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MATHCAD ZA INŽENJERE: OPTIMIZACIJA RJEŠENJA PROBLEMSKOG ZADATKA IZ DINAMIKE KRUTIH TIJELA

MATHCAD FOR ENGINEERS: OPTIMIZATION OF RIGID BODY DYNAMICS EXAMPLE PROBLEM

Katarina Pisačić, Marko Horvat, Vinko Višnjić

Stručni članak

Sažetak: *U radu je na problemskom zadatku prikazana primjena MathCad-a u rješavanju zadataka iz dinamike krutih tijela. U zadatku je potrebno odrediti koliko je kutno ubrzanje za zadani moment i koliki je moment za minimalno ubrzanje, stoga ovaj problem predstavlja optimizacijski problem. Klasično analitičko rješenje zadatka je postavljeno koristeći D'Alembertov princip te je dana sintaksa MathCad koda kojim je dobiveno rješenje. U nastavku je za određivanje momenta definirana funkcija ovisnosti kutnog ubrzanja o momentu te je minimizacijom odredjen rezultantni moment. Usporedba rješenja optimizacije s klasičnim rješenjem potvrdila je učinkovitost ovog načina rješavanja zadatka. Za ovaj postupak su korištene MathCad simboličke funkcije.*

Ključne riječi: *MathCad, optimizacija, dinamika krutih tijela, symbolics*

Professional paper

Abstract: *In this paper, an example problem was used to demonstrate the application of Mathcad in solving the tasks of the dynamics of rigid bodies. The task was to determine angular acceleration for a given moment and the moment value at minimum acceleration, which makes this problem an optimization problem. Classical analytical solution of the problem is obtained using the D'Alembert principle. Mathcad code is given by which the solution is obtained. Functional dependence of angular acceleration to frictional moment is determined and final solution is calculated by the minimization function. Comparison of optimization solutions to classical solution confirmed the effectiveness of this method of problem solving. For this procedure Mathcad symbolic functions were used.*

Key words: *MathCad, optimization, rigid body dynamics, symbolics*

1. INTRODUCTION

In the recent studies of rigid body dynamics various programming languages and software are been used. MathCad is equation solver and graphing software used by scientists for solving various scientific problems [1,2]. This article presents a concise example of rigid body dynamics problems solved in MathCad and presents simple optimization problem.

D'Alemberts principle is used to create artificial equilibrium state by balancing active forces with inertial force which equals to $-m \cdot a$. This principle enables solving dynamics problems similarly to statics and is well known in studies of vector and engineering mechanics [3,4].

Furthermore when acceleration equals 0 dynamic problem becomes static for $-m \cdot a = 0$ and D'Alembert's force becomes 0. Analytical solution for this case is easily obtained and it is used to validate optimized dynamic solution.

2. DYNAMIC EXAMPLE PROBLEM

Problem solved for this presentation is modified textbook example [4].

2.1. Sample problem

Due to rotation frictional moment $M_O = 2.5$ Nm appears at pivot O (see Figure 1.). Drum mass is $m_\mathrm{D} = 6$ kg and radius of gyration $i_s = 225$ mm while weights masses are $m_1 = 12$ kg and $m_2 = 7$ kg, drum radiuses are $r_1 = 200$ mm and $r_2 = 300$ mm.

1. Determine angular acceleration of the grooved drum.
2. Determine maximum frictional moment when weights are still moving

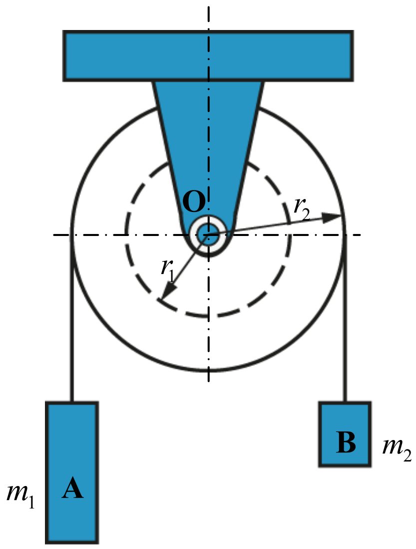

Figure 1. Rigid body dynamics problem

To obtain solution following will be done:
1) Free body diagram
2) Dynamic equilibrium equations
3) Static equilibrium equations
4) Solving equations using MathCad

2.2. Free body diagram

Identifying correct relations between the bodies is necessary to draw corresponding free body diagram.

Free body diagram is shown in Figure 2.

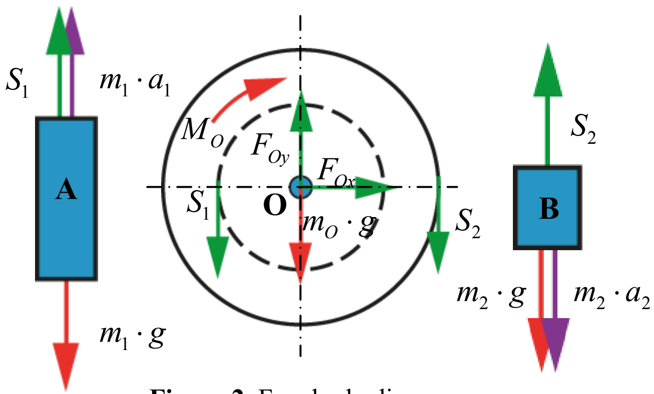

Figure 2. Free body diagram

2.3. Dynamic equilibrium equations

It is requested to determine angular acceleration and analytical solution will be obtained by the use of D'Alembert's principle. Vector equations are following:

For weight A:

$$\sum F_y = 0 \quad -m_1 \cdot g + m_1 \cdot a_1 + S_1 = 0 \tag{1}$$

For weight B:

$$\sum F_y = 0 \quad -m_2 \cdot g - m_2 \cdot a_2 + S_2 = 0 \tag{2}$$

For drum:

$$\sum M_O = 0 \quad S_1 \cdot r_1 - S_2 \cdot r_2 - I_O \cdot \alpha - M_O = 0 \tag{3}$$

Kinematic relationships are:

$$a_1 = \alpha \cdot r_1 \tag{4}$$

$$a_2 = \alpha \cdot r_2 \tag{5}$$

2.4. Static equilibrium equations

Second problem statement requests the calculation maximum value of frictional moment M_O. If frictional moment is large enough, thy system will become still and the problem becomes static. Therefore static equilibrium equations will be used to obtain solution.

For weight A:

$$\sum F_y = 0 \quad -m_1 \cdot g + S_1 = 0 \tag{6}$$

For weight B:

$$\sum F_y = 0 \quad -m_2 \cdot g + S_2 = 0 \tag{7}$$

For drum:

$$\sum M_O = 0 \quad S_1 \cdot r_1 - S_2 \cdot r_2 - M_O = 0 \tag{8}$$

3. MATHCAD SOLUTION

One way of solving system of linear equations in MathCad is using given-find block [5].

For successful calculation four steps should be made:
1) Define variables
2) Define initial values
3) Given-Find solve block
4) Store solutions

3.1. Mathcad solution of dynamic equilibrium

To enable proper calculation all constants must be defined [6, 7], local variable definitions are used for easier manipulation. Unlike global definitions, local definitions are valid from the place where are written to the right and below.

3.1.1. Defining variables

Green wavy line indicates that some previous definitions are overridden. MathCad recognizes dozens physical constants and units. If variable name is underlined then is recommended to rename it (Figure 3.).

$$g := 9.81\frac{m}{s^2} \quad M_O := 2.5N\cdot m$$
$$r_1 := 200mm \quad r_2 := 300mm \quad i_s := 225mm$$
$$m_1 := 12kg \quad m_2 := 7kg \quad m_D := 6kg$$
$$I_O := m_D\cdot i_s^2 = 0.304m^2\cdot kg$$

Figure 3. MathCad code defining variables

3.1.2. Initial values

For all unknowns initial values and units should be defined (Figure 4.). Similar as defining constants local variable definition is used. For nonlinear systems it is more effective to define 1 rather than 0, in some cases 0 fails performance of Given-Find block.

$$S_1 := 0kN \quad S_2 := 0kN$$
$$a_1 := 0\frac{m}{s^2} \quad a_2 := 0\frac{m}{s^2} \quad \alpha := 0\frac{rad}{s^2}$$

Figure 4. Defining initial values

3.1.3. Solving system

Given-Find solve block is numerical solve block where the number of unknown variables must be same as number of equations in the block. Solution of given system is shown in Figure 5.

Given

$$-m_1\cdot g + m_1\cdot a_1 + S_1 = 0$$
$$-m_2\cdot a_2 - m_2\cdot g + S_2 = 0$$
$$S_1\cdot r_1 - S_2\cdot r_2 - I_O\cdot\alpha - M_O = 0$$
$$a_1 = \alpha\cdot r_1$$
$$a_2 = \alpha\cdot r_2$$
$$\begin{pmatrix} S_1 \\ S_2 \\ a_1 \\ a_2 \\ \alpha \end{pmatrix} := \mathrm{Find}(S_1, S_2, a_1, a_2, \alpha)$$

Figure 5. Solution block Given-Find

3.1.4. Storing solution

Solve function Find is used to obtain solution. In this example vector is defined with same number of elements as given equations, containing variable names in same order as in Find function and thus solutions are stored. This is optional, but usually done for further calculation needs.

If solutions have various physical quantities, numerical values are separately recalled (Figure 6.).

$$S_1 = 116.968\ N \quad S_2 = 69.328\ N \quad \alpha = 0.313\ \frac{1}{s^2}$$

Figure 6. Recalling solutions

3.2. Mathcad solution of static equilibrium

For static equilibrium solution similar procedure is done. Constants are same as in dynamic equilibrium and this step is unnecessary.

3.2.1. Initial values

Although mostly same constants are used, number of equations is changed and different initial values are defined using appropriate physical units, Mathcad syntax for this procedure is shown in Figure 7.

$$S_1 := 0kN \quad S_2 := 0kN \quad M_O := 1N\cdot m$$

Figure 7. Defining initial values for second solution

3.2.2. Solving system and storing solutions

In this step solve block Given-Find is used. Solutions are stored and numerical values are recalled separately because different physical units are used. Different units can be recalled inside same matrix only when symbolical evaluation is used (arrow).

$$S_1\cdot r_1 - S_2\cdot r_2 - M_O = 0$$
$$-m_1\cdot g + S_1 = 0$$
$$-m_2\cdot g + S_2 = 0$$
$$\begin{pmatrix} S_1 \\ S_2 \\ M_O \end{pmatrix} := \mathrm{Find}(S_1, S_2, M_O) \rightarrow \begin{pmatrix} \frac{117.72kg\cdot m}{s^2} \\ \frac{68.67kg\cdot m}{s^2} \\ \frac{2943.0kg\cdot m\cdot mm}{s^2} \end{pmatrix}$$
$$M_O = 2.943J$$

Figure 8. Second solution block

4. OPTIMIZATION OF DYNAMIC SOLUTION

Another approach is possible to obtain solution for maximum frictional moment M_O. When frictional moment reaches high value weights decelerate, subsequent angular acceleration reaches minimum value. Solution of problem is finding frictional moment for minimum angular acceleration. Although many known optimization procedures are known [8, 9], for this purpose built in MathCad functions will be used.

4.1. Defining solution function

In this step function for obtaining range of solutions is defined. System of equations solved in chapter 3.2.2 is optimized using symbolical calculation in MathCad.

4.1.1. Defining variables

When running symbolic calculations use of units requires some additional calculation steps and in this calculation physical units are omitted for faster calculation (Figure 9.).

$$r_1 := 0.200 \qquad r_2 := 0.300 \qquad i_s := 0.225$$
$$m_D := 6 \qquad m_1 := 12 \qquad m_2 := 7$$
$$I_O := m_D \cdot i_s^2 = 0.304 \qquad g := 9.81$$

Figure 9. Defined variables with omitted units

4.1.2. Defining initial values

For effective use of symbolic calculation initial values for unknown variables are defined to override any stored numerical value (Figure 10.).

$$M_O := M_O \qquad a_1 := a_1 \qquad a_2 := a_2$$
$$\alpha := \alpha \qquad S_1 := S_1 \qquad S_2 := S_2$$

Figure 10. Initial values for symbolic calculation

When full symbolic solution is wanted, all used physical quantities are to be defined this way, regardless if they are already defined as constants.

4.1.3. Defining solution function

Following is function formed of symbolic solution of dynamic equilibrium equations and function argument is frictional moment.

For obtaining solution solve function is used combined with simplify and float keywords for optimizing result. Solution of system is five elements function vector (Figure 11.).

$$rj(M_O) := \begin{pmatrix} S_1 \cdot r_1 - S_2 \cdot r_2 - I_O \cdot \alpha - M_O = 0 \\ -m_1 \cdot g + m_1 \cdot a_1 + S_1 = 0 \\ -m_2 \cdot a_2 - m_2 \cdot g + S_2 = 0 \\ a_1 = \alpha \cdot r_1 \\ a_2 = \alpha \cdot r_2 \end{pmatrix} \left| \begin{array}{l} \text{solve}, \begin{pmatrix} S_1 \\ S_2 \\ a_1 \\ a_2 \\ \alpha \end{pmatrix} \rightarrow \\ \text{simplify} \\ \text{float} \end{array} \right.$$
$$\rightarrow \left(1.698 \cdot M_O + 112.724 \quad -1.485 \cdot M_O + 73.042 \quad -0.141 \cdot M_O + 0.416 \quad -0.212 \cdot M_O + 0.625 \quad -0.707 \cdot M_O + 2.082\right)$$

Figure 11. Defining whole solution function

4.2. Optimizing solution function

Angular acceleration as stated in solve function is fifth element. Default starting index (MathCad built in variable ORIGIN) is 0 so the function alfa(x) representing angular acceleration dependent on momentum is defined. This function is used to extract single solution value from solution vector (Figure 12.).

$$\text{alfa}(M) := rj(M)_{0,4}$$

Figure 12. Defining single solution function

4.2.1. Solution plot

Range variable is defined for plotting solution an linear plot of solution is made.

mome := 0,0.1..3

0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
...

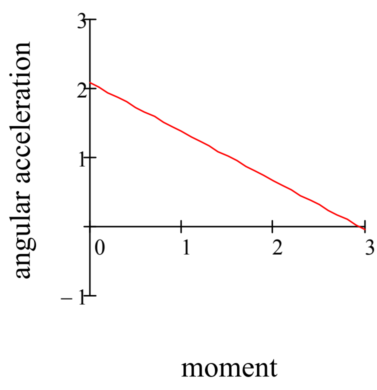

Figure 13. Solution plot

Mathematical solution of problem shows linear (Figure 13.) dependence between moment and angular acceleration, thus solution plot is represented with line and minimum of such function in this case would be $-\infty$.

4.2.2. Minimizing function

For obtaining physically correct solution, minimum value of angular acceleration equals zero and maximum frictional moment equals root of function alfa(M).

Root function in this form requires defining initial value for solution and syntax is as follows in Figure 14.

$$M_0 := 3 \qquad M_0 := \mathrm{root}\left(\mathrm{alfa}\left(M_0\right), M_0\right) = 2.943$$

Figure 14. Root function solution

5. CONCLUSION

Short theoretical problem is used to show one approach to solving optimization problems by use of symbolic calculations in MathCad engineering software. Physics laws demand that real conditions are considered while solutions are validated. Use of mathematical software shortens the process of solution verification and enables devising more theoretical problems and obtaining reliable solutions in shorter time even so without the use of graphical software.

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Kontakt autora:

Katarina Pisačić, dipl. ing.
Sveučilište Sjever
Sveučilišni centar Varaždin
104. brigade 3
42000 Varaždin
e-mail: katarina.pisacic@unin.hr

Marko Horvat, dipl. ing.
Sveučilište Sjever
Sveučilišni centar Varaždin
104. brigade 3
42000 Varaždin
e-mail: marko.horvat@unin.hr

Izv. prof. dr. sc. Vinko Višnjić.
Sveučilište Sjever
Sveučilišni centar Varaždin
104. brigade 3
42000 Varaždin
e-mail: vinko.visnjic@unin.hr