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MODELLING OF ASSEMBLY SEQUENCES USING HYPERGRAPH AND DIRECTED GRAPH

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Original scientific paper
The work concerns the problem of modelling technological process of product assembly, with particular stress on the sequences of connecting its parts and units. The work presents the most important parameters of modelling assembly sequences, including the bases of heuristic proceeding which has to simplify finding its rational variation. Suggested method of determining the sequences of assembly of parts and units of machines using hypergraphs and directed graphs consists of: selection of the main base part and base parts of particular assembly units, recording of construction structure of assembly unit in the form of directed hypergraph and minimization of the number of its edges to the form of digraph, matrix recording of construction structure of assembly unit in the form of state matrix and graph matrix, the selection of extreme path in digraph. An important element of the work is a detailed algorithm of determination of assembly sequences using the matrix of hypergraph and directed graph, state matrix and the graph which was implemented to computer software "Msassembly".

Keywords: *directed graph, hypergraph, mechanical assembly sequence, modelling*

Modeliranje sekvenci montaže uporabom hipergrafa i usmjerenog grafa

Izvorni znanstveni članak
Rad se bavi problemom modeliranja tehnološkog procesa montaže proizvoda s posebnim naglaskom na sekvence kod povezivanja dijelova i cjelina. U radu se predstavljaju najvažniji parametri modeliranja sekvenci montaže, uključujući osnove heuristčkog postupka kojim bi se trebalo pojednostavniti pronalaženje racionalne varijante. Predložena metoda određivanja sekvenci montaže dijelova i cjelina strojeva primjenom hipergrafova i usmjerenih grafova sastoji se od: izbora glavnog osnovnog dijela i osnovnih dijelova pojedinih jedinica montaže, bilježenja strukture konstrukcije jedinice montaže u obliku usmjerenog hipergrafa i minimiziranja broja njegovih rubova do oblika digrafa, bilježenja matrice strukture konstrukcije jedinice montaže u obliku matrice stanja i matrice grafa, izbora ekstremne putanje u digrafu. Važan element rada je detaljni algoritam određivanja sekvenci montaže primjenom matrice hipergrafa i usmjerenog grafa, matrice stanja i grafa koji je primijenjen na računalnom softveru "Msassembly".

Ključne riječi: *hipergraf, modeliranje, sekvenca strojne montaže, usmjereni graf*

1 Introduction

Technological processes of assembly influence in most cases costs of production and quality of a product. Determining proper sequences of connecting parts during the construction of a product or reorganization of its assembly technological process is a very important task for its designer, i.e. constructor or technologist.

We assume that the most important parameters of determining assembly sequence are [1, 2]:

- orientation of assembly parts and units in space and directions of their connecting,
- method of determining (basing) assembly parts and units.

Taking into account construction limitations which are the basis of further analysis relating to indicating assembly sequences, decreases significantly the number of possible solutions of connecting assembly parts and units. Guarantee of optimum solution is provided by algorithmic methods which consider all possible solutions, but due to its large labour intensity heuristic method was suggested, taking into account:

- distribution of a large task of determining assembly sequences into some number of smaller ones including determination of structure of conduct (relations between the main goal and its sub-goals),
- the choice of proper conduct, among the possible ones, taking into account conclusions resulting from the analysis of all additional data (the results of division of a product into assembly units or the choice of base parts),
- using the recurrent method of conduct to solve the problem.

2 Model of construction structure of product

Determination of base part is started with connecting subsequent assembly parts and units. To do that there has been made an aggregation of parts in sets and on this basis there are selected base parts of the main assembly and assembly units of a lower rank (sets, sub-assemblies).

The selection of the base part can be referred to two areas: 1) base part connecting singular assembly parts of higher ranks (subsets, sets) of the so called main body and 2) base part which are a body for singular parts and assembly parts of lower rank (Fig. 1). Description of the set of possible variations of a sequence of the assembly technological process and description presenting construction structure including relations between connected parts, created on the basis of construction documentation, consists of hypergraph and directed graph. They enable construction of directed hypergraphs of construction limitations after previous separation from the product assembly units and independent parts (e.g.: clamping screws, washers, etc.). It was assumed that parts or states of assembly are marked in hypergraph as hyper vertices, and hyperarchs present possible sequences (paths) of their connections. Assembly is made through adding to assembly state of the n^{th} equation of singular part and in case of adding a unit consisting of a larger number of parts, there is no discussion about possible sequences of their assembly.

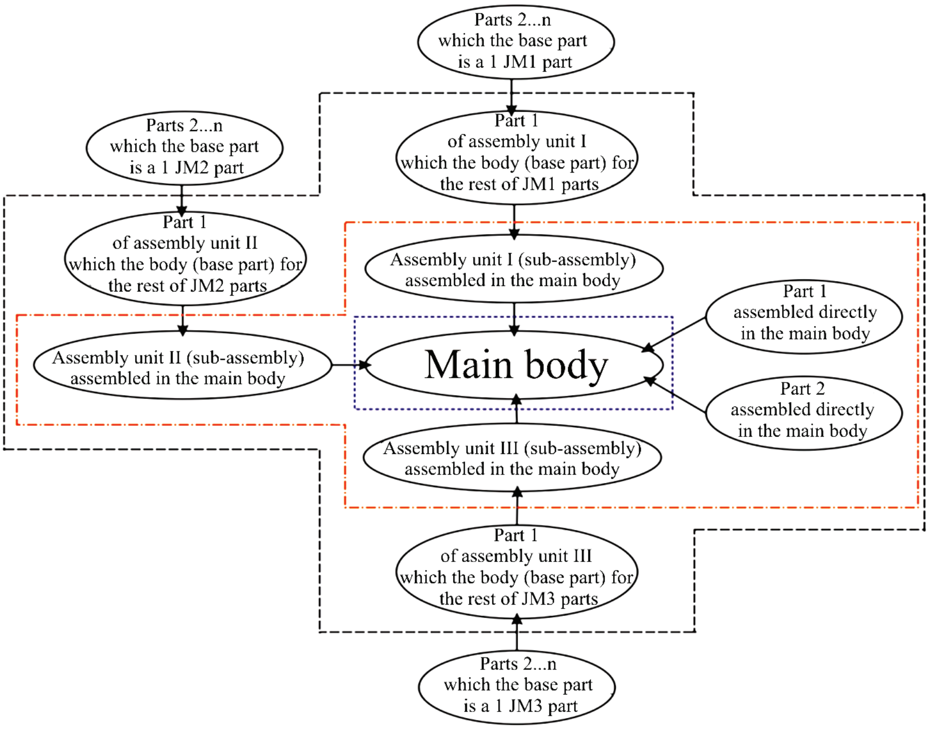

Figure 1 Bodies as base parts with different "depths" of assembly

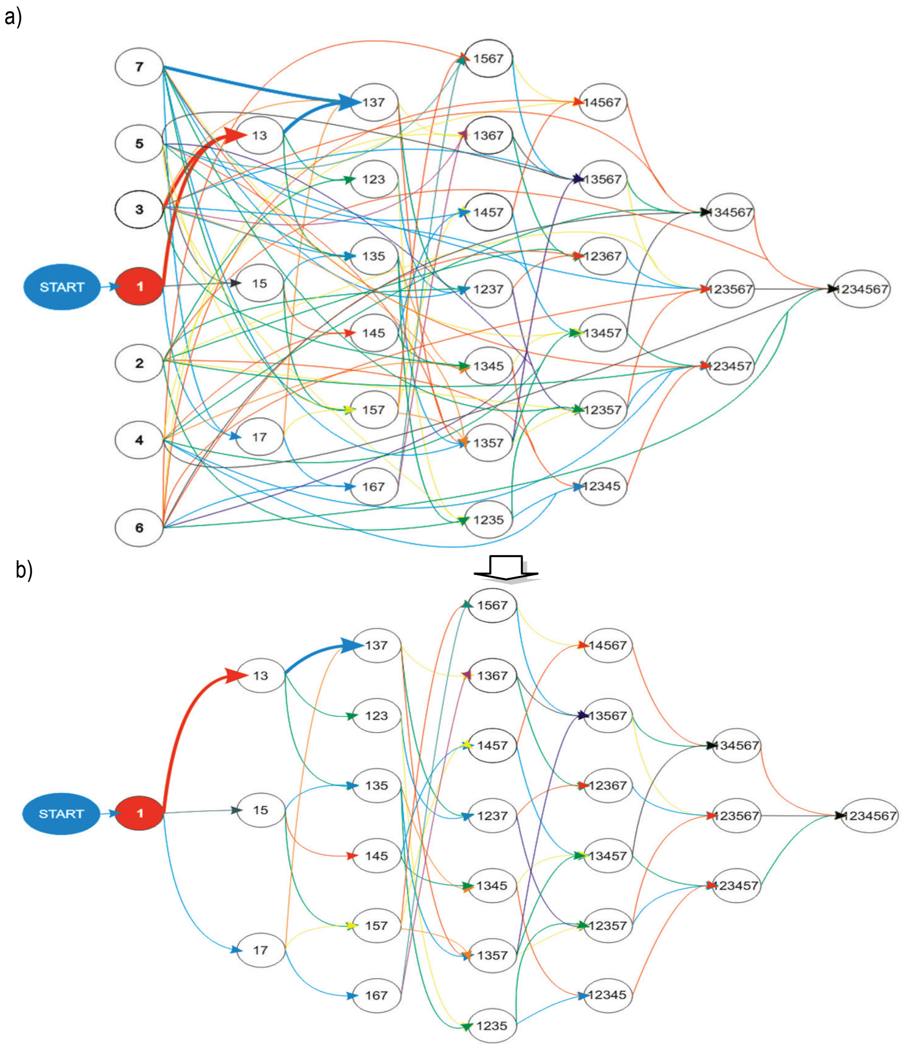

Figure 2 Example of transition from directed hypergraph of construction structure of assembly unit (a) to digraph of construction structure (state graph) of the same unit (b)

In Fig. 2a there is directed hypergraph of construction structure in separated typical assembly unit JM1, consisting of 7 parts JM1 $\subset \{1, 2, 3, 4, 5, 6, 7\}$. After removing from directed hypergraph construction structure of this assembly unit, and using one of entries for each of the hyperarches, it becomes a classic directed graph, i.e. digraph without loop (Fig. 2b), where each branch joins only two vertices. Output (out) of hyperarch still contains information about parts whose connection caused transition to another state of assembly (different depth of assembly), where hyperarch becomes a classic arch and directed graph becomes a digraph. For example, vertex 137 in Fig. 2b, with one entering directed edge from assembly unit 13, contains information that another entry of hyper edge is vertex 7 (state 137 is formed as a result of connection of state 13 and part 7; arches in bold in Fig. 2a). The same information is included in directed hyperarch, according to the notation accepted earlier, beginnings and endings of hyperarch: $A_{E1} = 13$, $A_{E2} = 7$ and $B_E = 137$ (hyperarches in bold in Fig. 2b). In the digraph of assembly states initial vertex x_p has an output value $d^+(x_p) \geq 1$ and input value $d^-(x_p) = 0$, and vertex x_k characterizes $d^+(x_k) = 0$ and $d^-(x_k) \geq 1$, while both input and output steps for other vertices are included in interval from $<1, n>$.

To determine assembly sequences of a product recorded in the form of digraph there were suggested matrixes of states of assembly $L = [h_{ij}]_{n\times n}$ and graph $H = [h_{ij}]_{n\times(n+4)}$ [3, 4, 5]. The method of completing states of assembly matrix which was presented in works [6, 7] is a basis of algorithm of searching graph and finding the best assembly sequence with previously accepted criteria of evaluation of transitions between its particular states. Graph matrix should present in a clear way the possibilities of transitions from lower to higher assembly state with hierarchical presentation of solutions. Each line of matrix includes the lists of: 1) successors of i-vertex (informs which parts can be added to a given state), 2) predecessors (informs which parts were added to $n-1$ state to achieve desired state of assembly) and 3) non-incidental vertices (informs which assembly states – vertices are not directly connected with the current state).

One of the basic tasks of graph matrix is providing technologist with information related to the best assembly sequence of parts or assembly units in any process stage.

3 Selection of extreme path in directed graph

The way in which none of the vertices appears in a graph more than once is called the path. End vertices of the path in suggested method have input or output degree equal to 0 (for x_p $d^+(x_p) \geq 1$, $d^-(x_p) = 0$; for x_k $d^+(x_k) = 0$ and $d^-(x_k) \geq 1$), and other vertices which belong to it have both input and output degree bigger than 0 ($<1, n>$).

Determining the most favourable assembly sequence of a product using assumed conditions, e.g. criteria of connection evaluation, comes to finding extreme directed path in graph $H = (X, U)$, where each edge $u \in U$ is allocated certain real number, called weight of edge. The path in considered digraph has a form of sequence of commutative vertices (x) and edges (u): x_p, u_1, x_1, u_2,…x_k, where x_p is an initial vertex, and x_k final vertex. A graph with allocated weights of edge becomes a network $S = <G, \varnothing, \{l\}>$ (where G is a digraph, and l real function defined in the set of its arches) and is described in discussed case by the matrix of states in the form of value a_{ij}. The solution is then to find an extreme path $[\mu_{extr}(x_p, x_k)]$ for determined vertices x_p (base part) and x_k (assemble product), for which:

$$F[\mu_{extr}(x_p, x_k)] = \underset{\mu\, \in D\,(x_p,\, x_k)}{\mathrm{extr}} F[\mu(x_p, x_k)], \tag{1}$$

where

$$F(\mu) = \sum_{u \in U(\mu)} l(u)\,, \tag{2}$$

where:
$D(x_p, x_k)$ – the set of paths connecting vertices x_p and x_k,
$\mu(x_p, x_k)$ – the path connecting vertices x_p and x_k,
$\mu(x_p, x_k) = \{x_p, u_1, x_1, u_2,\ldots x_k\}$.

In the method of determining assembly sequences of the product, the extreme was defined as minimum, which means the shortest path, and the digraph is acyclic. The authors assumed that cyclicality and coherence of the presented network is a result of the method used during determination of assembly sequences and resigned from checking the properties using appropriate algorithms. An extreme in minimum sense path from initial vertex x_p to x_k in the network with nonnegative weights of the edge can be determined using Dijkstra algorithm (the shortest path without remembering its form).

4 Algorithm of assembly sequences determination

Suggested way of generating assembly sequence of assembly unit/product (Fig. 3) includes the following activities:

- completing input data related to the product (technological specification of a product, models of a product),
- division of a product into assembly units (sets, subsets, independent parts),
- choice of base part of the main assembly and base parts of assembly units of lower ranks,
- construction of directed hypergraphs of constructional limitation of assembly units,
- minimization of a number of edges of hypergraphs and moving to the form of digraphs,
- recording of digraphs structure in symmetric state matrixes,
- completion of the state matrix of assembly unit due to, e.g. selected criteria of evaluation of transitions between particular assembly states,
- searching of a state matrix as regards the accepted criteria according to Dijkstra algorithm,
- finding possible sequences of connection of parts (extreme path) in particular assembly unit,

- choice and analysis of assembly sequence of assembly units (analysis of possibility of parallel assembly of separated assembly units),
- recording of determined assembly sequences in graph matrixes,
- repetition of the procedure for assembly units of higher rank.

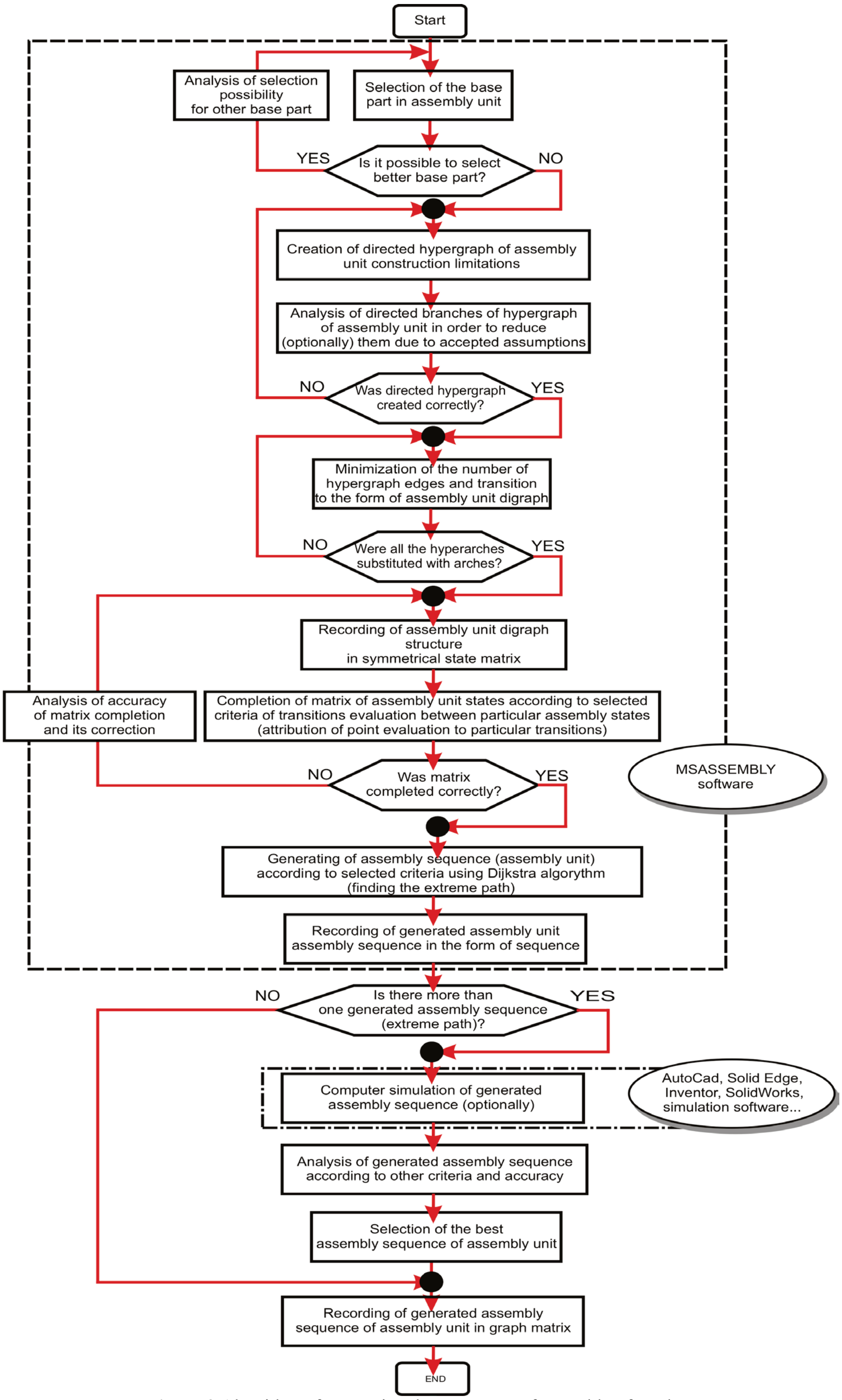

Figure 3 Algorithm of generating the sequences of assembly of product

5 Algorithm implementation into Msassembly program

An essential part of the proposed assembly sequences modelling is combining it with processing ability of a computer. The result is supportive software Msassembly that incorporates hypergraph, directed graph, state matrix and Dijkstra algorithm. The main menu of the programme and the commands available are presented in Fig 4. Six bookmarks are particularly important: 1) parts list, 2) construction of hypergraph, 3) graph, 4) evaluation of transitions, 5) state matrix, 6) assembly sequence. They facilitate entering and editing data necessary to generate assembly sequence paths in accordance with proper proceedings (Fig. 3).

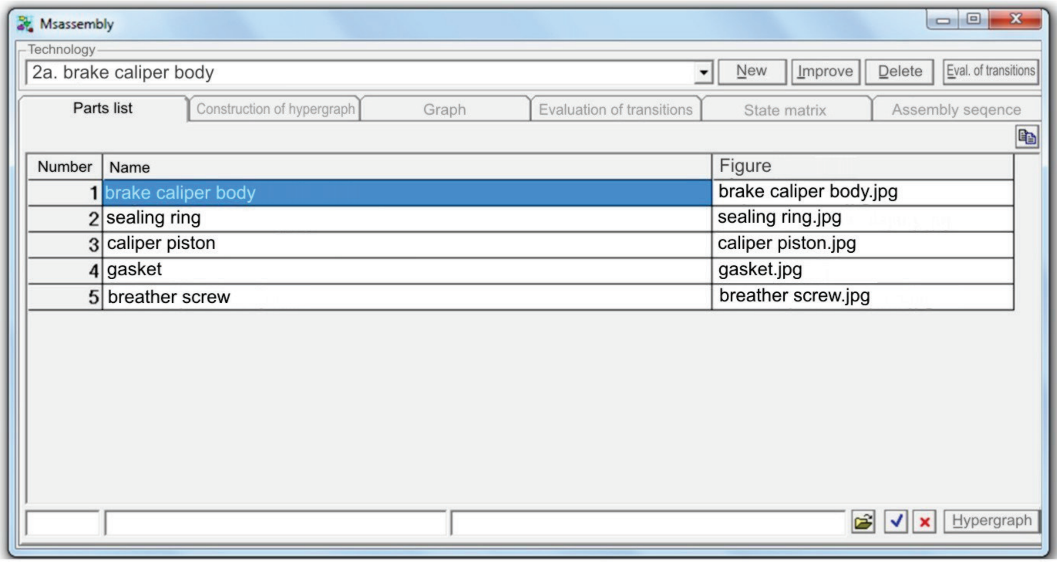

Figure 4 Msassembly program main menu

6 Conclusions

The results of the work proved that modelling assembly sequences of a product can be effectively performed using hypergraph and directed graph. The digraph which was suggested for creating the model of constructional structure model matches some advantages of directed hypergraph and classical directed graph, allowing for including the same number of information as directed hypergraph, using at the same time more simple matrix recording. It is important because so far it has not been decided and there have not been used formalized rules, allowing technologists of assembly to determine different variations of assembling parts and sets of a product sequence. In industrial practice it is assumed that assembly sequences in an obvious way result from the constructional solution which according to the research of the authors is not always the best. The division of a product into assembly units (sets, subsets) included in its constructional documentation is mostly based on separation from the whole separate, constructional closed sets fulfilling certain functions. Therefore it does not meet demands of technological process of assembly where it is essential to divide into sets, subsets and parts.

It was also concluded that modelling assembly sequences of a product using a hypergraph and directed graph enables using effective algorithms and procedures to find its possible variations, including search for following criteria.

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