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VIBRATIONS AND RESPONSE TO SEISMIC EXCITATIONS OF FLEXURE AND SHEAR BEAMS ON HINGED END SUPPORTS AND A CONTINUOUS DEFORMABLE FOUNDATION

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Summary

Simple formulas are derived for the fundamental periods of the free lateral vibrations and the response to sine loads of flexure beams and shear beams on laterally fix hinged end supports and a continuous elastic foundation. It is shown how the mechanical charasteristics and the response quantities of systems with flexure-shear beams can be approximated by those of a flexure or a shear beam by appropriately defining their equivalent cross-sectional stiffnesses. Applications of the developed theories in the design and analysis of some engineering structures are also dealt with and two numerical examples elucidate the procedures.

Key-words: vibrations, response to seismic excitations, flexure beams, shear beams, flexure-shear beams, periods

1. Introduction

Beams with laterally fix hinged end supports and a continuous elastic foundation along its length often occur in engineering structures. The beam can be either a flexure beam, i.e. a beam whose deformation is dominantly due to flexure, a shear beam, i.e. a beam whose deformation is dominantly due to shear or a flexure-shear beam whose deformation is due to both flexure and shear so that neither of the two contributions is negligible compared with that of the other. The elastic foundation is supposed to be of the Winkler type.

Simple formulas are derived for the fundamental period of the systems' free lateral vibrations and its response to a sine load.

A possible practical application of the developed theories are bracing structures of some flat-roofed buildings subjected to earhtquake excitations; the derived formulas can be applied in both their preliminary-design and final-analyses phases. Numerical examples of two typical structures with a roof truss beam and a roof decking consisting of stressed-skin corrugated metal sheeets, respectively, elucidate the systems' mechanical behavior.

2. Reduced cross-sectional stiffnesses of the simple flexure and shear beams

Fig 1A shows the simple flexure-shear beam subjected to an uniform load and the corresponding displacement line. The meaning of the symbols is: L..span, K_M^o and K_V^o.. cross-sectional flexure and shear stifnesses, respectively, q..load intensity, w^o.. mid-span displacement.

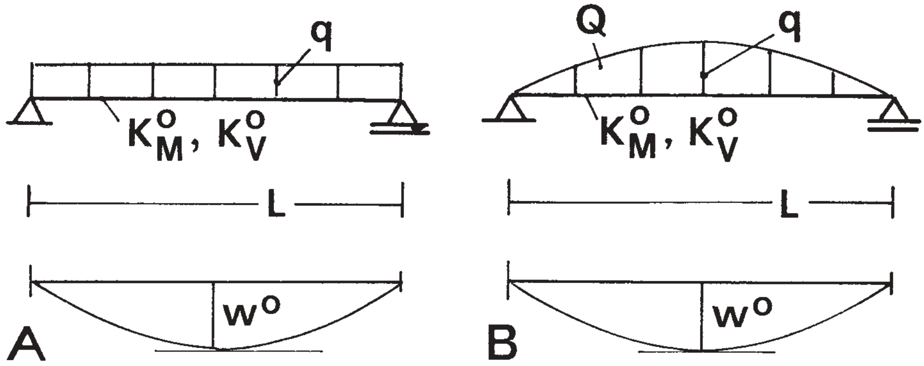

Fig. 1 Flexure-shear beams subjected to A) an uniform load and B) a sine load and corresponding displacement lines

The mid-span displacements due to flexure and shear and their sum are easily found to be

$$w_M^o = \frac{5}{384}\frac{qL^4}{K_M^o}\,,\quad w_V^o = \frac{qL^2}{8K_V^o}\,,\quad w^o = w_M^o + w_V^o \tag{1}$$

If the beam is approximated by a flexure beam, its mid-span displacement can be written down as

$$w^o = \frac{5}{384}\frac{qL^4}{K_M}\,, \tag{2}$$

where

$$K_M = \frac{K_M^o}{1+\dfrac{9{,}6K_M^o}{L^2\,K_V^o}} \tag{3}$$

is the beam´s reduced or equivalent cross-sectional stiffness corresponding to an uniform load. Of course, if $K_V^o \to \infty$, then $K_M \to K_M^o$.

If the beam is approximated by a shear beam. its mid-span displacement is

$$w^o = \frac{qL^2}{8K_V}, \tag{4}$$

where

$$KV = \frac{K_V^o}{1+\dfrac{L^2 K_V^o}{9{,}6K_M^o}} \tag{5}$$

is the beam´s reduced or equivalent cross-sectional stiffness corresponding to an uniform load. Of course, if $K_M^o \to \infty$, then $K_V \to K_V^o$.

Fig. 1B shows the simple flexure-shear beam subjected to a sine load. If the total load is Q, te mid-span intensity and the intesity at x are

$$q = \frac{\pi}{L}\frac{Q}{2}, \quad q_x = q\sin\left(\pi\frac{x}{L}\right). \tag{6}$$

The mid-span displacements due to flexure and shear are

$$w_M^o = \left(\frac{L}{\pi}\right)^3 \frac{Q}{2K_M^o}, \quad w_V^o = \frac{L}{\pi}\frac{Q}{2K_V^o}, \tag{7}$$

their sum

$$w^o = \frac{L}{\pi}\left[\left(\frac{L}{\pi}\right)^2 \frac{1}{K_M^o} + \frac{1}{K_V^o}\right]\frac{Q}{2}. \tag{8}$$

If the beam is approximated by a flexure beam, its mid-span displacement can be formulated as

$$w_M^o = \left(\frac{L}{\pi}\right)^3 \frac{Q}{2K_M}, \tag{9}$$

where

$$K_M = \frac{K_M^o}{1+\left(\dfrac{\pi}{L}\right)^2 \dfrac{K_M^o}{K_V^o}} \tag{10}$$

is the beams reduced or equivalent cross-sectional stiffness corresponding to a sine load. Of course, if $K_V^o \to \infty$, then $K_M \to K_M^o$.

If the beam is approximated by a shear beam, its displacement is

$$w_M^o = \frac{L}{\pi} \frac{Q}{2K_V} \tag{11}$$

where

$$K_V = \frac{K_V^o}{1 + \left(\frac{\pi}{L}\right)^2 \frac{K_V^o}{K_M^o}} \tag{12}$$

is the beam's reduced or equivalent cross-sectional stiffness corresponding to a sine load. Of course, if $K_M^o \to \infty$, then $K_V \to K_V^o$.

Comparing Eqs. (5) and (12) and considering that $\pi^2 = 9{,}87$ it becomes obvious, that the equivalent stiffnesses only insignificantly depend on the distribution of the load along the beam's span.

3. Fundamental free-vibrations' periods of flexure beams

Fig. 2 A shows a simple flexure beam subjected to an uniform load and the corresponding displacement line. K_M denotes the beam´s reduced flexural stiffness (Eq. (10)), which indireclty approximately takes into account the effect of shear onto the deformation, q the load intensity numerically equal to the system's weight intensity and w_M^o the coresponding mid-span displacement.

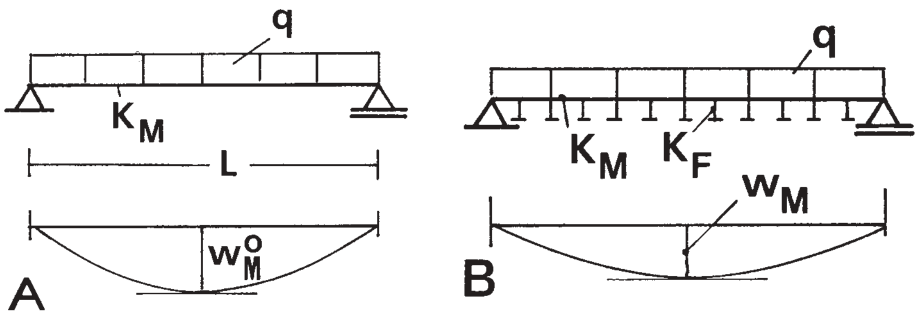

Fig. 2. Flexure beams subjected to an uniform load on A) hinged end supports and B) hinged end supports and a continuous deformable foundation and corresponding displacement lines

The exact value of the fundamental period of the beam's free vibrations follows from [1] to be

$$T_M^o = \frac{2}{\pi\sqrt{g}} L^2 \sqrt{\frac{q}{K_M}} = 0{,}2033\, L^2 \sqrt{\frac{q}{K_M}} = 1{,}781\sqrt{w_M^o}\,, \tag{13}$$

where g is the gravitational acceleration (9,81 m/sec^2). In the second and the third expressions on the right-hand side of Eq. (13) lenghts must be expressed in meters and the period is obtained in seconds.

To obtain the period of the simple flexure beam with an additional continous elastic foundation (Fig. 2B) two simple approximate methods are proposed.

Firstly, in the last expression for T_M^o on the right-hand side of Eq. (13) the displacement w_M^o of the simple beam is replaced by the displacement $R_{w,M}\ w_M^o$ of the beam with the additional foundation. Herein, the displacement-reduction coefficient amounts to

$$R_{w,M} = \frac{1,2}{U^4}\left(1 - \frac{2\cosh U \cos U}{\cosh(2U) + \cos(2U)}\right), \tag{14}$$

where

$$U = \frac{L}{2}\sqrt[4]{\frac{K_F}{4K_M}} \tag{15}$$

is the systems dimensionless parameter [2]. In practice, U assumes values in the domain (0; $\pi/2$); for $U = \pi/2$ the discharging effect of the foundation is nearly nil.

Herewith, the period to be determined is

$$T_M = 1{,}781\sqrt{R_{w,M}\, w_M^o} = t_M\, T_M^o\,, \tag{16}$$

where the dimensionless period-reduction coefficient with respect to the period of the simple flexure beam amount to

$$t_M = t_M(U) = \frac{1}{U^2}\sqrt{1{,}2\left(1 - \frac{2\cosh U \cos U}{\cosh(2U) + \cos(2U)}\right)}. \tag{17}$$

In the left part of the Table numerical values of t_M are listed for some values of U.

Secondly, the method of splitting the given system into subsystems [3] is applied. The two subsystems are the simple beam and the continous foundation. With their periods

$$T_M^o = \frac{2}{\pi\sqrt{g}} L^2 \sqrt{\frac{q}{K_M}}, \quad T_F = \frac{2}{\sqrt{g}} \sqrt{\frac{q}{K_F}} \tag{18}$$

the relationship

$$\frac{1}{T_M^2} \geq \frac{1}{T_M^{o2}} + \frac{1}{T_F^2} \tag{19}$$

holds, so that the maximal possible value of the period looked for is

$$T_M = t_M T_M^o ; \tag{20}$$

the dimensionless period-reduction coefficient with respect to the period of the simple flexure beam is

$$t_M = \sqrt{\frac{1}{1+(\frac{L}{\pi})^4 \frac{K_F}{K_M}}}. \tag{21}$$

Hence, with both methods, the period T_M^o of the simple flexure beam is the reference value. For the period-reduction coefficient t_M two values are derived (Eqs. (17) and (21)); for the systems parameters L = 42 m, K_M = 8,21 GNm^2 and K_F = 49,6 kN/m^2 for example the first method gives, with U = 0,73619, the approximate value t_M = 0,9152, whilst according to the second there is $t_M \leq$ 0,9156. Obviously, both values excellently agree.

4. Fundamental free-vibrations' periods of shear beams

Fig. 3A shows a simple shear beam subjected to an uniform load and the corresponding displacement line. K_V denotes the beam's reduced shear stiffness. (Eq. (12)), which indirectly approximately takes into account the effect of flexure onto the deformation, q the load intensity numerically equal to the system's weight intensity and w_V^o the corresponding mid-span displacement.

The exact value of the fundamental period of the beam's free vibrations is, starting from [1], found to be

$$T_V^o = \frac{2}{\sqrt{g}} L \sqrt{\frac{q}{K_V}} = 0{,}6386\, L \sqrt{\frac{q}{K_V}} = 1{,}806 \sqrt{w_V^o} \tag{22}$$

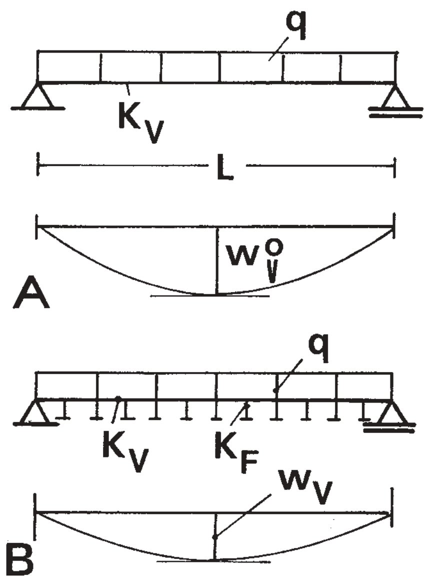

Fig. 3. Shear beams subjected to a sine load on A) hinged end supports and B) hinged end supports and a continuous deformable foundation and corresponding displacement lines.

In the second and third expressions on the right-hand side of Eq. (22) lenghts must be expressed in meters so that the period is obtained in seconds. The author's simple energy approach [4] leads to practically the same result, concretely to the factor 0,6344 sec/√m instead of 0,6386 sec/√m in the second expression on the right-hand side of Eq. (22).

To obtain the period of the simple shear beam with an additional continuous elastic foundation (Fig. 3B) again two simple approximate methods are proposed.

Firstly, in the last expression for T_V^o on the right-hand side of Eq. (22) the displacement w_V^o of the simple beam is replaced by the displacement $R_{w,V}\, w_V^o$ of the beam with the additional foundation. Herein, the displacement-reduction coefficient amounts to

$$R_{w,V} = \frac{2}{A^2}\left(1 - \frac{1}{\cosh A}\right), \tag{23}$$

where

$$A = \frac{L}{2}\sqrt{\frac{K_F}{K_V}} \tag{24}$$

is the systems dimensionless parameter [5]. Theoretically, A is defined in the domain (0; ∞), but a considerable discharging effect of the foundation is obtained only if $A \leq 5$.

Herewith, the period to be determined is

$$T_V = 1{,}806\sqrt{R_{w,V} w_V^o} = t_V T_V^o, \tag{25}$$

where the dimensionless period-reduction coefficient amounts to

$$t_V = t_V(A) = \frac{1}{A}\sqrt{2(1 - \frac{1}{\cosh A})}. \tag{26}$$

In the right part of the Table numerical values of t_V are listed for some values of A.

Secondly, the method of splitting the given system into subsystems [3] is applied again. The two subsystems are the simple beam and the contionous foundation. With their periods

$$T_V^o = \frac{2}{\sqrt{g}} L\sqrt{\frac{q}{K_V}},\ T_F = \frac{2}{\sqrt{g}}\sqrt{\frac{q}{K_F}} \tag{27}$$

the relationship

$$\frac{1}{T_V^2} \geq \frac{1}{T_V^{o2}} + \frac{1}{T_F^2} \tag{28}$$

holds, so that the maximal possible value of the period looked for is

$$T_V = t_V\, T_V^o; \tag{29}$$

the dimensionless period-reduction coefficient with respect to the period of the simple shear beam is

$$t_V = \sqrt{\frac{1}{1 + (\frac{L}{\pi})^2 \frac{K_F}{K_V}}}. \tag{30}$$

Hence, with both methods, the period T_V^o of the simple shear beam is the reference value. For the period-reduction coefficient t_V two values are derived (Eqs. (26) and (30)); for the system's parameters L = 36 m, K_V = 67 MN akd K_F = 84,2 kN/m^2 for example the first method gives, with A = 0,6381, the approximate value t_V = 0,9244, whilst according to the second there is $t_V \leq$ 0,9265. Againg, both values excellently agree.

5. Response of flexure beams

Fig. 4A shows one half of the simple flexure beam subjected to a sine load, its reaction, the shear-force and bending-moment diagrams and the displacement line.

If S denotes the total load, its - maximal - intensity at the mid-span and at an arbitrary x are

$$s = \frac{\pi}{L}\frac{S}{2}, \quad s_x = s\sin(\pi\frac{x}{L}). \tag{31}$$

The maximal values of the beam's response quantities are easily found to amount

$$F^o = V^o = \frac{S}{2}, \quad M^o = \frac{L}{\pi}\frac{S}{2} = \frac{L}{\pi}V^o, \tag{32}$$

$$w^o = (\frac{L}{\pi})^3 \frac{S}{2K_M} = (\frac{L}{\pi})^2 \frac{M^o}{K_M}, \tag{33}$$

whilst at an arbitrary x there is

$$V_X^o = V^o \cos(\pi\frac{x}{L}), \quad M_X^o = M^o \sin(\pi\frac{x}{L}), \tag{34}$$

$$w_x^o = x^o \sin(\pi\frac{x}{L}). \tag{35}$$

Fig. 4B shows one half of the simple flexure beam with the additional continous elastic foundation subjected to a sine load, its end-support's reaction, the beam's load, its shear-force and bending moment diagrams, the foundation´s load and the displacement line.

In [2] it is shown that the beam's load and its response quantities are equal to those of the reference beam (Fig. 4A) multiplied by the response-reduction factor

$$R = \frac{1}{1+(\frac{L}{\pi})^4 \frac{K_F}{K_M}}; \tag{36}$$

herein, the second term in the denominator is the dimensionless system's parameter corresponding to a sine load. Herewith there is

$$s_B = R\, s,\ F = V = RV^o,\ M = RM^o, \tag{37}$$

$$s_F = (1 - R)\, s, \tag{38}$$

$$w = R\, w^o \tag{39}$$

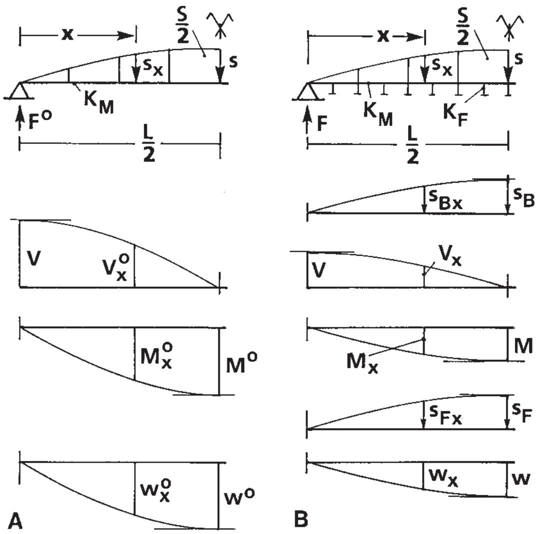

Fig. 4. A) One half of a flexure beam on hinged end supports subjected to a sine load, corresponding reaction, shear-force and bending-moment diagrams and displacement line. B) One half of a flexure beam on hinged end supports and a continuous deformable foundation subjected to a sine load, corresponding end reaction, beam load, shear-force and bending-moment diagrams, foundation load and displacement line

and for an arbitrary x

$$s_{Bx} = s_B \sin(\pi \frac{x}{L}), \quad V_x = V \cos(\pi \frac{x}{L}), \quad M_x = M \sin(\pi \frac{x}{L}), \tag{40}$$

$$s_{Fx} = s_F \sin(\pi \frac{x}{K}), \tag{41}$$

$$w_x = w \sin(\pi \frac{x}{L}). \tag{42}$$

Numerical example. As an engineering application of the developed theories let consider the bracing structure (Fig. 5A) consisting of a longitudinal roof truss beam, rigid gables and elastic transverse frames of a flat-roofed building subjected to a seismic load in the building's transverse direction.

Data. Buildings lenght: L = 42 m. Panel length: l = 7 m. The roof beam acts as a flexural beam its cross-sectional stiffness being K_M = 8,21 GNm^2. The lateral stiffness of a frame is K_R = 347 kN/m, so that the stiffness of the roof-beam's continuous

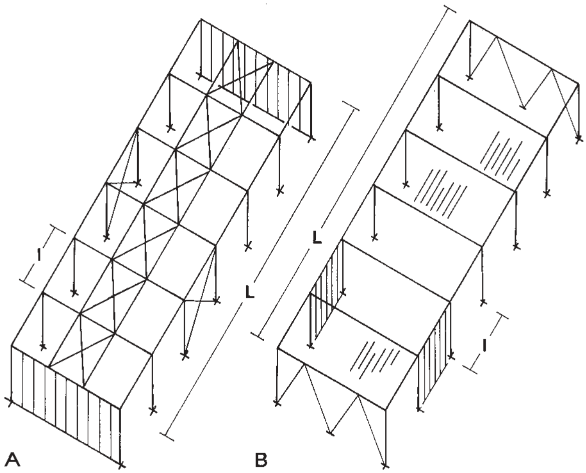

Fig. 5. Two examples of bracing structures consisting of a longitudinal roof beam, rigid gables and deformable transverse frames of a flat-roofed building subjected to a transverse sine load. The roof beam is A) a flexure and B) a shear beam

foundation is $K_F = K_R / l = 49{,}6$ kN/m^2. The roof's weight per unit building's length including a reasonable part of the snow load is $q = 30$ kN/m.

Free-vibrations period. $T_M^o = 0{,}6855$ sec, $t_M = 0{,}915$, $T_M = 0{,}627$ sec.

Total seismic load. Assuming that it amounts to 5% of the governing roof's weight, there is $S = 0{,}05 \cdot 30 \cdot 42 = 63$ kN. Load's mid-span intensity: $s = 2{,}36$ kN/m.

Maximal response values of the reference beam: $F^o = V^o = 31{,}5$ kN, $M^o = 421$ kNm, $w^o = 9{,}17$ mm.

Response-reduction factor: $R = 0{,}838$. Hence the maximal values of the systems response are: $F = V = 26{,}4$ kN, $M = 353$ kNm, $w = 7{,}68$ mm, $s_F = 0{,}382$ kN/m.

6. Response of shear beams

To determine the response of shear beams to seismic loads let first consider an infinitesimal element or cutout in the region (x, x +dx) of the beam (Fig. 6). The symbols s_x, V_x, M_x, w_x and w_x' denote the load intensity, the shear force, bending moment, displacement and slope of the displacement line at x, respectively. Primes generally denote derivatives with respect to the abscissa x. The governing equations are

$$V_x = M_x', \quad V_x' = -s_x, \quad w_x' = \frac{V_x}{K_V}; \tag{43}$$

the first and the second are the equilibrium requirements of the element, the third is the constitutive equation. Integrating the third equation yields

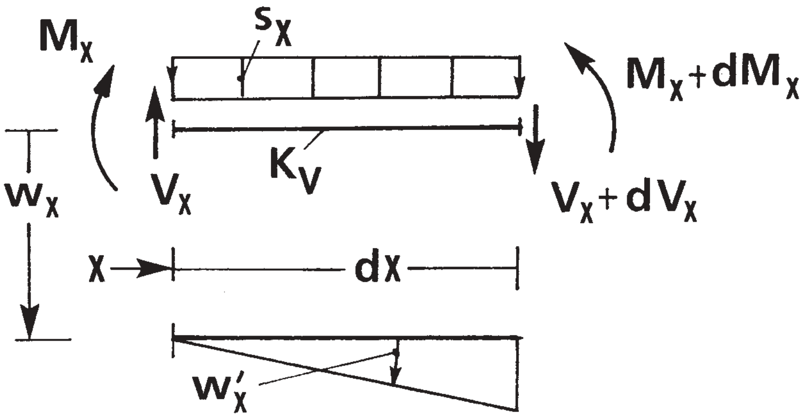

Fig. 6. Free-body diagram of an infinitesimal element of a shear beam and corresponding displacement line

$$w_x = \frac{1}{K_V}\int V_X \; dx + D = \frac{M_X}{K_V} + D, \tag{44}$$

D being an integration constant, so that the profile of w_x corresponds to that of M_x provided, if necessary, the boundary condition is considered.

Fig. 7A shows one half of a simple shear beam subjected to a sine load, the raction, its shear-force and bending-moment diagrams and the displacement line.

The maximal value of the response quantities are easily found to be

$$F^o = V^o = \frac{S}{2}, \quad M^o = \frac{L}{\pi}\frac{S}{2} = \frac{L}{\pi}V^o, \quad w^o = \frac{L}{\pi}\frac{S}{2K_V} = \frac{M^o}{K_V}, \tag{45}$$

whilst at an arbitrary x there is

$$V_x^o = V^o \cos(\pi \frac{X}{L}), \quad M_x^o = M^o \sin(\pi \frac{x}{L}), \tag{46}$$

$$w_x^o = w^o \sin(\pi \frac{x}{L}). \qquad (47).$$

Of course, the results for the internal forces are the same as those of the flexure beam - they follow from only equilibrium requirements.

Fig. 7B shows one half of a shear beam with hinged end supports and a continuous elastic foundation subjected to a sine load, its end-support's reaction, the beam's load, its shear-force and bending moment diagrams, the foundation's load and the displacement line.

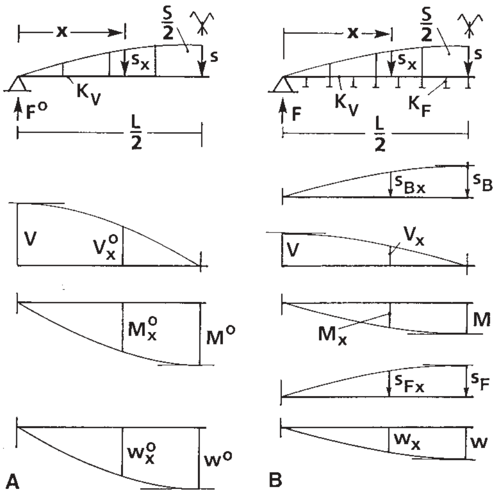

Fig. 7. A) One half of a simple shear beam subjected to a sine load, its reaction, its shear-force and bending-moment diagrams and displacement line. B) One half of the simple shear beam with an additional continuous elastic foundation subjected to a sine load, its end reaction, the beam load, its shear-force and bending-moment diagrams, the foundation load and the displacement line.

To formulate the problem, the basic Eqs. (43) must be supplemented by the requirement, that the sum of the beam's load intensity and the foundation's load intensity must at any x be equal to the total load's intensity,

$$s_{Bx} + s_{Fx} = s. \tag{48}$$

Combining Eq. (48) with the equilibrium condition of the vertical forces on an infinitesimal beam's cutout,

$$V_x' = -s_{Bx}, \tag{49}$$

the intensity of the foundation's load can be expressed as

$$s_{Fx} = s_x + V_x'. \tag{50}$$

The complementary energy of the system per unit length is

$$U = \frac{V_x^2}{2K_V} + \frac{1}{2K_F}(s_x + V_x')^2; \tag{51}$$

the first and the second term on the right-hand side represent the contributions of the beam and the foundation, respectively.

The Euler equation

$$\frac{\delta U}{\delta V_x} - \frac{\mathrm{d}}{\mathrm{d}x}\frac{\delta U}{\delta V_x'} = 0 \tag{52}$$

of the calculus of variations associated with the functional U yields, after the derivations are performed, the second-order differential equation

$$-V_x'' + \frac{K_F}{K_V} V_x = \frac{\pi}{L} s \cos(\pi \frac{x}{L}) \tag{53}$$

of the beam's shear force. The corresponding boundary requirements are that 1) at $x = 0$ the beam's load intensity vanishes, because there both the total load's intensity and the foundation's load intensity vanish and 2) at $x = L/2$ due to the symmetry the beam's shear force vanishes. So,

$$V_0' = 0, \;\; V_{L/2} = 0. \tag{54}$$

D-Eq. (53) and B.Eqs. (54) uniquely formulate the boundary-value problem dealt with.

The solution of D-Eq. (53) consists of a complementary and a particular part,

$$V_H = Z_1 \sinh(\sqrt{\frac{K_F}{K_V}} x) + Z_2 \cosh(\sqrt{\frac{K_F}{K_V}} x), \tag{55}$$

$$V_P = C \cos(\pi \frac{x}{L}). \tag{56}$$

The constant C is found by inserting V_P and its second derivative into D-Eq. (53) to be

$$C = R\frac{S}{2}, \tag{57}$$

where

$$R = \frac{1}{1 + (\frac{L}{\pi})^2 \frac{K_F}{K_V}}. \tag{58}$$

Herewith

$$V_P = R\frac{S}{2}\cos(\pi \frac{x}{L}). \tag{59}$$

Now the complete solution

$$V = V_H + V_P \tag{60}$$

of D-Eq. (53) is inserted into the B-Eqs. (54); it is easily shown that both constants Z_1 and Z_2 are equal to zero, so that V_H identically vanishes and thus

$$V = R\frac{S}{2}\cos(\pi\frac{x}{L}). \tag{61}$$

To check this result it is proven that V satisfies both the D-Eq. (53) and the B-Eqs. (54).

Herewith, the maximal values of the system's response are

$$s_B = Rs, \quad F = V = R\,V^{o}, M = R\,M^{o,} \tag{62}$$

$$s_F = (1 - R)\,s, \tag{63}$$

$$w = R\,w^{o}, \tag{64}$$

whilst at an arbitrary x there is

$$s_{Bx} = s_B\,\sin(\pi\frac{x}{L}), \tag{65}$$

$$V_x = V\cos(\pi\frac{x}{L}), \quad M_x = M\sin(\pi\frac{x}{L}), \tag{66}$$

$$s_{Fx} = s_F\,\sin(\pi\frac{x}{L}), \tag{67}$$

$$w_x = w\sin(\pi\frac{x}{L}). \tag{68}$$

Obviously, the response-reduction factor R translates the response quantities of the simple beam (Fig. 7A) into those of the beam with the additional continous fooundation (Fig. 7B).

Numerical example. As an engineering application of the developed theories let consider the bracing structure (fig. 5B) consisting of a longitudinal roof stressed-skin decking made of corrugated metal sheets, rigid gables and elastic transverse frames of a flat-roofed building subjected to a seismic load in the buildings transverse direction.

Data. Building's lenght: L = 36 m. Panel length: l = 4,5 m. The roof beam acts as a shear beam its cross-sectional stiffness being K_V = 67 MN. The lateral stiffness of a frame is K_R = 379 kN/m, so that the stiffness of the roof-beam's continous foundation is $K_F = K_R/l = 84{,}2$ kN/m^2. The roof's weight per unit building's lenght including a reasonable part of the snow load is $q = 30$ kN/m.

Free-vibration's period: $T_V^o = 0{,}487$ sec, $t_V = 0{,}924$, $T_V = 0{,}450$ sec.

Total seismic load. Assuming that it amounts to 5% of the governing roof's weight, there is $S = 0{,}05 \cdot 30 \cdot 36 = 54$ kN. Loads mid-span intensity: $s = 2{,}36$ kN/m^2.

Maximal response values of the reference beam: $F^o = V^o = 27{,}0$ kN, $M^o = 309$ kNm, $w^o = 4{,}62$ mm.

Response-reduction factor: $R = 0{,}858$. Hence the maximal values of the system's response are: $F = V = 23{,}2$ kN, $M = 266$ kNm, $w = 3{,}96$ mm, $s_F = 0{,}334$ kN/m.

7. Conclusion

Starting from the known results for the free-vibration's fundamental period of the simple flexure beam and applying the method of splitting the given system into subsystems, the free-vibration's fundamental period of the simple flexure beam with an additional elastic foundation is determined. For practical reasons, the result is expressed as a product of the period of the reference beam and a dimensionless reduction coefficient which depends on the system's parameters, i.e. its span, beam stiffness and foundation stiffness.

Analogously, starting from the known result for the free-vibration's fundamental period of the simple shear beam and again using the method of splitting the system into subsystems, the free-vibration's fundamental period of the simple shear beam with an additional elastic foundation is found.

Knowing the governing period, the total seismic load can be determined on the basis of the relevant code. With respect to its distribution along the system's length it is assumed to be a sine load.

In the analysis of the response of the flexure system to a sine load the shear forces, bending moments and displacements of the reference's beam are determined first. Then, the beam's load, shear forces, bending moments and displacements and the foundation load of the flexure system are found. It is shown that the beams response quantities are equal to those of the reference beam multiplied by a dimensionless reduction coefficient which depends on the system's parameters.

The response of the shear system is analyzed using the stationary complementary energy theorem. The problem is formulated by the second-order differential equation of the beam's shear force and the corresponding boundary conditions. The practical analysis algorithm is shown to be formally analogous to that of the flexure system.

The effect of shear in the flexure system and the effect of flexure in the shear system are introduced by appropriately defining the beam's cross-sectional stiffnesses.

An example of engineering structures to which the developed theories and the derived simple formulas can be applied are those of flat-roofed buildings having gables, cross frames and a longitudinal roof diaphragm formed by either a truss or a corrugated metal stressed skin decking. So the paper also intends to contribute to an easier design and preliminary and simple final dynamic analyses of some spatial building structures.

Table. Numerical values of the period-reduction coefficients t_M for flexure beams and t_V for shear beams with hinged end supports and a continuous elastic foundation

U	t_M	A	t_V
0	1	0	1
0,25	0,999	0,25	0,987
0,5	0,980	0,5	0,952
0,6	0,960	0,75	0,900
0,65	0,946	1	0,839
0,7	0,929	1,25	0,776
0,75	0,909	1,5	0,715
0,8	0,887	1,75	0,658
0,85	0,863	2	0,606
0,9	0,835	2,25	0,559
0,95	0,806	2,5	0,518
1	0,776	2,75	0,480
1,05	0,744	3	0,447
1,1	0,713	3,25	0,418
1,15	0,680	3,5	0,392
1,2	0,649	3,75	0,368
1,25	0,618	4	0,347
1,3	0,587	4,25	0,328
1,35	0,559	4,5	0,311
1,4	0,530	4,75	0,295
1,45	0,503	5	0,281
1,5	0,477	5,25	0,268
1,55	0,454	5,5	0,256
$\pi/2$	0,444	5,75	0,245

References

[1] Rogers, G.: Dynamics of Framed Structures. Wiley, New York, 1959

[2] Rosman, R.: Tragverhalten des Dachverband-Rahmen-Wände-Systems typischer Hallen. Stahlbau 67 (1998), 926-935

[3] Rosman, R.: Näherungsweise Lösung von Eigenwertaufgaben der Baumechanik durch Aufspalten in Teilaufgaben. Bautechnik 67 (1990), 375-382.

[4] Rosman, R.: Ein praktischen Verfahren zur Lösung von Aufgaben der Baudynamik. Beton- und Stahlbetonau 64 (1969), 161-167

[5] Rosman, R.: Tragverhalten des Systems Dachverband-Rahmen-Wände für typische Hallen mit Trapezprofildach. Stahlbau 70 (2001) 464-473

Vibracije i odziv na potresne uzbude savojnih greda i posmičnih greda oslonjenih na krajevima na zglobne ležaje a duž raspona na deformabilnu podlogu

Riko Rosman

SAŽETAK

Izvedeni su jednostavni obrasci za osnovne periode vlastitih bočnih vibracija te za odziv na sinusno opterećenje savojnih greda i posmičnih greda oslonjenih na krajevima na zglobne ležaje a duž raspona na deformabilnu podlogu. Pokazano je, da mehaničke karakteristike i odzivne veličine savojno-posmične grede mogu biti aproksimirane odgovarajućim veličinama savojne grede odnosno posmične grede ako se prikladno definiraju njene ekvivalentne karakteristike poprečnog presjeka. Nadalje je prikazana primjena razrađenih teorija u dizajnu i analizi nekih inženjerskih konstrukcija, a dva brojčana primjera objašnjavaju algoritam proračuna.

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