those that stay unchanged by the Lorentz transformation. In going from Newtonian physics to special theory of relativity the notions of absolute rest, absolute velocity, and absolute simultaneity are no longer invariant. There still remain absolute acceleration and rotation, which disappear in the general theory of relativity. Cassirer points out that above all it is the general form of natural law that is the real invariant and the real logical framework of nature in general. This is what the general theory of relativity achieves by its general principle of relativity, the invariance with regard to all transformations of the system of reference.

The general theory of relativity makes it clear that we are not to seek for the ultimate constants, the invariants of the system in particular given things but always in certain fundamental relations and functional dependencies retained in the symbolic language of our mathematics and physics, in certain equations.

Cassirer thus rejects the negative views of the theory of relativity, namely, that it brings an element of the subjective into the formulation of the laws of nature and thus destroys the concept of nature. In fact the theory of relativity does just the opposite, it teaches that only those relations and values can be called truly objective that do not change from one reference system to another.

»What we call the system of nature only arises when we combine the measurements, which are first made from a standpoint of a particular reference body, with those made from other reference bodies, and in those made from all 'possible' reference bodies, and bring them ideally into a single result. How there can be found in this assertion any limitation of the 'objectivity' of physical knowledge is not evident; obviously it is meant to be nothing but a definition of this very objectivity.« (Cassirer, 1953: 380)

Cassirer refers to Kant here and his definition of the object as that X in which we produce the synthetic unity of the manifold of intuition. We gain the object by unifying the totality of observations and measurements into a single complete whole. The objectivity of physical knowledge lies in that the measurements are mutually coordinated according to definite rules, in this function of determination itself. It is a particular group of transformations that plays this role of connecting particular measurements into a whole. The group of Newtonian mechanics joins all coordinate systems at rest relative to absolute space, the Lorentz group joins all inertial systems and the group of general relativity joins all reference systems.

The theory of invariance and the concept of group play an essential role in Cassirer's consideration of the theory of relativity and his modifications of the transcendental philosophy of space and time. In taking the concept of group as a priori, he is able to retain the unity of space as the pure form of coexistence and include non-Euclidean geometries in the determination of the physical reality. Even more physical theories prove to be theories of invariants with regard to certain groups of transformations and it is exactly the invariance that secures the objectivity of a physical theory.

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Die Rolle der Invarianz in Cassirers Interpretation der Relativitätstheorie

Zusammenfassung

Der Artikel setzt sich mit Cassirers Erklärung für die durch die Relativitätstheorie aufgestellten philosophischen Probleme auseinander. Die Hauptfrage richtet sich darauf, wie Cassirer als Neukantianer auf Einsteins Entdeckungen antwortet. Das Problem, das hierbei aufkommt, ist die Präsupposition von der aprioristischen Natur der euklidischen Geometrie. Cassirers Antwort liegt in der Begründung, dass Kants Philosophie ausreichend breit angelegt sei, um auch nicht-euklidische Geometrien in die Determinierung der physikalischen Welt mit einzubeziehen. Er tut es, indem er aufweist, dass Kant, auch wenn er sie für Formen der reinen Intuition hielt, Zeit und Raum mit bestimmten theoretischen Faktoren, mit Erkenntnisregeln in Verbindung brachte. Der Raum als reine Koexistenzform und die Zeit als reine Sukzessionsform implizieren keine besonderen Messrelationen, und es wäre folglich falsch, eine aprioristische Natur der Euklid'schen Geometrie anzunehmen. Die Art der Anwendung verschiedener Geometrien bei der Determinierung der physikalischen Welt wird erklärt in Anlehnung an Kleins Geometrieansatz, bei dem geometrische Eigenschaften vor dem Hintergrund einer bestimmten Gruppe von Transformationen als invariant bleibende angesehen werden. Das Konzept der Gruppe stellt das eigentliche Konzept a priori dar. Die Gruppentheorie spielt sogar eine größere Rolle sowohl in physikalischen Theorien als auch in Cassirers Epistemologie. Denn mit der Relativitätstheorie wird offenkundig, dass physikalische Theorien gegenüber einer Transformationsgruppe Invarianztheorien sind. Cassirer behauptet, dass die allgemeine Lehre von der Unveränderlichkeit bestimmter Werte in irgendeiner Form in jeglicher Theorie der Natur zurückkehren müsse, weil sie zur logischen und epistemologischen Natur einer solchen Theorie gehöre.

Schlüsselwörter

Relativitätstheorie, Ernst Cassirer, Transzendentalphilosophie von Raum und Zeit, nicht-euklidische Geometrie, Invarianztheorie

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Le rôle de l'invariance dans l'interprétation de Cassirer de la théorie de la relativité

Sommaire

Cet article traite les explications de Cassirer sur les problèmes philosophiques que soulève la théorie de la relativité. La question principale posée par cet article est de voir comment Cassirer en tant que néokantien répond aux découvertes d'Einstein. Il s'agit surtout du problème de la présupposition de la nature a priori de la géométrie euclidienne. La réponse de Cassirer démontre que la philosophie de Kant est suffisamment étendue pour y inclure aussi les géométries non-euclidiennes dans la détermination du monde physique. Cassirer le fait en révélant que Kant tout en concevant l'espace et le temps comme des formes de pure intuition les a déjà reliées à certaines facteurs théoriques, aux règles de l'entendement. L'espace en tant que forme pure de coexistence et le temps en tant que forme pure de succession n'impliquent aucunes relations spéciales de mesure. Ainsi il est erroné de présumer la nature a priori de la géométrie euclidienne. La manière dont les géométries différentes peuvent figurer dans la détermination du monde physique se réfère à l'approche géométrique de Klein selon laquelle les propriétés géométriques sont définies comme celles qui restent invariantes par rapport à un certain groupe de transformations. Et c'est justement le concept de groupe qui est le vrai concept a priori. La théorie du groupe a même un rôle plus important dans les théories physiques, comme c'est le cas de l'épistémologie de Cassirer. Avec la théorie de la relativité il est devenu évident que les théories physiques sont des théories des invariances en relation aux groupes de transformations. Cassirer avance que la théorie générale de l'invariabilité de certaines valeurs doit revenir sous une certaine forme dans toute théorie de la nature parce qu'elle appartient à la nature logique et épistémologique d'une telle théorie.

Mots clés

théorie de la relativité, Ernst Cassirer, philosophie transcendantale de l'espace et du temps, géométrie non-euclidienne, théorie de l'invariance