UDK 512.53
515.14
Izvorni znanstveni članak

Prihvaćeno za tisak: 26.2.2002.

ON SUMMABILITY OF *m*-TUPLES OF CONVOLUTION SEQUENCES OF PROBABILITY MEASURES ON LOCALLY COMPACT SEMIGROUPS

Cvetan Jardas

Abstract

In this paper we study the problem of convergence in the weak and the vague topology of the sequence

$$\left(\sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty}\cdots\sum_{i_m=1}^{\infty} a_{i_1 i_2\cdots i_m}^{(n)} \mu_1^{i_1} * \mu_2^{i_2} * \cdots * \mu_m^{i_m}, n \in \boldsymbol{N}\right)$$

where $\mu_1, \mu_2, \ldots, \mu_m$ are probability measures on locally compact commutative semigroup S and $A_n = [a_{i_1 i_2\cdots i_m}^{(n)}](i_1, i_2, \ldots, i_m, n \in \boldsymbol{N})$ is a sequence of stochastic m-matrices satisfying some additional conditions. The results of this paper are a natural generalization of the results in [3].

Key words and phrases: probability measure on semigroup, convolution sequence, weak and vague topology, stochastic *m*-matrix, locally compact space, polish space.

Introduction and preliminaries

We follow the notation from [2] and [3].

By S we denote a locally compact commutative (Hausdorff) second countable semigroup. By a measure on S, we mean a finite regular non-negative measure on the class B_S of all Borel sets in S. $P(S)$ denotes the set of all regular probability measures on S. We put $Q(S) = \{\mu : \mu \text{ is a measure on } S \text{ and } \mu(S) \leq 1\}$.

Let $C(S) \supset K(S)$ be the spaces of all (real-valued) bounded continuous functions, and all continuous functions with compact support, respectively. For $f \in C(S)$ or $f \in K(S)$ we denote by $\|f\|$ the sup norm of f.

The weak and the vague topology and the convolution $\mu * \nu$ of two measures μ, ν we define in the usual way (see [2]). By the Banach-Alaoglu's theorem the set $Q(S)$ is compact in the vague toplogy.

The following facts are well known (see [2]): the set $P(S)$ is a commutative topological semigroup with respect to the convolution (i.e. the mapping $* : P(S) \times P(S) \to P(S)$ is jointly continuous in the vague topology). $Q(S)$ although compact is not a topological semigroup. In the general case the convolution in $Q(S)$ is not even vaguely separately continuous. $P(S)$ is compact if and only if S is compact. Otherwise, the closure of $P(S)$ in the vague topology is equal to $Q(S)$. If the multiplication in S satisfies

AMS (1991) Subject Classification: 60B05, 60B10, 43A05.

$$\{(x,y): xy \in K', y \in K\} \text{ is compact for all compact } K, K' \subset S, \tag{1}$$

then the convolution is vaguely separately continuous. Separate continuity in the weak topology takes place without any condition on the multiplication.

$\mu * \mu * \ldots * \mu$ (with n terms) we denote by μ^n.

Let $A = [a_{nj}](n, j \in \boldsymbol{N})$ be a real infinite matrix. We say that A is **strongly regular** if it is a Toeplitz matrix and if in addition it satisfies the following condition

$$\lim_{n\to\infty} \sum_{j=1}^{\infty} \left| a_{nj} - a_{n,j+1} \right| = 0. \tag{2}$$

The following result has been proved in [2], Theorem 2 (without assumption of commutativity of S): If the multiplication in S satisfies (1) and if $A = [a_{nj}](n, j \in \boldsymbol{N})$ is a stochastic strongly regular matrix, then for each $\mu \in P(S)$, the sequence $(\mu^n, n \in \boldsymbol{N})$ is **vaguely A-convergent** to some $\mu_0 \in Q(S)$, i.e. we have

$$\lim_{n\to\infty} \sum_{j=1}^{\infty} a_{nj} \int_S f du^j = \int_S f d\mu_0, f \in K(S). \tag{3}$$

Moreover, the limiting measure μ_0 doesn't depend on matrix A. If we put $s_n(\mu) = \sum_{j=1}^{\infty} a_{nj} \mu^j \ (n \in \boldsymbol{N})$ we can write

$$\mu_0 = \varphi(\mu) = (v) A - \lim_{n\to\infty} \mu^n = (v) \lim_{n\to\infty} s_n(\mu) \tag{4}$$

By Theorem 2 in [2], we have

$$\mu * \varphi(\mu) = \varphi(\mu) * \mu = \varphi(\mu) = \varphi(\mu)^2, \tag{5}$$

and $\varphi(\mu)$ is either a null-measure or $\varphi(\mu) \in P(S)$.

The examples of stochastic strongly regular matrices have been given in [2].

Results

Let $m \geq 2$ be given natural number. By $i = (i_1, i_2, \ldots, i_m)$ we denote a multi index.

Definition. *A function* $A: \boldsymbol{N}^m \longrightarrow \boldsymbol{R}$, $A = [a_i](i \in \boldsymbol{N}^m)$ *is a* **stochastic *m*-matrix** *if* $a_i \geq 0 (i \in N^m)$ *and if*

$$\sum_{i \in \boldsymbol{N}^m} a_i = 1. \tag{6}$$

By S_m we denote the set of all stochastic m-matrices.

Let $\mathcal{A}=(A_n, n\in \boldsymbol{N})$ be a sequence in $S_m, A_n=[a_i^{(n)}](i\in \boldsymbol{N}^m, n\in \boldsymbol{N})$ and let $\mu=(\mu_1,\mu_2,\ldots,\mu_m)\in \mathcal{P}(S)^m$. For $i=(i_1,i_2,\ldots,i_m)\in \boldsymbol{N}^m$ we write $\mu^i=\mu_1^{i_1}*\mu_2^{i_2}*\ldots*\mu_m^{i_m}$. If there exists $\lambda\in P(S)$ such that

$$\lim_{n\to\infty}\sum_{i\in \boldsymbol{N}^m} a_i^{(n)}\int_S fd\mu^i=\int_S fd\lambda, f\in C(S), \tag{7}$$

then we say that the m-tuple $(\mu_1^{i_1}, i_1\in \boldsymbol{N}),(\mu_2^{i_2}, i_2\in \boldsymbol{N}),\ldots,(\mu_m^{i_m}, i_m\in \boldsymbol{N})$ of convolution sequences is **weakly $\mathcal{A}$-summable** to λ, and we put $\lambda=(w)\mathcal{A}-\lim\mu^i=(w)\mathcal{A}-\lim(\mu_1^{i_1},\ldots\mu_m^{i_m})$.

If (7) holds true for each $f\in K(S)$ (with $\mu_1,\mu_2,\ldots,\mu_m,\lambda\in Q(S)$) we say that the m-tuple $(\mu_1^{i_1}, i_1\in \boldsymbol{N}),(\mu_2^{i_2}, i_2\in \boldsymbol{N}),\ldots,(\mu_m^{i_m}, i_m\in \boldsymbol{N})$ is **vaguely $\mathcal{A}$-summable** to λ, and we put $\lambda=(v)\mathcal{A}-\lim\mu^i=(v)\mathcal{A}-\lim(\mu_1^{i_1},\ldots,\mu_m^{i_m})$.

If we put

$$s_n(\mu)=s_n(\mu_1,\ldots,\mu_m)=\sum_{i\in \boldsymbol{N}^m} a_i^{(n)}\mu^i=\sum_{i_1=1}^{\infty}\cdots\sum_{i_m=1}^{\infty} a_{i_1\ldots i_m}^{(n)}\mu_1^{i_1}*\ldots*\mu_m^{i_m}, n\in \boldsymbol{N},$$

then we have

$$\begin{aligned}(w)\mathcal{A}-\lim\mu^i&=(w)\lim_{n\to\infty}s_n(\mu)\\ (v)\mathcal{A}-\lim\mu^i&=(v)\lim_{n\to\infty}s_n(\mu).\end{aligned}\tag{8}$$

For a sequence $\mathcal{A}=(A_n, n\in \boldsymbol{N})\subset S_m, A_n=[a_{i_1\ldots i_m}^{(n)}](i_1,\ldots,i_m, n\in \boldsymbol{N})$ set:

$$\alpha_r^{(n)}=\sum_{i_1=1}^{\infty}\cdots\sum_{i_{r-1}=1}^{\infty}\sum_{i_{r+1}=1}^{\infty}\cdots\sum_{i_m=1}^{\infty} a_{i_1\cdots i_{r-1}1 i_{r+1}\cdots i_m}^{(n)}, n\in \boldsymbol{N}, r=1,2,\ldots,m, \tag{9}$$

$$K_r^{(n)}=\sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty}\cdots\sum_{i_m=1}^{\infty}\left|a_{i_1\cdots i_{r-1}i_r+1 i_{r+1}\cdots i_m}^{(n)}-a_{i_1\cdots i_{r-1}i_r i_{r+1}\cdots i_m}^{(n)}\right|, n\in \boldsymbol{N}, r=1,2,\ldots,m. \tag{10}$$

Theorem 1 *Let S be a locally compact commutative (Hausdorff) second countable semigroup. Let $\mu=(\mu_1,\mu_2,\ldots,\mu_m)\in \mathcal{P}(S)^m$ be such that the set $\{\mu^i: i\in \boldsymbol{N}^m\}$ is tight and let $\mathcal{A}=(A_n, n\in \boldsymbol{N})\subset S_m, A_n=[a_{i_1\ldots i_m}^{(n)}](i_1,\ldots,i_m, n\in \boldsymbol{N})$ be such that*

$$\lim_{n\to\infty}\alpha_r^{(n)}=0, \lim_{n\to\infty}K_r^{(n)}=0, r=1,2\ldots,m. \tag{11}$$

Then the m-tuple $(\mu_1^{i_1}, i_1\in \boldsymbol{N}),(\mu_2^{i_2}, i_2\in \boldsymbol{N}),\ldots,(\mu_m^{i_m}, i_m\in \boldsymbol{N})$ of convolution sequences is weakly $\mathcal{A}$-summable. If we put $(w)\mathcal{A}-\lim\mu^i=\lambda\in P(S)$, then λ doesnt't depend on sequence $\mathcal{A}$, and we have

$$\mu_r*\lambda=\lambda=\lambda^2, r=1,2,\ldots,m. \tag{12}$$

Proof. The tightness of the set $[\mu^i : i \in \boldsymbol{N}^m]$ implies the tightness of the sequence $(s_n(\mu), n \in \boldsymbol{N}) \subset P(S)$. It follows by Prokhorov's theorem that the set $(s_n(\mu), n \in \boldsymbol{N})$ is relatively compact in *P(S)* in the weak topology. Hence, there exists $\lambda \in P(S)$ and a subsequence

$$\left(s_{n_k}(\mu) = \sum_{i \in \boldsymbol{N}^m} a_i^{(n_k)} \mu^i,\ k \in \boldsymbol{N}\right),$$

such that

$$\lambda = (w) \lim_{k \to \infty} s_{n_k}(\mu) \tag{13}$$

For $f \in C(S)$ and $r = 1, 2, \ldots, m$, we have

$$\left| \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{\infty} \cdots \sum_{i_m=1}^{\infty} a_{i_1, i_2 \ldots i_m}^{(n_k)} \int_S f d(\mu_1^{i_1} * \ldots * \mu_{r-1}^{i_{r-1}} * \mu_r^{i_r+1} * \mu_{r+1}^{i_{r+1}} * \ldots * \mu_m^{i_m}) - \right.$$

$$\left. - \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{\infty} \cdots \sum_{i_m=1}^{\infty} a_{i_1, i_2 \ldots i_m}^{(n_k)} \int_S f d(\mu_1^{i_1} * \mu_2^{i_2} * \ldots * \mu_m^{i_m}) \right| =$$

$$= \left| \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{\infty} \cdots \sum_{i_m=1}^{\infty} (a_{i_1, i_2 \ldots i_m}^{(n_k)} - a_{i_1 \ldots i_{r-1} i_r+1 i_{r+1} \ldots i_m}^{(n_k)}) \int_S f d(\mu_1^{i_1} * \ldots * \mu_{r-1}^{i_{r-1}} * \mu_r^{i_r+1} * \mu_{r+1}^{i_{r+1}} * \ldots * \mu_m^{i_m}) - \right.$$

$$\left. - \sum_{i_1=1}^{\infty} \cdots \sum_{i_{r-1}=1}^{\infty} \sum_{i_{r+1}=1}^{\infty} \cdots \sum_{i_m=1}^{\infty} a_{i_1 \ldots i_{r-1} 1 i_{r+1} \ldots i_m}^{(n_k)} \int_S f d(\mu_1^{i_1} * \ldots * \mu_{r-1}^{i_{r-1}} * \mu_r * \mu_{r+1}^{i_{r+1}} * \ldots * \mu_m^{i_m}) \right| \le$$

$$\le (K_r^{(n_k)} + \alpha_r^{(n_k)}) \|f\|,$$

so by (11) we conclude

$$\begin{aligned} \lim_{k \to \infty} & \left| \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{\infty} \cdots \sum_{i_m=1}^{\infty} a_{i_1, i_2 \ldots i_m}^{(n_k)} \int_S f d(\mu_1^{i_1} * \ldots * \mu_{r-1}^{i_{r-1}} * \mu_r^{i_r+1} * \mu_{r+1}^{i_{r+1}} * \ldots * \mu_m^{i_m}) - \right. \\ & \left. - \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{\infty} \cdots \sum_{i_m=1}^{\infty} a_{i_1, i_2 \ldots i_m}^{(n_k)} \int_S f d(\mu_1^{i_1} * \mu_2^{i_2} * \ldots * \mu_m^{i_m}) \right| = 0. \end{aligned} \tag{14}$$

It follows from (13) and (14) that for $r = 1, 2, \ldots, m$, we have

$$\lambda = (w) \lim_{k \to \infty} \sum_{i_1=1}^{\infty} \cdots \sum_{i_m=1}^{\infty} a_{i_1 \ldots i_m}^{(n_k)} \mu_1^{i_1} * \ldots * \mu_{r-1}^{i_{r-1}} * \mu_r^{i_r+1} * \mu_{r+1}^{i_{r+1}} * \ldots * \mu_m^{i_m}. \tag{15}$$

Now, by separate continuity of the convolution in the weak topology, we get

$$\mu_r * \lambda = \mu_r * (w)\lim_{k\to\infty} s_{n_k}(\mu) = (w)\lim_{k\to\infty}(\mu_r * s_{n_k}(\mu)) = (\text{by}\,(15)) = \lambda,\ r = 1,2,\ldots,m,$$

and therefore $\mu^i * \lambda = \mu_1^{i_1} * \ldots * \mu_m^{i_m} * \lambda = \lambda$ for $i = (i_1,\ldots,i_m) \in \boldsymbol{N}^m$. It follows $s_{n_k}(\mu) * \lambda = \lambda (k \in \boldsymbol{N}) \Rightarrow \lambda^2 = \lambda$, so λ is an idempotent in *P(S)*.

Considering now an another subsequence $\left(s_{m_k}(\mu) = \sum_{i\in N^m} a_i^{(m_k)} \mu^i,\ k \in \boldsymbol{N}\right)$ converging weakly to $\lambda_1 \in P(S)$ we conclude again that λ_1 is an idempotent such that $\mu_r * \lambda_1 = \lambda_1 (r = 1,2,\ldots,m)$ and therefore $\mu^i * \lambda_1 = \lambda_1 (i \in \boldsymbol{N}^m)$ so we have

$$s_{n_k}(\mu) * \lambda_1 = \left(\sum_{i\in N^m} a_i^{(n_k)}\right)\lambda_1 = \lambda_1, \text{ for each } k.$$

Analogously we get $\lambda * s_{m_k}(\mu) = \lambda (k \in \boldsymbol{N})$. It follows

$$\lambda_1 = (w)\lim_{k\to\infty}(s_{n_k}(\mu) * \lambda_1) = \lambda * \lambda_1 = (w)\lim_{k\to\infty}(\lambda * s_{m_k}(\mu)) = \lambda.$$

Thus, every weakly convergent subsequence of the sequence $(s_n(\mu), n \in \boldsymbol{N})$ converges to the same element in *P(S)*, denote it by λ. Since $(s_n(\mu), n \in \boldsymbol{N})$ is relatively compact in the weak topology, it follows easyly that $\lambda = (w) - \lim_{n\to\infty} s_n(\mu)$. We also have proved that (12) holds true.

Let $\mathcal{B} = (B_n, n \in \boldsymbol{N}), B_n = [b_{i_1\ldots i_m}^{(n)}](i_1,\ldots,i_m, n \in \boldsymbol{N})$ be another sequence in S_m satisfying (11) (with $b_{i_1\ldots i_m}^{(n)}$ instead of $a_{i_1\ldots i_m}^{(n)}$), set $\tilde{s}_n(\mu) = \sum_{i\in N^m} b_i^{(n)} \mu^i (n \in \boldsymbol{N})$ and let $\lambda_1 = (w)\mathcal{B} - \lim \mu^i = (w)\lim_{n\to\infty} \tilde{s}_n(\mu)$. For arbitrary $i \in \boldsymbol{N}^m$ we have again $\mu^i * \lambda_1 = \lambda_1$, and therefore $s_n(\mu) * \lambda_1 = \left(\sum_{i\in N^m} a_i^{(n)}\right)\lambda_1 = \lambda_1$. Analogously we get $\tilde{s}_n(\mu) * \lambda = \lambda$. Now we have

$$\lambda_1 = (w)\lim_{n\to\infty}(s_n(\mu) * \lambda_1) = \lambda * \lambda_1 = \lambda_1 * \lambda = (w)\lim_{n\to\infty}(\tilde{s}_n(\mu) * \lambda) = \lambda$$

Q.E.D.

Example. For a natural number $k>1$ we define a sequence $\mathcal{A} = (A_n, n \in \boldsymbol{N})$, $A_n = [a_i^{(n)}](i \in \boldsymbol{N}^m, n \in \boldsymbol{N})$ of *m*-matrices by

$$a_i^{(n)} = \begin{cases} \dfrac{1}{k^{mn}}, & i_1, i_2, \ldots, i_m \le k^n \\ 0, & \text{otherwise.} \end{cases}$$

It is easy to verify that the sequence $\mathcal{A}$ defined in that way is in S_m and it satisfies conditions (11).

Remark. Let us make some changes on the topological assumptions on S, i.e. suppose that S is a polish space. Prokhorov's theorem holds on all polish spaces, and joint continuity of $(\mu,\nu)\mapsto\mu*\nu$ holds in that case by [5], Theorem 1, p. 70. Therefore, Theorem 1 holds on all polish spaces with the same proof.

In the case of the vague topology we have the following theorem.

Theorem 2 *Let S be a locally compact commutative (Hausdorff) second countable semigroup with the multiplication satisfying (1) and let $\mathcal{A}=(A_n, n\in \mathbf{N})$, $A_n=(a^{(n)}_{i_1\ldots i_m})(i_1,i_2,\ldots,i_m,n\in\mathbf{N})$ be a sequence in S_m satisfying conditions (11). Then the m-tuple $(\mu_1^{i_1}, i_1\in\mathbf{N}),(\mu_2^{i_2}, i_2\in\mathbf{N}),\ldots,(\mu_m^{i_m}, i_m\in\mathbf{N})$ of convolution sequences is vaguely $\mathcal{A}$-summable for every $\mu=(\mu_1,\mu_2,\ldots,\mu_m)\in P(S)^m$. If we put $\varphi(\mu)=\varphi(\mu_1,\ldots,\mu_m)=(v)\mathcal{A}-\lim\mu^i$, then $\varphi(\mu)\in Q(S)$ is an idempotent not depending on sequence $\mathcal{A}$ and $\varphi(\mu)$ is either a null-measure or $\varphi(\mu)\in P(S)$.*

Moreover, if $\varphi(\mu_r)\in P(S)(r=1,2,\ldots,m)$, then we have

$$\varphi(\mu_1,\mu_2,\ldots,\mu_m)=\varphi(\mu_1)*\varphi(\mu_2)*\ldots*\varphi(\mu_m) \tag{16}$$

Proof. We have $\left(s_n(\mu)=\sum_{i\in N^m} a_i^{(n)}\mu^i, n\in\mathbf{N}\right)\subset Q(S)$ and since $Q(S)$ is compact in the vague topology, the set $(s_n(\mu), n\in\mathbf{N})$ is relatively compact in the vague topology. Condition (1) implies that the convolution is vaguely separately continuous on $Q(S)$. Thus, in the same way as in Theorem 1, we prove that the m-tuple $(\mu_1^{i_1}, i_1\in\mathbf{N}),\ldots,(\mu_m^{i_m}, i_m\in\mathbf{N})$ is vaguely $\mathcal{A}$-summable to $\varphi(\mu)=\varphi(\mu_1,\ldots,\mu_m)\in$ $\in Q(S)$ not depending on sequence $\mathcal{A}$ and satisfying

$$\mu_r*\varphi(\mu)=\varphi(\mu)=\varphi(\mu)^2, r=1,2,\ldots,m. \tag{17}$$

Analogously as in Theorem 2 in [3] we prove that $\varphi(\mu)=0$ or $\varphi(\mu)\in P(S)$.

By Theorem 2 in [2] the sequences $(\mu_1^{i_1}, i_1\in\mathbf{N}),\ldots,(\mu_m^{i_m}, i_m\in\mathbf{N})$ are vaguely A-convergent for every stochastic strongly regular matrix A and by the assumption the limiting measures $\varphi(\mu_1),\ldots,\varphi(\mu_m)$ are in $P(S)$ and do not depend on A. The Cesaro $(c,1)$ summability matrix $C=[c_{nj}]$ defined by $c_{nj}=\frac{1}{n}$ for $j\le n$ and $c_{nj}=0$ for $j>n$ is obviously stochastic and strongly regular. Therefore, if we put

$$\mu_r^{(n)}=\frac{1}{n}\sum_{j=1}^{n}\mu_r^j, r=1,2,\ldots,m. \tag{18}$$

we have

$$\varphi(\mu_r)=(v)\lim_{n\to\infty}\mu_r^{(n)}, r=1,2,\ldots,m. \tag{19}$$

Then by use of the above Example and the fact that *P(S)* is commutative topological semigroup (in the vague topology) we get

$$\varphi(\mu_1,\dots,\mu_m)=(v)\lim_{n\to\infty}\sum_{i_1=1}^{k^n}\dots\sum_{i_m=1}^{k^n}\frac{1}{k^{mn}}\mu_1^{i_1}*\dots*\mu_m^{i_m}=$$

$$=(v)\lim_{n\to\infty}\left[\left(\frac{1}{k^n}\sum_{i_1=1}^{k^n}\mu_1^{i_1}\right)*\dots*\left(\frac{1}{k^n}\sum_{i_m=1}^{k^n}\mu_m^{i_m}\right)\right]=$$

$$=\varphi(\mu_1)*\dots*\varphi(\mu_m).$$

Q.E.D.

In the same way as the Corollary in [3] we can prove the following result.

Corollary. *Let the conditions of Theorem 2 are fulfilled.*

(i) Let $p_r\in\mathbf{N}$ *and let* $\mu_r^{p_r}=\nu_r,(r=1,2,\dots,m)$. *Then we have*

$$\varphi(\mu_1,\mu_2,\dots,\mu_m)=\mu_1^{(p_1)}*\mu_2^{(p_2)}*\dots*\mu_m^{(p_m)}*\varphi(\nu_1,\nu_2,\dots,\nu_m). \tag{20}$$

(ii) If there exist $p_r\in\mathbf{N}(r=1,2,\dots,m)$ *such that* $\mu_r^{p_r+1}=\mu_r$, *then we have*

$$\varphi(\mu_1,\mu_2,\dots,\mu_m)=\mu_1^{(p_1)}*\mu_2^{(p_2)}*\dots*\mu_m^{(p_m)}. \tag{21}$$

(iii) If there exist $\lambda_r=(v)\lim_{n\to\infty}\mu_r^n(r=1,2,\dots,m)$ *then we have*

$$\varphi(\mu_1,\mu_2,\dots,\mu_m)=\lambda_1*\lambda_2*\dots*\lambda_m. \tag{22}$$

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O sumabilnosti *m*-torki nizova konvolucija vjerojatnosnih mjera na lokalno kompaktnim polugrupama

Cvetan Jardas

SAŽETAK

U članku se proučava problem konvergencije u slaboj i neodređenoj topologiji niza

$$\left(\sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty}\cdots\sum_{i_m=1}^{\infty} a^{(n)}_{i_1 i_2 \cdots i_m}\mu_1^{i_1} * \mu_2^{i_2} * \cdots * \mu_m^{i_m}, n \in \boldsymbol{N}\right)$$

gdje su $\mu_1, \mu_2, \ldots, \mu$ vjerojasne mjere na lokalno kompaktnoj komutativnoj polugrupi S i $A_n = [a^{(n)}_{i_1 i_2 \cdots i_m}](i_1, i_2, \ldots, i_m, n \in \boldsymbol{N})$ je niz stohastičkih *m*-matrica koje zadovoljavaju neke dodatne uvjete. Rezultati ovog članka su prirodna popćenja rezultata iz [3].

Cvetan Jardas
Ekonomski fakultet Sveučilišta u Rijeci,
51000 Rijeka, Ivana Filipovića 4, Croatia
cvetan.jardas@efri.hr