CARLOS A. SILVA, Ph.D.
E-mail: c.silva@centec.tecnico.ulisboa.pt
C. GUEDES SOARES, Ph.D.
E-mail: c.guedes.soares@centec.tecnico.ulisboa.pt
Centre for Marine Engineering and Technology (CENTEC)
Instituto Superior Técnico
Universidade de Lisboa
Av. Rovisco Pais, 1049-001 Lisbon, Portugal

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SIZING A FLEET OF CONTAINER SHIPS FOR A GIVEN MARKET

ABSTRACT

The growth in the short sea shipping sector motivated the development of a methodology used as a decision support tool in which both the parameters regarding the demand of markets and the characteristics of the fleet may be tested and appraised. It is also possible to determine the fleet deployment, establishing its routes and scales in the ports for a particular scenario. The considered methodology is divided into two parts, the first being the one related to the generation of all feasible routes, with all the parameters specific to each route for each vessel class. The second part introduces a linear programming model that maximizes the shipping operation total profit, according to a given set of restrictions. The models were structured according to three main criteria: the evaluation of the fleet for each vessel class; the optimal route for each vessel and the frequency in each port. To provide the methodology validation, the developed models shall be submitted to a fictitious operational scenario, considering three different situations: the fleet normal operation; the fleet response to different demand scenarios; an evaluation of several fleet compositions for the same demand level.

KEY WORDS

containerships; fleet design; linear programming; mixed integer programming

1. INTRODUCTION

The need for container ship owners to size their fleet in order to meet a required demand level motivated the development of a methodology sustained by a decision support tool, in which the parameters associated with the demand for goods in different markets and fleet characteristics can be tested (for example, the assignment of routes, port calls, vessels specifications or loading plans).

By definition, "Fleet Sizing" is the activity of quantifying the number and type of vehicles that will be used in a transport operation once set on a specific market. Subsequently, there is also the problem of quantifying the fleet properties, affected by factors such as the size of operation, the internal capacity of the company, the jurisdiction of the sector, the availability of return cargo or the logistic model to be used. The basic principles for selecting and negotiating with the transporters must be envisaged with a freight consolidation policy.

List et al. [1] discussed this approach and proposed a conceptual model linking the various components in the strategic and operational levels of fleet management. The authors established not only the fleet size, but its assignment as well, considering the minimum level of service to be fulfilled for a given demand, network and cost parameters. This structure shows a relationship between the various components, where the demand is met and the requests for transportation are determined according to the shipment prominence or priority. These affect how the fleet is assigned considering loaded and empty transport, so that the restriction of demand is met.

On the other hand, the term "fleet employment" is used when determining the fleet productivity of a given size, i.e. the maximum utilization of a fleet of vehicles will dictate the maximum amount of demand that they are able to meet, leading to the quantification of the units needed for a given operation. Ronen [2] published a review of major studies published up to 1993 regarding fleet scheduling with an emphasis on maritime transport. Along with this review the author discusses the need for integration decisions which are taken at strategic and operational levels.

These two concepts directed the current bibliographic review, in hoping to find models that not only assess the use of the fleet in a refined manner, but were also able to perform the correct fleet sizing and employment to meet the objectives of the operation on a constrained environment. It was determined that this problem has been hardly tackled, except for the preliminary work by Silva et al. [3], which will serve as foundation for the present problem. In that paper, the authors present the basic model which will be reformulated to consider the introduction of several additional concepts, such as time varying cargo demand, vessel operational profits or port physical limitations.

The following sections elaborate on all of the main particulars of the developed model describing the main concepts and structure (Section 2), the detailed explanation of the model components (Sections 3 and 4), model validation considering parametric variations of several input parameters (Sections 5, 6 and 7) and final conclusions.

2. FLEET MANAGEMENT OPTIMIZATION

In recent years, the size of container vessels has shown considerable growth resulting from economies of scale. Cullinane et al. [4] present a model that quantifies the economies of scale during the operation of container ships of large tonnage. A sensitivity analysis is conducted to test the effect of various scenarios, as well as determining the optimal size of a vessel for each situation. Later, some considerations are made regarding the determination of the allocation of existing vessels, considering trends and the impact they will have on container operations, logistics and port operations.

Wong et al. [5] also emphasized the importance of analyzing the effect of various fleet compositions, on the performance level and costs for operating the system, as a manner to correctly direct the optimization of the system. This approach contemplates in the objective function a penalty related to the level of service and considers the effect of various fleet configurations in service quality while searching for the optimal solution. Shapiro [6] emphasizes that the basis of the concept of integrated logistics is the total cost analysis, which seeks to minimize the set of all parcels of logistics costs, while maintaining a level of service desired. Additional studies regarding fleet sizing and scheduling are presented by Kochel [7], Petering [8], Lau [9].

Fagerholt [10] developed a model for the dimensioning of a fleet of ships on a feeder system, by defining the route of vessels, subject to the trip maximum duration. Routes were constructed using a thorough procedure and were incorporated in an integer programming model of "set partitioning" type, which assigns ships to routes to minimize the fixed cost of a variable operation. Fagerholt [11] further developed a methodology based on the transformation of the constraint related with the rigid time window in a more flexible restriction, while evaluating the impacts on the vessel schedule (service level) and transport costs.

Taillard et al. [12] defined vehicle routing for a heterogeneous fleet, solving a succession of problems with this issue. They applied the method of column generation supported with a taboo search in order to generate only the most promising routes, solving then a model of "set partitioning" type. Salhi [13] also developed a solution for a heterogeneous fleet of vehicles routing through a heuristic improvement: an exchange of customers between routes, evaluation of the relationship that relates the fixed and operational costs. Among others, Taner et al. [14], Caric et al. [15], Karlaftis et al. [16] or Jarpa et al. [17] also address the problem related to vehicle routing.

Bearing in mind the objective of planning the transport operation of the fleet in a refined manner, the developed model and methodology intend to serve as a decision support tool for container cargo transport shipping entities, i.e. the whole approach of the model development is from the point of view of the ship owner. Consequently, the model overall objective is to maximize the operation's profit, while maintaining a given minimum service level. It is assumed that the navigation company will not operate using vessels chartered for a specific voyage or time chartering (Sherali et al. [18]), since the objective function would not apply to these particular situations. The model will assist the decision-making process, considering a time period of five (5) years, with respect to the allocation ships, setting the best route for each vessel, while trying to reach the expected call frequency, subjected to a particular set of constraints.

The fleet design concept considers factors associated with the vessel characteristics such as speed, capacity and costs, among others, directly influencing the calculation (Bausch et al. [19], Brown et al. [20]). Therefore, it does not only characterize the fleet itself, but its operation as well. On the other hand, it is possible to define the routes and required cargo flows, allowing the depiction of the best overall operational scenario. The data related to the proposed vessels, routes, cargo flows, etc. shall be inserted in the model by means of input parameters allowing posterior sensitivity analysis on all parameters. The number of the initially available vessels is defined as a model input parameter, implying that a limited resources problem is considered.

The developed decision support tool is prepared by means of linear programming, which is a commonly used methodology to address the proposed problem. Although linear programming may seem simplistic, in fact it encompasses most existing reality related systems and its application is recommended for these cases. Most restricted problems (i.e. restricted optimizations) are of a linear nature, where there is a single objective function and various imposed restrictions. While the introduction of the simplex solver in 1947 brought a rapid evolution of using this method, still today there are methods being designed to solve linear programming problems.

The tool structure is divided into two parts as depicted in *Figure 1*: firstly, the evaluation of all feasible

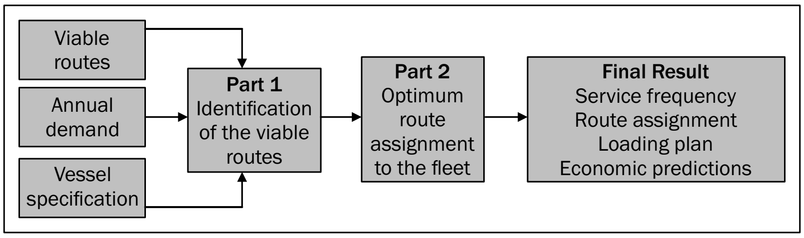


Figure 1 - Model structure [3]

routes and calculation of parameters related to each operation (route) and each type of vessel considered (such as the involved costs, revenues, number of round trips, etc.). Such results will provide valuable input to the second part, whose main objective is to establish a schedule of routes for each vessel to maximize profits while meeting the required service level.

The frequency of call for each of the ports is automatically defined with the choice of the best routes for all vessels, since the amount of possible travels has been calculated in the first model and the best route is defined in order to meet the required annual demand service.

In order to enable testing different scenarios, by varying the parameters mentioned above, the models shall present robust structures. As such, the final calibration and validation will be achieved through an example case and later the development of a parametric sensitivity analysis of the operations will be made.

3. IDENTIFICATION OF THE VIABLE ROUTES

Due to the need of providing reliable service, where customers may schedule their transport service with the remaining operations of their services, ship owners have come to the conclusion that it is preferable to consider cyclic trips with different routes on each way/return trip, while maintaining a constant port call frequency (despite not being the optimal solution, occasionally considering costs higher than desirable for ship owners). Following this basic concept it is presumed that a feasible route is composed of a sequence of ports or a number of conceivable voyages, represented by arcs, to be operated by a vessel. Since it is possible for the route to have distinct paths in both directions (commonly referred to as ascending-descending directions or North-South bounding), the route is established by two-way trip itineraries. In practical operations, in each voyage the vessel will not navigate in opposite directions towards the final destination port, i.e. if the vessel follows a trajectory towards the north or south she will continue in that direction until the final stretch of this itinerary. In order to maintain the continuity of the voyage, the cyclic feature of the route must be considered, meaning the arrival and departure ports must be the same.

The methodology used to calculate the number of viable routes is illustrated in *Table 1* (in the example, considering five ports). A similar logic is applied for different number of ports. The routes creation program was developed in a manner such that a binary variable “x” for each port defines if it is included or not on that specific route. This variable will present a value of “1” for the ports that are called and “0” otherwise. Through this method, the ordering of ports will imply a logical sequence of voyages, avoiding alternating directions, as described before.

The number of viable routes for each row is given in the column on the right of *Table 1*, this value being equal to the power of two (2) and exponent to the number of binary variables on the line. The formulation of Equation 1, which calculates the number of alternatives (R) for this problem, is shown in Expression 1 (where “n” is the number of ports considered).

$$R = \sum_{l=0}^{n-2} \sum_{i=0}^{i=l} 2^{2i} \qquad (1)$$

Consequently and based on this knowledge, it is possible to make an assessment of the aspects that distinguish each of these routes, (for example with respect to the travelled distances, travel time, number of round trips, maximum transport capacity during the adopted time horizon and, ultimately, the route fixed costs and profits).

Table 1 - Route definition [3]

Ascend					Descend						
1	2	3	4	5	5	4	3	2	1	Combinations	
1	X	X	X	1	1	X	X	X	1	=	64
1	X	X	1			1	X	X	1	=	16
1	X	1					1	X	1	=	4
1	1							1	1	=	1
	1	X	X	1	1	X	X	1		=	16
	1	X	1			1	X	1		=	4
	1	1					1	1		=	1
		1	X	1	1	X	1			=	4
		1	1			1	1			=	1
			1	1	1	1				=	1
										Total	112

4. DETERMINATION OF THE OPTIMUM ROUTES FOR THE FLEET

The results acquired in the previous model will serve as input basis for the optimization model, formulated as a mathematical model in mixed linear programming (MIP). The second part should be structured to select the best route for each fleet vessel, while meeting the restrictions of demand between each considered origin-destination pair of ports. The demand parameter considers five consecutive periods of one year (each annual demand may fluctuate between years), where the required cargo transportation can be accomplished in several trips by different routes and combining different ships.

The fleet optimum sizing will be obtained through the preparation of several operational scenarios, evaluating the balance between the overall cost and provided service level. Since a fixed number of ships with specific characteristics are given as input to the model, it is considered a problem of limited resources.

4.1 Indexes and parameters

Throughout the mathematical model presentation the following indexes and parameters will be used:

"r": indicates the route consisting of a sequence of ports in both directions (considering a round trip with different itineraries in each direction). It ranges from 1 to R, where R is the number of routes available to the model and depends on the amount of ports considered;

"i": indicates the ports of origin, receiving a specific port code in a standardized format;

"j": indicates the ports of destination, receiving a specific port code in a standardized format;

"k": indicates the vessel. It ranges from 1 to n, where n is the maximum number of vessels available;

"a": indicates the year. It represents one-year time periods for a 5-year limit;

"Cap_k": capacity of the vessel k in TEUs;

"$D_{i,j,a}$": total demand from port i to port j during year a, in TEUs;

"$N_{r,k}$": maximum number of round trips by vessel k on route r;

"$C_{r,k}$": annual cost of route r, when operated by vessel type k;

"CF_k": fixed annual operating cost for the k type vessel;

"$CV_{r,k,a}$": variable cost per voyage in the year a, on route r by vessel k;

"$L_{r,k,a}$": revenue per voyage in route r, operated by vessel type k, in year a;

"Cal_k": vessel k draft, in metres;

"Max_j": maximum draft for port j, in metres.

4.2 Decision variables

The selection of which route is assigned to each vessel is done using a binary variable ($X_{r,k}$), which takes the value "1" if route "r" is assigned to vessel "k" and "0" otherwise. It is therefore considered that the assigned routes are for the entire planning horizon of five years.

Variable $M_{r,k,i,j}$ defines the flow of cargo between the origin and the destination ports (indexes "i" and "j"), on route "r", in one voyage of vessel "k". This variable was created to control the amount of cargo within the vessel, in order to comply with the vessel maximum capacity restriction. Moreover, the model will consider the value for $M_{r,k,i,j}$ as being the same on all trips during the five-year planning time horizon.

4.3 Objective function

The model's main objective is to present an appropriate scheduling for each vessel, while respecting the restrictions, by maximizing the overall transport operational profit. The total profit to be maximized is the sum over all vessels (and consequently over all chosen routes), of the product of the profit (freight revenues minus variable and fixed costs) associated with each vessel on its assigned route. Since fixed costs exist, either the vessel is in operation or not, these do not depend on the assignment (for example, investment capital, crew, insurance or maintenance costs). Variable costs relate mainly with fuel consumption (either for the situations of cruising between ports or in port) and which ports are called by the vessel.

Maximize Profit =

$$= \sum_{a}^{A}\sum_{r}^{R}\sum_{k}^{K} X_{r,k}\cdot\left[\left(N_{r,k}\cdot L_{r,k,a}\right)-\left(N_{r,k}\cdot\left(CV_{r,k,a}\right)+CF_k\right)\right] \quad (2)$$

4.4 Restrictions

The vessel's exclusivity restriction to one route is given by Equation 3, ensuring that each unit is assigned to a single route, i.e., the vessel cannot be assigned to more than one route throughout the planning horizon. Moreover, the restriction allows more than one unit to be assigned to the same route.

$$\sum_{r}^{R} X_{r,k} = 1, \forall k, k = 1 \ldots n \quad (3)$$

The restriction for meeting the required demand is defined by Expression 4. The attendance of the annual demand rate from port i to port j considers all chosen routes, all vessels and all years. The number of vessels available for the model will be greater or equal to those required to cover all the required demand. Equation 3 considers that the sum for all years, routes and vessels, of the product between the number of trips and the amount of cargo transported between ports, must be greater or equal to the annual cargo demand between the ports of i and j index:

$$\sum_{a}^{A}\sum_{r}^{R}\sum_{k=1}^{k=n} N_{r,k} M_{r,k,i,j} \geq \sum_{a}^{A} D_{i,j,a}, \forall i,j \qquad (4)$$

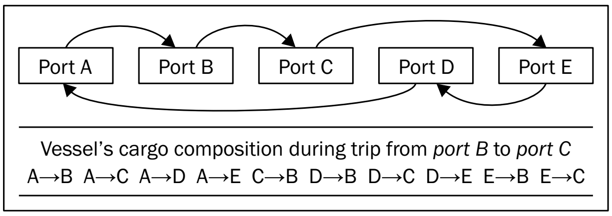
The restriction which establishes a correlation between the existence of flow variable "M" and the existence of the arc on the route is given by Equation 5. Only on the chosen routes will there be cargo transferred between ports (indexes i and j) by whichever vessel k assigned to the route of index r in year a. Mathematically, $M_{r,\,k,\,i,\,j}$ is greater than zero when $X_{r,k}$ receives the amount equal to one.

$$\sum_{i}\sum_{j} M_{r,k,i,j} \leq X^{*}_{r,k} \sum_{i}\sum_{j} D_{i,j,a}, \forall r,k,a \qquad (5)$$

The vessel capacity restriction is defined in Equation 6, ensuring that for each pair of ports i and j (between two consecutive ports on a single voyage), all transferred cargo does not exceed the maximum capacity of the vessel. The multiplier represented inside the sum of each pair of ports i and j, stipulates that for each voyage within the route, all the potential cargoes that may be present aboard the vessel must be considered.

$$\sum_{i}\sum_{j} M_{r,k,i,j} \cdot F(T(r,i),T(r,j)) \leq Cap_k, \forall r,k \qquad (6)$$

The great difficulty to mathematically express such a restriction is due to the cyclical factor of the route, i.e. the same port can be characterized as an origin or destination. *Figure 2* shows an example of a route including five ports in which the cargoes within the vessel during the voyage from port B to port C consider the depicted composition.



Vessel's cargo composition during trip from *port B* to *port C*

A→B A→C A→D A→E C→B D→B D→C D→E E→B E→C

Figure 2 - Cargo composition example [3]

In short, for the voyage between the port of index i and j, there is cargo loaded in port i to be shipped to port j and later, cargo loaded in the ports previous to i and shipped to port j and later. The sum for all origin-destination pairs of the cargo within a specific voyage should be less or equal to the capacity of the vessel assigned to the route.

The characterization of the route periodicity is achieved through vector T (r, I), where "r" identifies the route and "I" the voyage defined by the initial port. This vector takes the value "1" when the port is visited and "0" otherwise. Additionally, auxiliary vectors are introduced allowing the mathematical model not only to acknowledge the upper and lower limits of the route, as well as the direction of each specific voyage (ascending or descending).

The first vector "VO" defines the logical sequence of the visited ports, varying from 1 to n on the ascending direction and from n to 1 in the opposite direction (descending). The second vector "VI" defines the port of origin for each voyage of the route, presenting the value 1 for all ports included in the ascended direction and n for all ports in the descending direction. The third and final auxiliary vector "VF" relates to the destination port of each voyage, presenting the value n for all ascending ports and 2*(n-1) for ports in the descending direction. These vectors permit the model to identify and classify all intervals included on the chosen route with regard to origin, destination and direction, permitting the definition of the route as a cycle of voyages.

Let us now consider a generic interval "I" in any route, in which all potential loads within the vessel must be considered. The equation of constraint will be composed by the elements presented next. In the figures, included to facilitate the understanding of each component (Figures 3a through 3g), the upper rectangle represents a way/ascending trip and the lower rectangle represents the return/descending trip (if the rectangle is filled grey the port is called, if it is blank then the port is not called). The bold arrow represents a possible voyage, while the thin arrow stands for possible cargoes within the vessel cargo hold.

a) Cargoes originated in ports prior to voyage I and destined to ports visited and prior to the origin. This component corresponds to the loads originating from any port previous to the origin port D, which is visited only in the ascending direction (mathematically, $[T(r,I_i)] \cdot [Not[T(r,N\text{-}I_i)]]$), and destined to any port previous to the origin port C, in both directions (mathematically, $[T(r,I_j)\ or\ T(r,N\text{-}I_j)]$).

$$\sum_{I_i=VI(I)}^{I-1}\ \sum_{I_j=I+1}^{I_i-1} M_{r,k,i=VO(I_i),j=VO(I_j)} * \\ *[T(r,I_i)] \cdot [Not[T(r,N\text{-}I_i)]] * [T(r,I_j)\ or\ T(r,N\text{-}I_j)] \qquad (6a)$$

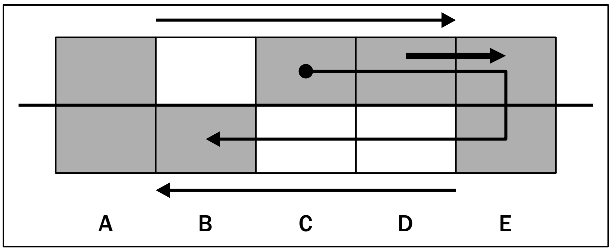


Figure 3(a) - Capacity restraint – Component "a" [3]

b) Cargoes originated in ports before voyage I and destined to ports visited, before voyage I, but after the origin. Cargoes originating from any port denoted with an index lower or equal to D-1 visited in whatever direction (mathematically, $[T(r,I_i)\ or\ T(r,N\text{-}I_i)]$) and destination to any port after the origin A, but before D and visited in the descending direction (mathematically, $[Not[T(r,l_j)]\cdot[T(r,N\text{-}l_j)]]$), as shown in *Figure* 3b.

$$\sum_{l_i=VI(I)}^{I-1}\ \sum_{l_j=l_i+1}^{I-1} M_{r,k,i=VO(l_i),j=VO(l_j)} *$$

$$*[T(r,l_i)\ or\ T(r,N\text{-}l_i)]*[Not[T(r,l_j)]\cdot[T(r,N\text{-}l_j)]] \quad (6b)$$

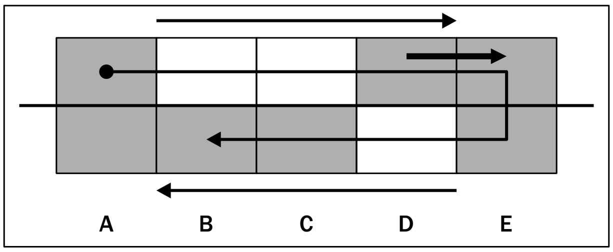


Figure 3(b) - Capacity restraint – Component "b" [3]

c) Cargoes originated in ports prior to voyage I and destined to all visited ports succeeding voyage I. Depicted in *Figure* 3c, it corresponds to all loads originated in any port prior to C (in both directions) and destined to any port posterior to C (in any direction). The sole condition for this component is that the ports of origin and destination must be visited.

$$\sum_{l_i=VI(I)}^{I-1}\ \sum_{l_j=l_i+1}^{VF(I)} M_{r,k,i=VO(l_i),j=VO(l_j)} *$$

$$*[T(r,l_i)\ or\ T(r,N\text{-}l_i)]*[T(r,l_j)\ or\ T(r,N\text{-}l_j)] \quad (6c)$$

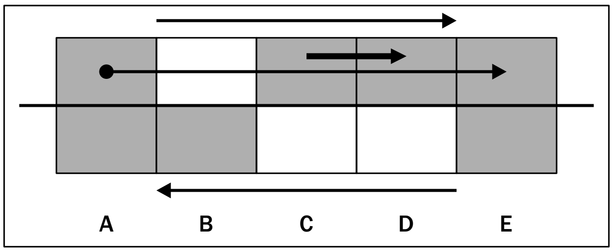


Figure 3(c) - Capacity restraint – Component "c" [3]

d) Cargoes with origin in voyage I and destined to previous ports. Refers to loads originated in C (visited only in the ascending direction) and destined to ports denoted with an index prior to C-1 (in any direction).

$$\sum_{l_i=VI(I)}^{I-1} M_{r,k,i=VO(I),j=VO(l_j)} * [T(r,l_j)\ or\ T(r,N\text{-}l_j)] *$$

$$*[Not\ T(r,N\text{-}I)] \quad (6d)$$

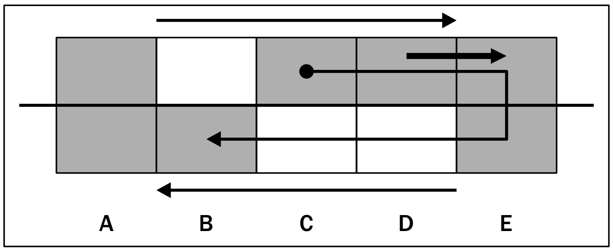


Figure 3(d) - Capacity restraint – Component "d" [3]

e) Cargoes originated in voyage I and destined to ports prior to voyage I, as represented in *Figure* 3e. Loads originated in C (on the ascending direction) and destined to ports denoted with an index after C+1 (in both directions), i.e., all ports posterior to C are part of this equation.

$$\sum_{l_i=I+1}^{VF(I)} M_{r,k,i=VO(I),j=VO(l_j)} * [T(r,l_j)\ or\ T(r,N\text{-}l_j)] \quad (6e)$$

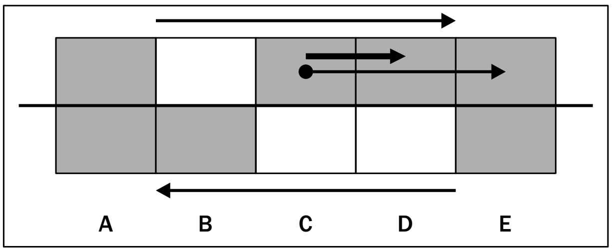


Figure 3(e) - Capacity restraint – Component "e" [3]

f) Cargoes originated in ports after voyage I and destined to ports after the origin. Corresponds to cargo loaded in ports after C (visited only on the descending direction) and destined to any port denoted with an index greater than C and the origin D (in any direction).

$$\sum_{l_i=I+1}^{VF(I)}\ \sum_{l_j=l_i+1}^{VF(I)} M_{r,k,i=VO(l_i),j=VO(l_j)} *$$

$$*[T(r,N\text{-}l_i)]\cdot[Not[T(t,l_i)]]*[T(r,l_j)\ or\ T(r,N\text{-}l_j)] \quad (6f)$$

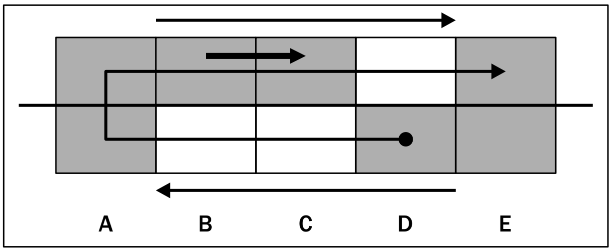


Figure 3(f) - Capacity restraint – Component "f" [3]

g) Cargoes originated in ports after voyage I and destined to ports prior to the origin. All cargo loaded in any port denoted with an index greater than C+1 (visited in any direction) and destined to ports before the origin E (only visited on the ascending direction, as represented in *Figure* 3g).

$$\sum_{l_i=I+1}^{VF(I)}\ \sum_{l_j=I+1}^{l_i-1} M_{r,k,i=VO(l_i),j=VO(l_j)} * [T(r,l_i)\ or\ T(r,N\text{-}l_i)] *$$

$$*[T(r,l_j)]\cdot[Not[T(r,N\text{-}l_j)]] \quad (6g)$$

Therefore, the final expression regarding the capacity restraint (previously presented as Equation 6) can be extended as the sum of all components, "a" through "g", as described in Equation 7.

$$(6a)+(6b)+(6c)+(6d)+(6e)+(6f)+(6g) <$$

$$< Cap_k, \forall r,k \quad (7)$$

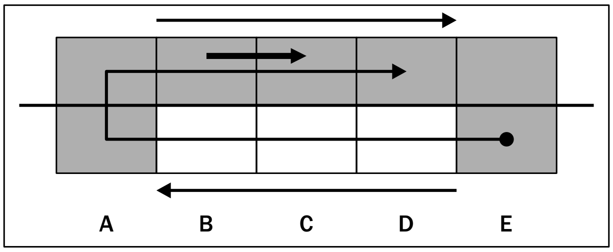


Figure 3(g) - Capacity restraint – Component "g" [3]

The non-negativity restriction ensures that all values for the decision variable regarding the transported cargo are positive (as represented by Equation 8).

$$M_{r,k,i,j} \geq 0 \forall r,k,i,j \tag{8}$$

The maximum draft restriction ensures that the allocation of a route considers that the draft of the vessel must be less than the maximum permissible at the destination port j (Equation 9).

$$X_{r,k} \leq \frac{Max_j}{Cal_k} \forall r,k,j \tag{9}$$

The binary values restriction only allows binary values to be assigned to the decision variable regarding the vessel allocation to a specific route.

$$X_{r,k} = binary \tag{10}$$

5. IMPLEMENTATION AND RESULTS

Since the model considers integer and real decision variables (variables $X_{r,\,k}$ and $M_{r,\,k,\,i,\,j}$ respectively) a mixed linear programming is considered. In order to solve this problem a model was prepared using the GAMS programming language and was structured to depend solely on external input. Input files will define the viable routes, cost data by type of vessel, number of round trips made by each unit on each route and, finally, the demand for cargo between the ports in the considered network.

Consider now a fictional scenario with four ports and two ships. This scenario has twenty-eight different routes that the model will be in charge of scheduling to each vessel. Some scenarios were run to validate and verify the sensitivity and impact of changing some of the input parameters. First, in order to verify the efficient utilization of the available fleet, scenarios were run varying the amount of cargo that should be moved, i.e., the demand will be modified according a parametric method. Then, scenarios were tested varying the capacity of the vessels, setting a fixed value of demand, with the aim of making the fleet more appropriate for that specific search and demand condition.

Table 2 displays the allocated route for each vessel, the resulting average occupation, plus the amount of total cost, which is the figure of merit of the model. Among the twenty-eight candidate routes included in the input data model, the optimal solution was reached with the selection of routes number 15 and number 9, for ships 1 and 2, respectively. *Figure 4* portrays that route 15, selected for vessel 1 (black arrows), considers trips visiting all ports in both ways, except port 2 on the ascending voyage and port 3 on the descending voyage. Route 9 nominated for vessel 2 (grey arrows), considers only voyages between port 2 and 4.

Table 2 - Four (4) ports and two (2) vessels optimization results

	Total profit	2,527,100 €
Vessel 1	Capacity (TEU)	700
	Route:	15
	Occupation	43%
Vessel 2	Capacity (TEU)	500
	Route:	9
	Occupation	100%

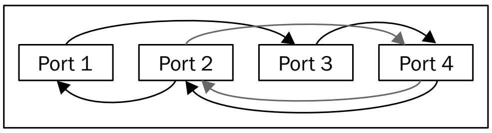


Figure 4 - Description of the selected routes

By discerning *Table 4*, it is possible to perceive that vessel 2 presents a high occupation level, while dedicated exclusively to the cargo transportation between ports 2 and 4. Regarding vessel 1, it is observed that several cargo compositions are included in the operational profile. It can be assumed, namely, that the vessel is dedicated to the cargo transportation between ports 1 and 3 (which are not called be vessel 2) and the reminiscent of the required cargo movement between ports 2 and 4 (i.e., cargo not transported by vessel 2).

One other aspect to be noted in *Table 3* relates to the occupation level for each trip within the voyage. For vessel 1 a higher occupation level is observed on the ascending trips than on the descending trips, leading to an unbalanced cargo composition, implying an over-dimensioned fleet cargo capacity.

With the objective of determining the optimum operational profile for the given fleet composition, in the following section the effects of the demand variation will be evaluated. Nevertheless, by considering these two previous tables / outputs it can be already assumed that the developed model helps in building the vessel cargo layout in order to meet its capacity and necessary cargo demand level maximizing total operations profit.

6. PARAMETRIC VARIATION OF THE DEMAND

A set of particular scenarios will now be considered by varying the demand for a fixed fleet in order

Table 3 - Vessel 1 route characteristics

Voyage	Cargo Origin	Cargo Destination	Cargo (TEUs)	Total cargo in voyage (TEUS)	Vessel occupation
1 → 3	Port 1	Port 2	40	457	65.3%
1 → 3	Port 1	Port 3	292		
1 → 3	Port 1	Port 4	125		
3 → 4	Port 3	Port 1	31	280	40.0%
3 → 4	Port 3	Port 2	46		
3 → 4	Port 3	Port 4	203		
4 → 2	Port 4	Port 1	75	115	16.4%
4 → 2	Port 4	Port 2	0		
4 → 2	Port 4	Port 3	40		
2 → 1	Port 2	Port 1	200	362	51.7%
2 → 1	Port 2	Port 3	162		
2 → 1	Port 2	Port 4	0		

Table 4 - Vessel 2 route characteristics

Voyage	Cargo Origin	Cargo Destination	Cargo (TEUs)	Total cargo in voyage (TEUS)	Vessel occupation
2 → 4	Port 1	Port 2	500	500	100%
4 → 2	Port 2	Port 4	500	500	100%

to assess what is the appropriate cargo handling level for the available fleet. The objective of this exercise is to provide satisfactory results in terms of average occupation, total profit, port attendance frequency and number of ships assigned for a given fleet. The vessel sizes for the creation of this set of scenarios are the same as the model previously studied, 700 and 500 TEUs. Additionally, ten percent interval variations were considered, starting first from fifty percent of the initial value (0.5*D) to the threshold value for a good and viable solution, which was thirty percent over current demand (1.3 *D).

To properly apprehend the best operational situation, while maximizing the total profit, several indicators are evaluated, namely the variation in the fleet total occupation level or the average round trip duration (*Table 5*). With the growing cargo demand, there is an increase in total profit due to the increase in the total cargo transported (hence an increase on the freight revenues); however, with the alteration of vessel 2 assigned route, the total operational cost increases significantly implying fluctuations on the final operation total profit. In other words, while there is an increase in total profit with the growing demand, whenever an additional call is inserted on vessel 2 assign schedule there is a significant increase in total operational cost.

Since vessel 1 preserves the same route assignment for the different demand levels, its occupation levels fluctuate drastically due to the variations assigned to vessel 2, i.e., vessel 1 occupation levels present a high dependency of vessel 2 routes and assigned cargo compositions.

For vessel 2, the route assignment, up to 90% of the original demand remains the same, being adjusted to another one with fewer ports of call (leading to an increase in the vessel occupation level). From

Table 5 - Demand parametric variation results

Demand	Total profit (m€)	Vessel 1 Route	Vessel 2 Route	Vessel 1 Occupation (%)	Vessel 2 Occupation (%)	Vessel 1 round trip (days)	Vessel 2 round trip (days)
50%	2,455	1-3-4-2-1	1-2-3-1	51%	99%	18.3	13.0
60%	2,475	1-3-4-2-1	1-2-3-1	51%	99%	18.3	13.0
70%	2,495	1-3-4-2-1	1-2-3-1	51%	99%	18.3	13.0
80%	2,515	1-3-4-2-1	1-2-3-1	52%	98%	18.3	13.0
90%	2,535	1-3-4-2-1	1-2-3-1	53%	97%	18.3	13.0
100%	2,527	1-3-4-2-1	2-4-2	43%	100%	18.3	14.0
110%	2,525	1-3-4-2-1	2-4-3-2	69%	91%	18.3	15.2
120%	2,580	1-3-4-2-1	2-4-3-2	75%	98%	18.3	15.2
130%	2,312	1-3-4-2-1	2-3-4-3-2	75%	79%	18.3	16.6

this demand level forward, the same basic route will be maintained, adding additional port calls in order to cope with the growing demand.

In *Figure 5* (which depicts the evolution of total profit when compared with the fleet total occupation), it is observed that the optimum cargo demand level is around 120%. Due to the significant operational costs increase (with the additional port call introduced in vessel 2), required to cope with 130% of cargo demand value there is a decrease in total profit, hence a reduction in total profit

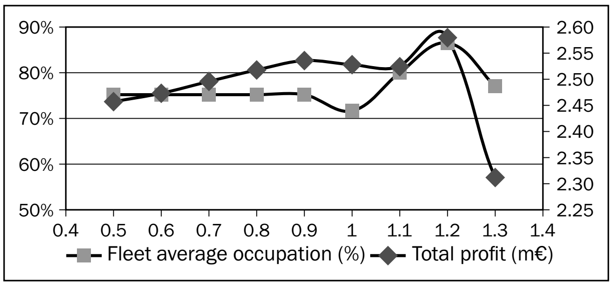


Figure 5 - Fleet occupation / total profit

Fleet occupation level was calculated as an average between the two vessels while considering their individual capacity, assigned cargo composition and total cargo transported. The scaling periodicity (mean time between port calls) is also very dependent on the order of port calls for each vessel and their geographical position, implying that with the alteration of the assigned route the scaling period will change as well. *Figure 6* represents the fluctuations on the average fleet occupation when compared with the variation of total operation profit (comparing with the original profit).

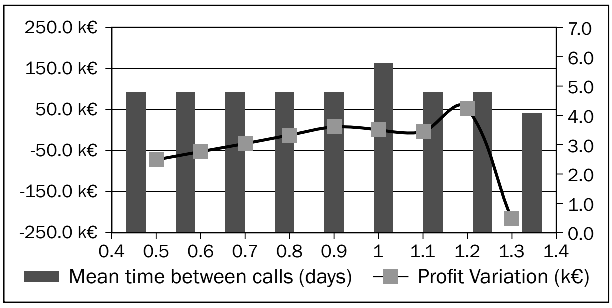


Figure 6 - Mean time between calls / Total profit variation

The conjugation of these results indicates that, for the original demand, the fleet is not dully regulated. By increasing the demand value, not only the total profit is maximized, but the total fleet occupation level will increase and without sacrificing too much of the time between port calls. In fact, due to the drastic adjustment of the elected route to vessel 2, between 90% and 100% (therefore, an increase of total costs), a greater total profit is observed for 90%, than for the original demand level.

This type of assessment, using the provided model, exemplifies how the developed model provides insight in the fleet operational and economic response when subjected to different market conditions. For this particular case it is perceived that the fleet is over-dimensioned for the current demand value. Therefore, with the objective of maximizing total profits and fleet occupation levels, while maintaining an acceptable service level, different fleet compositions will be evaluated considering the original demand.

7. FLEET CAPACITY PARAMETRIC VARIATION

For a given level of demand (assuming the default demand matrix), the system behaviour was verified by varying the size or capacity of vessels that could be attributed to the operation of maritime transport.

In practice, when searching for a vessel for a new line of operation in maritime transport, the owner must have information not only on this operation, but on the desired vessel class as well. The developed model provides valuable assistance to this process, allowing different scenarios to be confirmed by changing distinct fleet characteristics such as, speed, capacities, or costs. The following fleet configurations and respective assigned routes are presented in *Table 6*, where the heading column and row represent vessel 1 and 2 capacities and the centre values follow the "Vessel 1 route" // "Vessel 2 route" syntaxes.

With the growing total capacity of the fleet there is a tendency to either assign routes with fewer ports of call (routes 10 and 9) or to assign ports with a closer geographical profile (route 6). *Table 7* shows which ports are called on the assigned routes.

To determine the best combination regarding maximized profit, the optimization model will lock on the best route for vessel 1 while varying the vessel 2 route assignment, trying to achieve the required demand service level while maximizing profit. This is achieved by greatly modifying the cargo composition of each voyage within the assign route (namely the decision variable $M_{r,k,i,j}$).

Table 6 - Route assignment for different fleet configurations

Routes					
	500	600	700	800	900
500	15//10	15//10	15//10	15//10	15//10
600	15//10	15//10	15//10	15//10	15//10
700	15//9	15//9	15//9	15//9	15//9
800	15//6	15//6	15//6	15//6	15//6
900	15//6	15//6	15//6	15//6	15//6

Since the demand level is the same for different fleet combinations, the same freight revenues are expected (even if the cargo composition is different for

Table 7 - Sequence of ports on the selected routes

Route 15	Port 1	0	Port 3	Port 4	0	Port2	Port 1
Route 10	0	Port 2	0	Port 4	Port 3	Port 2	0
Route 9	0	Port 2	0	Port 4	0	Port 2	0
Route 6	Port 1	Port 2	Port 3	0	Port 3	0	Port 1

each voyage within the assigned route). On the other hand, the total costs will increase with the growing fleet capacity due to costs related with investment capital, crew, insurance, maintenance, fuel, etc. This means that for this particular case study, where the demand is unchanging, it actually is the varying total costs that decree the best route to be assigned.

Since distinct vessel velocities, number of crew members or maintenance policies is considered for each vessel, two different "sets" of costs are assumed. Therefore, despite the similar vessel capacity, a non-symmetric route assignment and total profit matrix are provided by the model (*Table 8*), where the best solution is achieved with two 500 TEU capacity vessels.

Table 8 - Total profit for different fleet configurations

Total profit (m€)					
	500	600	700	800	900
500	2.545	2.535	2.525	2.515	2.505
600	2.535	2.525	2.515	2.505	2.495
700	2.527	2.517	2.507	2.497	2.487
800	2.535	2.525	2.515	2.505	2.495
900	2.525	2.515	2.505	2.495	2.485

Despite the fact that costs increase with the fleet capacity, the evolutions of fleet occupation levels and average call time are not so linear. By consulting *Table 9*, it can be perceived that the occupation level is approximately the same for identical route assignments even if there is a greater fleet capacity. This reflects the balancing aspect regarding the cargo assignment composition within each voyage (variable $M_{r,\ k,\ i,\ j}$), i.e. the fleet occupation does not vary unless a different pair of assigned routes is considered.

Table 9 - Occupation levels for different "sizes of vessel 1 and vessel 2

Fleet average occupation (%)					
	500	600	700	800	900
500	84%	85%	85%	86%	86%
600	87%	87%	87%	87%	87%
700	72%	72%	72%	72%	72%
800	79%	80%	80%	80%	80%
900	75%	75%	75%	75%	75%

As expected, several situations arise as possible better situations when comparing with the original configuration of 700 and 500 TEUs. Considering a configuration of 600 // 500 TEUs for Vessel 1 // Vessel 2, respectively, not only do the occupation levels increase greatly, but also a better mean time between calls is expected. For this situation there is even a minor gain in the total operational profits.

The next step would be to evaluate this particular fleet response to different market conditions, by varying the demand and determining the fleet performance limits. This methodology would lead to a spiral regarding the determination of the best fleet route profile and cargo compositions, considering maximized profits while avoiding problematic operational situations (low occupation levels and poor service frequency). Furthermore, it is possible to verify the versatility of the developed model, allowing analysis of various aspects and points of view of the transport operation.

8. CONCLUSION

The presented scenarios show evidence of the model consistency, reaching optimal results for each configuration set and demonstrating how it will assist in the strategic decisions procedures. Because it is a model involving binary variables, any change in the input parameters can produce different solutions, allowing the development of a marginal analysis of the results. The introduced methodology provides a thorough evaluation of the employed strategies, while offering alternatives to increase service level and evaluating the basis for business development in this segment of maritime transport.

The model features solutions and outputs of a strategic, economical and operational nature. Not only is the overall operation profit optimized and different costs provided, but also the required fleet composition and specific routes to meet the minimum demand level are established presenting strategic long term information. With the vessel cargo hold composition for each voyage within the assigned route defined, i.e. characterization of all loading conditions and vessel occupation for each voyage, operational results are introduced as well.

The adopted methodology, in comparison with the complexity of the problem, allowed a simple, clean and efficient model to be prepared. The feeding of information on the model by means of aggregated data significantly reduced the degrees of freedom and therefore reduced greatly the number of model variables.

However, since the model is developed through mathematical programming, a major difficulty is to consider discrete, non-random variables to a largely randomized process. This means that the model operates under the assessment that all operations will run equally, without any problems and according to the developed planning for the given time period. The introduction of random factors, in the input variables (for example, costs, operation and travel times, etc.) and on external variables not defined in the model (such as the occupation of sea berths by other vessels, ports unavailability, queue times for commencing operations or meteorological effects) may be the next step in making the developed model closer to realistic fleet operations.

Dr. **CARLOS A. SILVA**
E-mail: c.silva@centec.tecnico.ulisboa.pt
Prof. **CARLOS GUEDES SOARES**
E-mail: c.guedes.soares@centec.tecnico.ulisboa.pt
Centro de Engenharia e Tecnologia Naval
Instituto Superior Técnico
Universidade de Lisboa
Av. Rovisco Pais, 1049-001 Lisbon, Portugal

RESUMO

DIMENSIONAMENTO DE UMA FROTA DE NAVIOS PORTA CONTENTORES PARA UM DADO MERCADO

O crescimento do transporte marítimo de curta distância motivou o desenvolvimento de uma metodologia a usar como uma ferramenta de apoio à decisão na qual ambos os parâmetros relacionados com a procura dos mercados e as características da frota podem ser testadas e avaliadas. Também é possível determinar a programação da frota, estabelecendo as suas rotas e escalas nos portos para um dado cenário. A metodologia considerada divide-se em duas partes, sendo a primeira relacionada com a geração de todas as possíveis rotas com todos parâmetros relacionados com cada rota para cada classe de navio. A segunda parte introduz um modelo de programação linear que maximiza o lucro total de operação de acordo com um conjunto de restrições. Os modelos foram estruturados de acordo com três critérios principais: a avaliação da frota para cada classe de navio, a rota óptima para cada navio e a frequência em cada porto. Para validar a metodologia, os modelos desenvolvidos são submetidos a um cenário operacional fictício que considera três situações diferentes: a operação normal da frota, a resposta da frota a diferentes cenários de procura e a avaliação de várias composições de frotas para um mesmo nível de procura.

PALAVRAS CHAVE

navios porta contentores; projecto da frota; programação linear; programação inteira mista

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