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Initial and time-dependent deformations in marl around small circular opening

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Preliminary note

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Initial and time-dependent deformations in marl around small circular opening

The basic property of standard laboratory tests is that they are conducted on samples of simple geometry and stress state. It is a common practice to use material parameters obtained from such tests in numerical models that describe more complex stress-dependent phenomena. For this reason, numerical modelling results have to be verified either through models or during construction of actual structures. The paper presents results of time-dependent deformations around the circular opening 10.8 cm in diameter, using the two-dimensional marl model measuring 60 x 60 x 10 cm

Key words:
creep, convergence, tunnel, model, circular opening

Prethodno priopćenje

Zvonko Tomanović

Inicijalne i vremenski ovisne deformacije oko malog kružnog otvora u laporu

Osnovna karakteristika standardnih laboratorijskih pokusa je ta da se provode na uzorcima jednostavnog geometrijskog oblika i stanja naprezanja. Uobičajeno je da se parametri materijala, dobiveni iz ovih pokusa, koriste u numeričkim modelima koji opisuju dosta složenije pojave ovisne o naprezanju. Zbog toga je nužno, na modelima ili tijekom gradnje stvarnih objekata, provjeriti rezultate numeričkih modela. U ovom radu prikazuju se rezultati deformacija u vremenu oko kružnog otvora promjera 10,8 cm, na dvodimenzionalnom modelu od lapora dimenzija 60 x 60 x 10 cm.

Ključne riječi:
puzanje, konvergencija, tunel, model, kružni otvor

Vorherige Mitteilung

Zvonko Tomanović

Anfängliche und zeitabhängige Verformungen von Mergelstein mit kreisförmiger Öffnung

Das Hauptmerkmal von Standardlaborversuchen ist, dass sie an Proben einfacher geometrischer Formen und Spannungszustände durchgeführt werden. Die durch solche Untersuchungen erhaltenen Materialparameter werden üblicherweise in numerischen Modellen angewandt, um bedeutend komplexere spannungsabhängige Erscheinungen zu beschreiben. Es ist daher notwendig, aufgrund von Modellen oder im Laufe des Aufbaus wirklicher Objekte, die Resultate numerischer Analysen zu überprüfen. In dieser Arbeit werden Resultate zeitabhängiger Verformungen um eine kreisförmige Öffnung mit 10,8 cm Durchmesser anhand des Modells einer 60 x 60 x 10 cm großen Mergelsteinprobe dargestellt.

Schlüsselwörter:
Kriechen, Konvergenz, Tunnel, kreisförmiger Öffnung

1. Introduction

In the past few decades, significant advancements have been made in the technology of tunnel construction and also in the design, numerical modelling and dimensioning of the tunnel support structures. Despite advanced construction and calculation technologies that are currently in use, the verification of balance established between the rock pressures and forces in the support structure is performed during construction, based on measurement of displacements of the tunnel opening contour. Namely, the excavation of a single tunnel sequence is immediately followed by the excavation contour displacement in radial direction, as a consequence of deformations caused by the change of stress in the rock mass. In addition to this instantaneous deformation, an increment of radial deformations of the excavation contour is observed over time, and the intensity of this deformation mostly depends on the type of rock in which the tunnel is constructed and the applied support structure.

The time-dependent behaviour of rock mass, and the yield of radial displacements of the excavation contour, have been observed soon after a wider application of flexible tunnel supports made of sprayed concrete, reinforcing meshes, and anchors [1, 2]. The flexible support structure itself also displays a significant time-dependent behaviour [3, 4]. Initial attempts to assess the time-dependent excavation contour displacements through application of theoretical solutions have not provided satisfying results. The issue of calculation and assessment of time-dependent deformations has been solved by application of empirical formulas which, in case of tunnels, have provided economically and technically acceptable results over a certain period of time. Rapid advancements in computers and numerical methods over the past few decades have led to the development of complex rheological models [5-8], numerical procedures, and software, which enabled better understanding of time-dependent behaviour of the support structure and rock mass around the tunnel opening [9, 10].

When developing mathematical models, it is also necessary to define characteristic rheological material coefficients and parameters. Generally, the material parameters and constants can be obtained from laboratory tests or in-situ tests. However, the definition of rheological behaviour of the rock mass is impeded by significant problems relating to the establishment of exact material constants and parameters that exist in the rheological model. The difficulties in defining material parameters come as a consequence of a specific structure of the rock material itself: discrete mineral composition, stratification plane, discontinuity, etc. In addition, the behaviour of both the rock matrix and the discontinuity has to be defined.

Conventional laboratory and in-situ tests typically involve specimens of simple geometry and a more or less simple state of stress. However, in different problems encountered in the actual rock mass, the geometry and stress state usually largely differ from the laboratory or in-situ tests. Therefore, in the analysis of the stress-deformation state around tunnel openings, it is a common practice to use material parameters obtained from simple tests in mathematical-rheological models, which describe far more complex stress-dependent phenomena, and other occurrence related to rock mechanics.

Differences between laboratory tests and actual stress-strain phenomena under study can cause, in terms of geometry and stress state, minor or major discrepancies between mathematical modelling results and the actual state of stress-strain. Therefore, measurement results obtained by simple conventional tests must be verified through models or through construction of actual structures, and this primarily to identify any major phenomenological discrepancies and, secondly, to check the accuracy of stress-strain calculations. Results of time-dependent deformations around the circular opening of 10.8 cm in diameter are presented below using the two-dimensional marl model measuring 60 x 60 x 10 cm, and the comparative analysis with creep results for uniaxial specimens is described.

2. Research objective

The behaviour of rock from the aspect of phenomenology, and under conditions of time-dependent deformations, must be determined to enable proper numerical calculation of the stress-strain state in the rock around the tunnel opening. It is also significant and necessary to provide proper material parameters of the rock matrix itself, this being the objective of experimental research. Uniaxial creep tests enable definition of material constants and parameters of the marl as a representative soft rock, which feature in the rheological model. An insight into phenomenology of the time-dependent behaviour around small circular openings can be gained through the study of time-dependent deformations around small circular openings on the 60 x 60 x 10 marl model exposed to bi-axial stress. Such behaviour can then be compared with the behaviour of the uniaxially compressed specimen.

The experimental research on marl specimens under conditions of short- and long-term loading and unloading must be performed due to insufficient experimental information about the behaviour of soft rocks and material parameters that are necessary to establish proper numerical models for stress and strain dependent phenomena in the rock mass. Namely, current rheological models and software based on such models are still unable to provide good estimation of time-dependent deformations in the rock around the tunnel opening, i.e. the estimation of the increment of radial movements at the the tunnel opening contour after excavation.

The rationale for the research of the topic under study lies in the need to develop numerical models for modern excavation processes and for construction of support systems of underground openings, as based on the interaction between the rock mass and the support structure in the time-dependent conditions, and to also enable estimation, already in the design phase, of the expected deformations in the excavation phase and during subsequent use of the structure. Analysis of time-independent (short-term) and time-dependent deformations (creep) represents a focal point of this research, and constitutes

the basis for establishment of numerical models of the stress-dependent phenomena in rock mechanics.
Results of this research are expected to reveal the phenomenology of the stress-strain behaviour of the rock material under short-term and long-term loading in the conditions of uniaxial and bi-axial compression. Experimental measurements should assist in defining a realistic range of material parameters of the rock material, and in developing proper empirical dependencies, by which it would be possible to approximate stress-strain relations at the short-term loading and the creep of the rock matrix in the long-term load conditions, and after partial or complete unloading.
A comparative analysis of the uniaxial creep test results and the results about the time-dependent behaviour of rock around the small tunnel opening on the model, should enable estimation of the possibility to use simple laboratory tests for the purposes of numerical models that would describe much more complex situations in terms of geometry and stress.

3. Research limitations

The analysis is limited to the scope in which the rock mass behaviour can be taken as quasi-elastic (visco-elastic) during the short-term loading at room temperature, and with a limited change of natural moisture. The quasi-elastic behaviour of rock means that deformations are fully reversible at the short-term loading and unloading, when the stress state is below the yield limit. In case of the short-term loading, the deformations also include the delayed – plastic component of deformation, although the state of stress has not reached the yield criterion as at the "short-term" loading (from several minutes to several hours). Consequently, the yield criterion at the long-term loading (several days, weeks or months) is lower than at the "short-term" loading. When time-dependent deformations occur even at the very low state of stress, then a zero yield stress criterion can be applied.
The scope of research is limited to the stress-strain behaviour of the rock below the yield surface at the short-term loading for two reasons. The first reason for this limitation is the fact that the stress state being the subject of the research is also characterized by the highest number of occurrences in the rock around the tunnel opening. Namely, if the stress state in the rock around the tunnel opening is below the yield surface, the tunnel opening should be supported or the lining that is not designed to take over any major force should be provided. When the yield surface is reached in the rock around the opening, the support structure is built to take over a portion of unbalanced forces in the rock, thus bringing the stress state in the rock below the yield surface and establishing the balance of forces.
The second reason for conducting this research is the fact that the described state of stress, which is below the yield surface, but exceeds the level for a correct approximation by linear elastic model, has not so far been sufficiently studied. Thus, the theory of elasticity was "well-rounded" in the last century. The theory of plasticity, and failure criteria in particular, have attracted a lot of interest over the centuries, which has resulted in a large number of theories – laws that provide a satisfying description of the behaviour of different materials after reaching the yield surface, and of the failure criterion itself.
The area that can be defined as a field of working-operating stresses, Figure 1, i.e. the area in which time-dependent deformations occur, represents a far less researched area of rheological behaviour of rock mass and materials in general. Namely, after establishment of satisfying theories with regard to material failure, it was possible to define safety factors to enhance structural safety or, in this particular case, to increase the safety of rock structure against failure. The differences between the computed state of stress (more or less based on the theory of elasticity) and actual state of stress have been solved by introduction of different factors of safety for different materials and types of structures, which has provided satisfying results in practice.

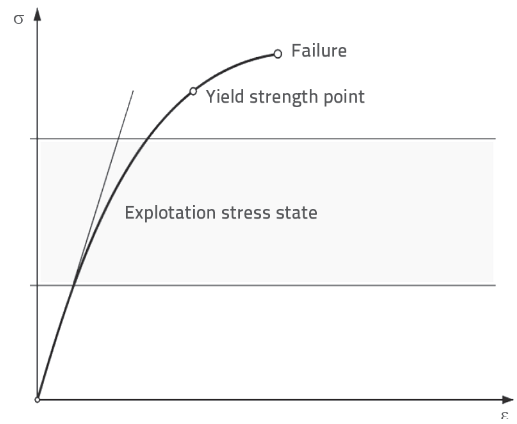

Figure 1. Stress-strain diagram

4. Laboratory tests

By definition, the basic requirement of the creep test is to maintain a constant stress state over a longer period of time. The uniaxial and biaxial specimen testing was conducted for the purposes of studying the marl creep phenomena. The marl was selected as a representative material of the soft rock group in order to study the very phenomenology of behaviour, and to obtain material parameters of a rheological model, which describe deformation behaviour of this material under long-term load.
According to its chemical content, the tested marl contains 48.10-48.30 % of $CaCO_3$, while the content of insoluble residues (clayey + quartzite residues) varies from 51.03 to 51.87 %. Regarding the mineralogical content, dominant mineral phases are the calcite (46-48 %) and quartz (12-13 %), while the clayey phase is formed of the illite, smectite, montmorillonite, kaolinite, glauconite, transformed feldspar, and mica. The sample moisture ranged from 8 to 11 %, and the uniaxial strength of the material amounted to approximately σ_c = 8,8 MPa.
Uniaxial creep tests were performed on prismatic specimens (15 x 15 x 40 cm) while biaxial creep tests were performed on plate-shaped specimens (60 x 60 x 10 cm). The uniaxial tests were performed after the loading, and the stress relaxation creep tests were performed after the complete or partial unloading. In the first phase, biaxial tests were performed at different vertical and

horizontal compression strain ratios. In the second phase, circular openings of 10.8 cm in diameter were cut into plate-shaped specimens, thus modelling the tunnel excavation in the rock mass, while maintaining the constant load as applied in the first phase of the test. This enabled the study of the stress-strain state around the small opening in the rock under conditions of the short-term (time-independent) and time-dependent deformations.

4.1. Uniaxial test

The most commonly used and the simplest creep test is performed on cylindrical or prismatic specimens at an uniaxial state of stress. The rheological model formulation for the purposes of this research is based on creep tests performed on uniaxial prismatic specimens of marls – soft rock.

The test was performed in three phases: loading, unloading, and reloading to a higher level of stress, with the value maintained constant after stress change. Prismatic samples measuring 15 x 15 x 40 cm were used for the uniaxial creep test. They can be considered as isolated prismatic rock segments in the vicinity of the tunnel excavation contour, according to the scheme presented in Figure 2, where the stress state is approximated to the uniaxial stress [8, 11].

4.2. Plate-shaped specimen testing

The test on plate specimens was conducted using a load device consisting of the primary frame for applying dead-load, and the secondary frame for applying horizontal load (Figure 3). Plate deformations were measured by applying a grid of measurement points (nodes) pertinent to both sides of the plate specimen, consisting of interconnected triangles, which allowed measurement of deformation fields in the plate plane through all test phases. The measuring base network of 100 mm was placed over the entire surface of the specimen, while the measuring base network of 60mm was formed in the central part of the specimen, as illustrated in Figures 2 and 3. A network with higher density of measuring points covered the immediate area around the opening, which was bored in phase 2 of the test. Measurement of deformation on the sides of the network triangles enables measurements in any direction required.

Strain measurements were conducted using the mechanical deformeter Pfender (displacement readings 1 / 1000 mm on the 100 mm measuring base). After long lasting tests, the measurement error of 2-3 / 1000 mm was identified for this type of device. Measured changes in distance between reference points during the creep testing of marl varied from 0.5 to 1 mm. Taking into consideration strain measurements, the measurement error of 3 / 1000 mm can be considered as acceptable.

In the first phase of the plate specimen testing, the uniaxial and biaxial loads were applied to the plate in its own plane (Figures 4.a.). The load was applied in 0.5 MPa increments for an hour until the vertical stress of 2.0 MPa was reached. This was about 25 % of the peak strength of the examined marl. Three specimens were loaded uniaxially, while the ratio, $k = \frac{\sigma_h}{\sigma_v}$, between horizontal and vertical load components was varied in the following three groups of specimens: 0.3, 0.5 and 1.0. The initiated stress state (with the constant ratio maintained between horizontal and vertical stresses) was retained in the following 45 days, with measurement of deformation fields on both sides of the plate specimen.

The first phase of the test was designed to determine lateral stress effects on the creep in vertical direction, and to determine variations of the measured deformations (instantaneous and time-dependent) depending on the shape of specimen, i.e. prismatic – plate shaped. Plates loaded biaxially in the first phase of the test can be considered as isolated segments of rock at an arbitrary distance from the tunnel excavation at different horizontal to vertical stress ratios, Figure 2.

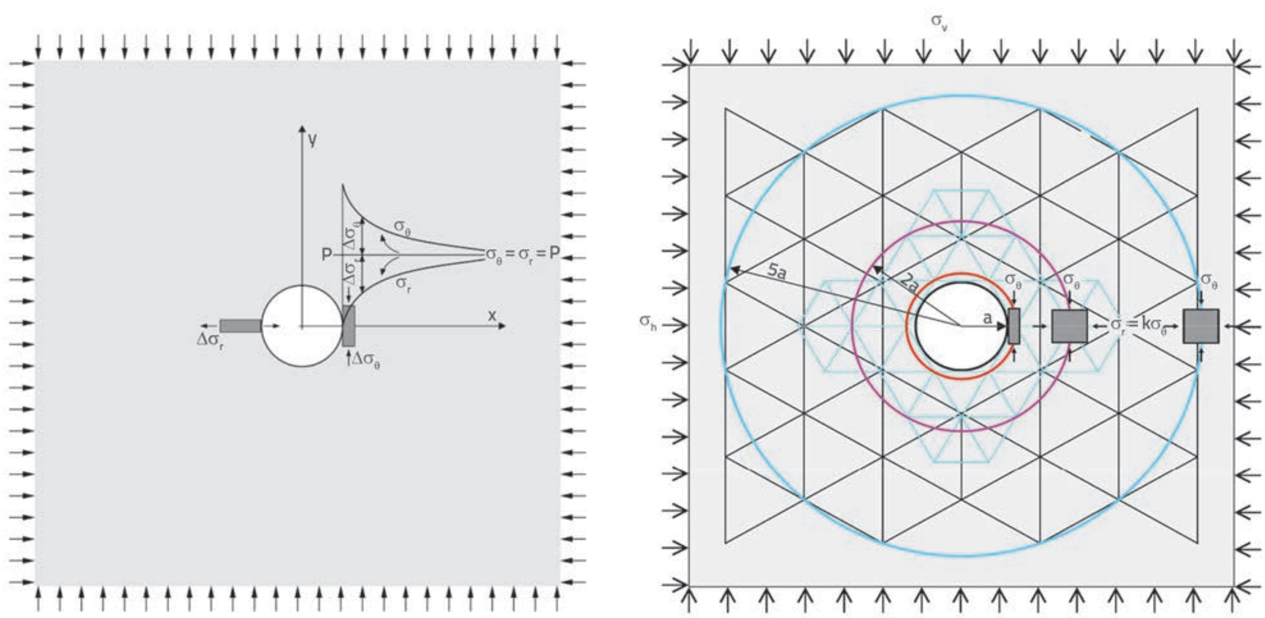

Figure 2. Stress state around the circular tunnel excavation

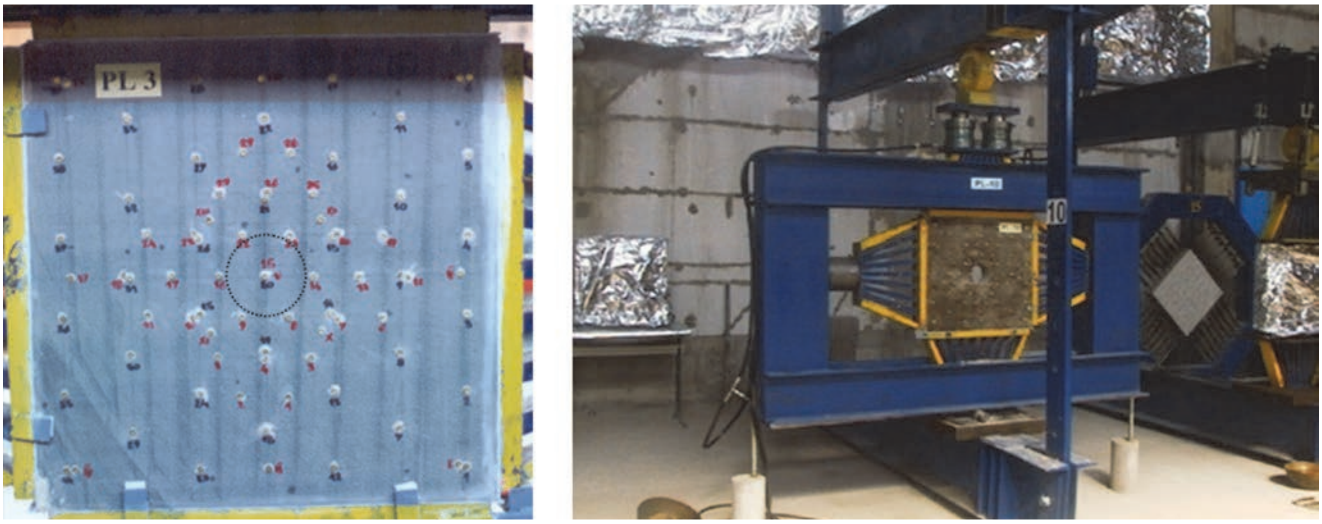

Figure 3. Biaxial loading apparatus: a) first and second test phases; b) third test phase

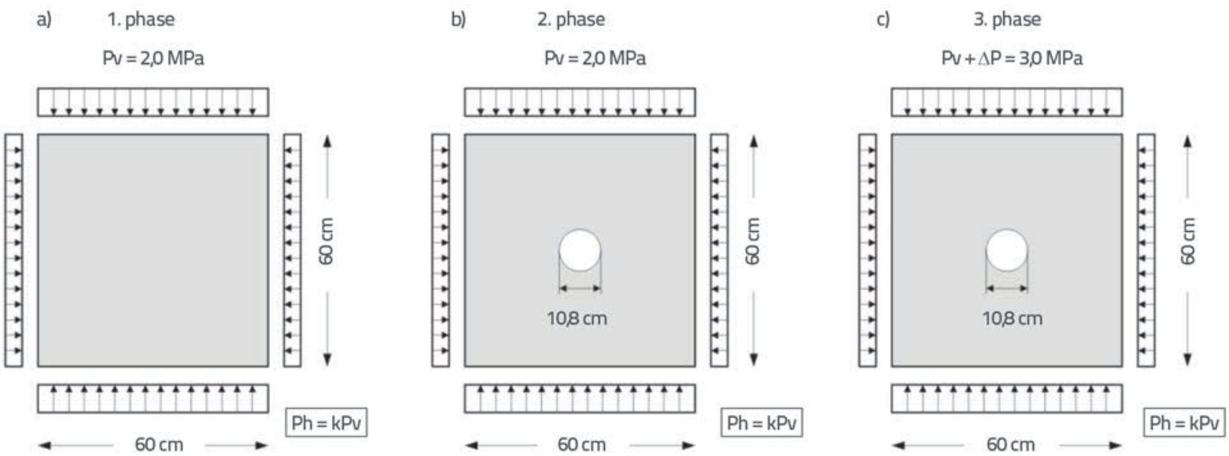

Figure 4. Phases of biaxial stress testing

The simulation of excavation in the rock mass was assumed in the second phase of the testing; a circular opening 10.8 cm in diameter was bored into the plate specimen as illustrated in Figure 4.b. After the opening was made, the model was maintained under initial stress state for the ensuing 30 days. Instantaneous deformations after the opening boring should point to the zone of changed stress – stress rearrangement around the opening and lateral stress (in this case radial stress) effects on axial deformation. The deformation field, i.e. the material creeping in the zone around the opening was monitored by measuring deformation over the period of 30 days.

In the third phase of the test (Figure 4.c.), the model was additionally loaded by 1.0 MPa in vertical direction and the corresponding ratio was used to apply the load in horizontal direction. After inducing the additional load, the model was maintained at the new stress level over the next 45 days. This test phase simulated the stress increment which could be induced by excavation of the next section of the existing tunnel tube in the course of construction.

After the stress state is changed, the creep deformations are measured on the network of measurement points (in all three phases of the test). The measurements are performed at 1, 6 and 24 hours after application of load, then after 3 and 7 days, and then after every 15 days in the ensuing period. Measurement intervals were selected to approximately even out the differences of the previous and currently measured deformations.

5. Initial - short-term deformations around circular opening

Prior knowledge of the initial secondary stress state around the opening is needed for proper analysis of the time-dependent deformations around the opening, as the intensity of the time-dependent deformations largely depends on the stress state. Taking this into consideration, Section 5 deals with the analysis of the secondary stress state around the small circular opening, and defines its intensity and distribution in order to enable the analysis and interpretation of time-dependent components of deformation that will occur once the opening is drilled through the rock material.

Once the small circular opening is made in the plate shaped specimens, in the second phase of the test, the specimens take on characteristics of the model of tunnel opening in the rock mass. Although it involves a simple circular shape of the opening, the model appears to be a very suitable basis for studying the stress-dependent phenomena, and for verifying basic assumptions of the constitutive model that is formed based on uniaxial tests. In addition to this, the circular opening enables a comparative analysis of measurement results and generation of solutions provided in the traditional theory of elasticity for stress-strain deformations around a circular opening.

The results of measurements performed on the model can directly be applied only for the analysis of stress relaxation in the boreholes, for the study of primary stress in the rock mass, or for similar analyses. On the other hand, in the analysis of stress state around the tunnel opening of realistic dimensions and in a natural rock mass, the results have a qualitative significance and point to the stress-dependent phenomena and processes occurring around the tunnel opening [12-14].

It should be noted that the load is applied on the plate-shaped specimen in its own plane, which means that the plate face is not loaded. Hence, the model corresponds to the plain state of stress. The plain stress state is not a common stress state in the rock mass. In fact, the stress state which is approximate to the plain state of deformations is more frequently observed. The stress state in the rock mass around the tunnel opening is approximate to the stress state in the model concerned only in the case when excavation is carried out perpendicular to the vertically oriented rock layers, with open discontinuities. This idealisation had to be made as it was physically impossible to make simultaneous measurements of displacement and application of load on the model face to establish the brittle stress state or simulate the plain stress state of deformation, which would correspond more to the "in-situ" state.

The impact of the third component of stress, perpendicular to the midplane of the plate, can be considered and quantified through the diagrams of impact of lateral stress under the rotationally symmetric load exerted via the traditional tri-axial compression test [8]. In the actual rock mass, the real stress state around the tunnel is to be found between these two boundary cases.

The boring of the openings disturbs the primary stress field, as induced by the lateral loading of the plate-shaped specimen in the first phase of the test, and induces the secondary state of stress. The stresses around the opening increase in the tangential direction on the contour, and decrease in the radial direction, Figure 2 (left). Results of the comparative analysis of the stress-strain around the circular opening are presented below, and this by application of the Kirsch's theoretical solution for a homogenous, isotropic and linear-elastic material: numerical solution with application of the stress-dependent modulus of elasticity and measured deformation values on the plate-shaped model under the same vertical and horizontal primary compression strain.

5.1. Kirsch's analytical solution for stress state around circular opening in homogenous, isotropic and elastic material

The Kirsch's analytical solution for the state of stress and strain around the circular opening in a homogenous, isotropic and elastic material is well known in the literature. In addition to the assumption that $a << z$ (where a is the the radius of the opening, while z is the depth from the surface to the axis of the tunnel-opening, Figure 5), the basic assumption of the solutions is that the linear-elastic material is described by Young's modulus of elasticity E and Poisson's ratio ν. Thus the solution relies on the traditional theory of elasticity that is based on the constant material parameters, i.e. on parameters that are not dependent on the stress state in the material. In cylindrical coordinates, Kirsch's equations for the component stresses read as follows:

$$\sigma_r = \frac{P_v}{2}\left[(1+\lambda)\left(1-\frac{a^2}{r^2}\right)+\left(1-4\frac{a^2}{r^2}+3\frac{a^3}{r^3}\right)\cos 2\theta\right] \quad (1)$$

$$\sigma_\theta = \frac{P_v}{2}\left[(1+\lambda)\left(1+\frac{a^2}{r^2}\right)-(1-\lambda)\left(1+3\frac{a^4}{r^4}\right)\cos 2\theta\right] \quad (2)$$

$$\tau_{r\theta} = -\frac{P_v}{2}(1-\lambda)\left(1+2\frac{a^2}{r^2}+3\frac{a^3}{r^3}\right)\sin 2\theta \quad (3)$$

where:

a - radius of circular opening,

$P_v = \gamma z$ - vertical compression of the dead weight of material,

$\lambda = P_h / P_v$ - ratio between horizontal and vertical primary stres,

r, θ - cylindrical coordinates of the point concerned.

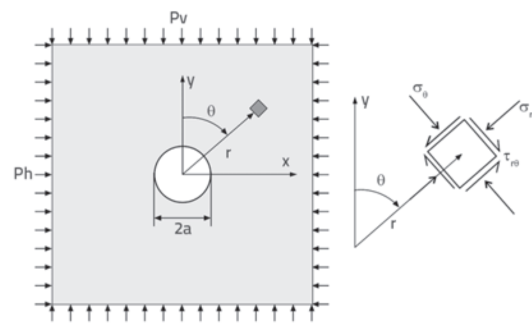

Figure 5. Geometry and marks of the Kirsch's analytical solution

Analytical solutions of this type are presently most often used for calibration of numerical models, that is, for verification of accuracy and for finding an optimal structure of the finite elements network, by applying software packages for the stress-strain analysis. The Kirsch's analytical solution is presented herein for the purpose of comparison of the results obtained by means of the classical theory of elasticity (based on Young's modulus), with the results obtained using the stress-dependent modulus of elasticity, and with the results obtained b< measurements conducted on the model.

5.2. Numerical solution for stress state around circular opening at stress-dependent modulus of elasticity

The numerical model of the plate-shaped specimen with circular opening was formed using the software package SAP2000, and was established on the basis of constant material parameters, i.e. Young's modulus of elasticity, Figure 6. However, at the triaxial state of stress, the modulus of elasticity depends on lateral stresses in certain directions, and so the moduli of elasticity that are function of the lateral stress are introduced in the analysis, [8, 12].

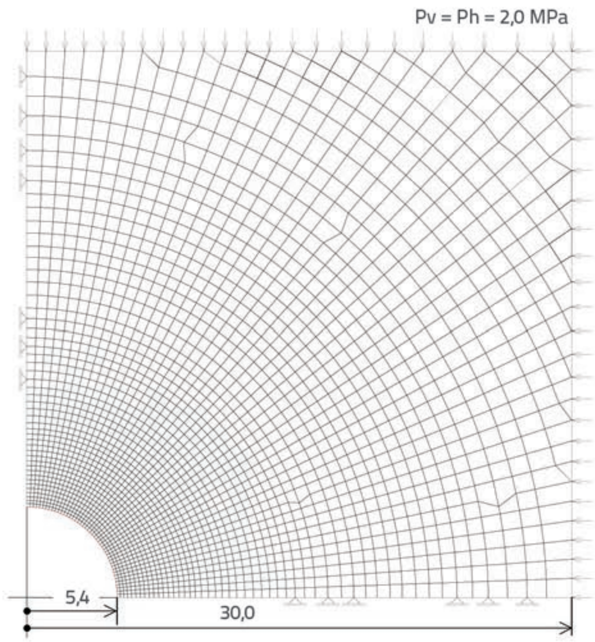

Figure 6. Finite elements grid for calculation using software package SAP2000

Dependencies of the initial modulus of deformability on the lateral stress become more relevant in case of the spatial state of stress, even in the range of the low stress state, when the stress-dependent modulus of elasticity can be expected. The initial modulus of elasticity of marl that is obtained at the unrestrained lateral expansion (meaning Young's modulus) is nearly two times smaller compared to the initial modulus of the lateral stress of 2.0 MPa (2.0 MPa represents approximately 25 % of the uniaxial peak strength).

When the Young's modulus *E* in the stress-strain relation for the linear-elastic material is replaced with the stress-dependent modulus of elasticity $E(\sigma_3)$, the following equation is obtained:

$$\sigma = E(\sigma_3)\varepsilon \tag{4}$$

Where (σ_3) can be the initial modulus of elasticity, tangential or secant modulus of deformability defined by equations.

Generally, if the assumption on linear-elasticity is abandoned, a non-linear relation between the stress and strain can be established as indicated below:

$$\varepsilon^n = k^n P_{(\sigma_3)} \left(\frac{\sigma}{\hat{\sigma}}\right)^m \text{ ili } \left(\frac{\sigma}{\hat{\sigma}}\right)^m = \frac{1}{k^n P_{(\sigma_3)}} \varepsilon^n \tag{5}$$

where *n* and *m* denote material parameters that are defined based on the experimental results, $\hat{\sigma}$ = 1,0 MPa (normalization), *k* means the stress state coefficient, and $P(\sigma_3)$ denotes a function that is dependent on lateral compression and, for the marl concerned, it reads as follows:

$$P_{(\sigma_3)} = 0.00022\left(\frac{\sigma_3}{\hat{\sigma}}\right)^{3.5} - 0.03453\left(\frac{\sigma_3}{\hat{\sigma}}\right) + 0.14727 \tag{6}$$

Strain - stress correlation:

$$\varepsilon_1 = \left[P_{(\sigma_3)}\left(\frac{\sigma_1}{\hat{\sigma}}\right)^3\right]^{\frac{1}{2}} \tag{7}$$

The diagram in Figure 7 shows curves of the dependence σ-ε according to the given empirical formula (5), and measured values obtained by means of a conventional triaxial apparatus. The empirical curve approximates well real deformation values practically up to the level of failure stress when $\sigma_3 \leq$ 4,0 MPa. When $\sigma_3 \leq$ 4,0 MPa (about 50 % of uniaxial strength), the empirical formula applies to limited stress levels defined by the line C-C. The empirical formula (5) developed for the description of stress-strain relationships is a smooth, continuous and differentiable function, and it can easily be incorporated into numerical procedures [8].

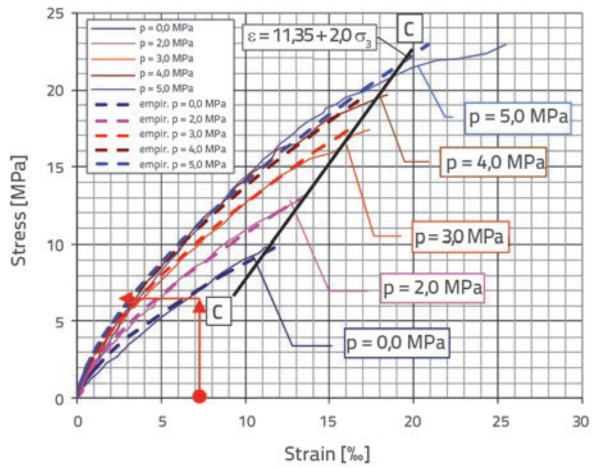

Figure 7. Comparison of stress-strain dependence of marl according to empirical formula and measured values (p-lateral cell pressure) [8]

The effect of the stress-dependent modulus of elasticity was incorporated iteratively. The first computation step involved a constant modulus of elasticity, and the results of the stress state were used for computing the stress-dependent modulus of elasticity, using formulas (4) to (7) and including the radial stress calculated in the previous step (see figure 7), as well as in every subsequent iteration, in each finite element, with different stress state. The increase of stress around the circular opening at rotationally symmetrical state of primary stresses occurs in the tangential direction with reference to the contour of the opening, while the decrease of stresses occurs in the radial direction and on the opening contour their value is zero, as shown in the diagram given in Figure 8. In the analysis of stress state around the circular opening, the smaller main stress is the radial stress, and hence it was adopted in the analysis that $\sigma_3 = \sigma_r$, by which the stress-dependent modulus of elasticity is defined, as presented in equation (2). A similar procedure was used to calculate stress based on strain measured on plate-shaped specimens in the second phase of the test, where the stress was obtained from known measured strain on the model, while the radial stress was obtained from numerical model, using formulas (4) to (7), Figure 7.

The diagram in Figure 8 gives a comparative presentation of the Kirsch's solution for stress for the geometry of experimentally treated plate-shaped specimens, at the ratio of primary stresses λ = 1,0, and the results of computation on the numerical model at stress-dependent modulus of elasticity, as described above. The radial stress increases from zero on the opening contour and rapidly approaches the primary stress value (at nearly 6·a, where a is radius of the opening). With the decrease of radial stress, we can also observe the change of the modulus of elasticity (it drops from the initial value of 1.37 GPa on the model contour to 0.87 GPa in the zone of the opening contour), as illustrated in Figure 8. Therefore, the rigidity of material relatively increases with the distance from the opening, which results in significant redistribution of stresses compared to the theoretically homogenous material by Kirsch, i.e. the traditional theory of elasticity. This effect is well known and involves creation of the zone of plasticisation around the opening, although it evidently exists even before occurrence of the zone of plasticisation, when the stress-dependent elastic moduli are taken into consideration.

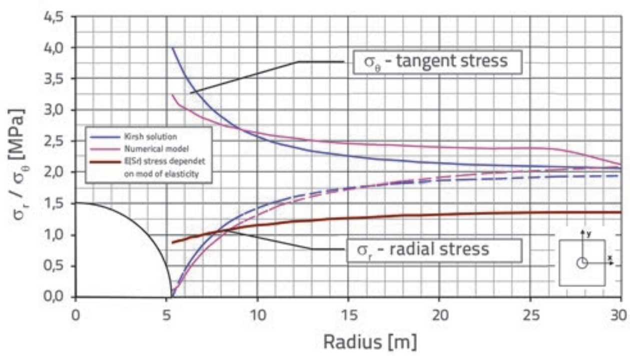

Figure 8. Comparative diagram of tangential and radial stresses according to Kirsch's solution and numerical model

Tangential stresses computed according to the numerical model are smaller than Kirsch's in the vicinity of the opening, but they become bigger as the radius exceeds the value of 1.75 *a*. The stress-dependent modulus of elasticity in the tangential direction does not exhibit any significant impact on radial stresses. Similar results were presented by Santarelli and others in their study of stress around the borehole [12].

5.3. Stress state in rock after boring the opening in plate-shaped model

Numerical model relies on the finite element method, and the iterative procedure of introducing the stress-dependent moduli of elasticity is conducted based on empirical correlations (3) and (4). As the stress state on the plate-shaped specimen is similar to the plain stress state $(\sigma_3 \cong 0, \sigma_1 > \sigma_2)$, the coefficient k in formula (3) has to be corrected. The stress state coefficient k is defined based on the criterion that the surface of the tangential stress diagram, at the secondary stress state, must correspond to the surface at the primary stress state. Namely, during the test, strain were measured in tangential direction at different radial distances from the centre of the opening, and by applying the formula (3), the tangential stress is calculated based on known strain in such discrete points (see Figure 6). The value of coefficient k = 0.6 is defined based on the criteria that the surface of the tangential stress diagram for θ = 90° must be equal to the diagram surface at the primary stress state.

Figure 9 shows a comparative diagram of tangential stresses according to the Kirsch's solution, numerical model at stress-dependent moduli of elasticity, and measurements performed on the model. The Kirsch's analytical solution, which relies on the Young's modulus, offers overrated values for tangential stresses in the vicinity of the opening contour. If the model also includes the effects of the stress-dependent modulus of elasticity, a solution can be obtained that significantly approaches tangential stress values computed from the measurements performed on the model.

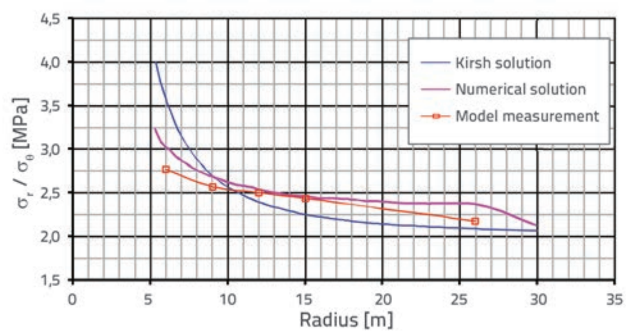

Figure 9. Comparison of tangential stresses according to the Kirsch's solution, numerical model at stress-dependent moduli of elasticity, and measurements performed on the model

The purpose of the tests performed on plate-shaped specimens subjected to short-term loading is to verify results obtained by numerical modelling, and to define the most influential parameters and phenomena that are incorporated in the mathematical model. The comparative analysis of solutions which were obtained based on mathematical models and model measurement results, suggests that significant improvements in the stress-strain analysis of the rock around the tunnel opening can be achieved if the stress-dependent moduli of elasticity are incorporated in the mathematical model.

In addition to this, the analysis of diagrams 8 and 9 suggests that the errors due to disregard of the stress-dependent moduli of deformability exceed by far the errors that occur due to linearization of the stress-strain correlation.

6. Time-dependent deformation test results

6.1. Creep test on uniaxial prismatic specimens

The creep test on uniaxial specimens was conducted in two phases. The first phase involved application of load up to the targeted stress level, and then the constant load was maintained for approximately 180 days. The second phase involved complete or partial unloading and measurement of deformations by relaxation creep during approximately 30 days, as shown in Figure 10.

Average deformation values obtained during the creep test conducted on prismatic specimens subjected to axial stresses of 2.0MPa and 4.0MPa, are shown in Figure 10. The diagram clearly shows the non-linear zone of intensive creep of material in the axial direction over the first twenty days after the loading. After the zone of intensive non-linear creep, the deformations are smaller and their increase is almost linear.

After the unloading, in the second phase of the test, the instantaneous reverse deformation is first observed, which is followed by development of time-dependent deformations. Completely unloaded specimens show convergence of the reverse time-dependent deformation. Partially unloaded specimens exhibit the reverse creep deformation in the first seven to ten days, and thereafter the change occurs in form of the time-dependent deformation, i.e. the creep of the material occurs under the impact of the remaining stress. Such behaviour is fully described by the rheological model (for more details see, [8, 15].

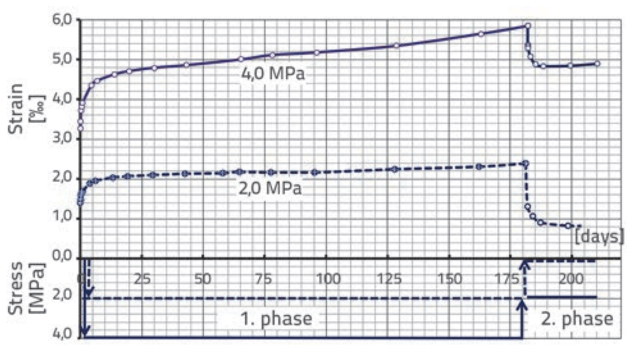

Figure 10. Creep diagram of axially compressed prismatic specimens

6.2. Time-dependent deformations around circular opening on plate-shaped specimens

A comparative analysis of time-dependent deformations in the rock around the opening in the plate-shaped model, and time-dependent deformations developed on rock specimens, should provide an answer to the question of whether there are significant phenomenological differences, and if the material parameters obtained by testing on specimens can be used for formulation of a rheological model. For this purpose, an analysis was conducted to determine time-dependent deformations within the model in the tangent direction at the radial distance of ρ ≅ 1a, 2a and 5a (concentric circles shown in Figure 2) in the rotationally symmetric state of primary stresses. The diagram of total deformations of the plate-shaped model in the tangent direction, in the rotationally symmetric field of primary stresses ($k = \frac{\sigma_h}{\sigma_v} = 1{,}0$), during three phases of the test, is shown in Figure 11. In the first phase of the test, when the opening was not as yet made, an instantaneous deformation developed as a result of the change in stress, at an even vertical and horizontal stress level (2.0 MPa). The strains were obtained from the changed length of measured deformations of the volumes of individual circles illustrated in Figure 2. Time-dependent even deformations developed at all radii in the first phase of the test, after application of initial load (the differences mainly result from discrepancies of initial deformations in the horizontal and vertical directions).

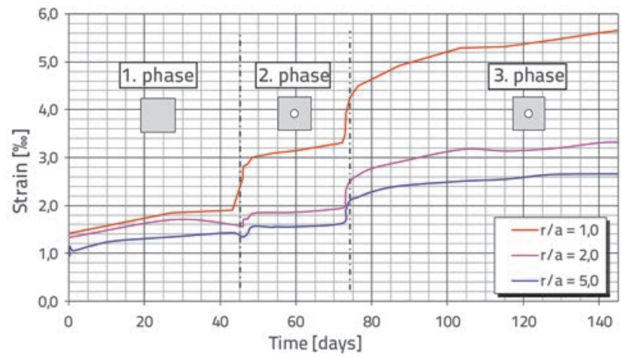

Figure 11. Total deformations of plate-shaped model (PL-3) in tangent direction at radial distance of ρ ≅ 1a, 2a and 5a with circular opening at different test phases

After the opening was bored in the second phase of the test, the instantaneous deformations were induced as a result of the changed state of stress. The instantaneous deformation is significant only in an immediate vicinity of the opening contour, and it rapidly drops with an increase in radius. Hence, after the radius ρ ≅ 2a, a slow increase in initial deformation can be observed. Time-dependent deformations in the tangential direction reveal the stress-dependence characteristics that are similar to those observed for prismatic specimens at the uniaxial creep test. Namely, the tangential stress increases in the vicinity of the opening and decreases with an increase in distance from the opening. In line with the stress state, the rate of increase of the time-dependent deformation grows faster in the zone closer to the opening compared to the zones that are on the larger radius where the stress increase is smaller (in compliance with the tangential stress show in Figure 9).

After the compression load was increased from 2.0 MPa to 3,0 MPa on the model in the third phase of the test, the same effect on the stress state was observed as in the case when the opening was bored (stress increase and faster increase of the time-dependent deformation close to the opening). Therefore, from the phenomenological aspect, the time-dependant rock deformations around the opening in the tangent direction, where the stress increases due to the boring of the opening, show almost the same stress-dependence as that observed for uniaxial tests on prismatic specimens.

7. Conclusion

The research presented in this paper is limited to stress-induced deformations (short-term deformations) and time-dependent deformations of rock at the constant state of stress, which is below the "short-term" yield surface. The analysis focused on the behaviour of the rock monolith (rock matrix) with micro fissures and crevices, but the crashed rock mass with the effects of significant volumetric change was not the subject of thorough considerations. The research presented in the paper is oriented toward defining material parameters on the basis of simple tests, which would enable estimation, during the design process, of the time-dependent stress-strain state during subsequent phases (tunnel excavation and operation).

The comparative analysis of the secondary stress results, obtained according to the traditional theory of elasticity, through application of the stress-dependent moduli of elasticity and based on the measurements performed on the plate-shaped specimen, suggests that there are significant discrepancies between the stress state that is calculated by means of the traditional theory of elasticity, and the values measured on the plate-shaped specimen. The margin of error, as a consequence of linearization of the stress-strain correlation at the exploitation stress level, is negligible compared to the error made by disregarding the impact of the lower main stress level on the deformability, i.e. on the initial modulus of elasticity.

The comparative analysis of the short-term and time-dependent deformations by the uniaxial creep test, and deformations around the small circular opening on the tunnel model, points to similar shapes of the creep diagram. Although the stress state around circular opening is more complex, the phenomenological time-dependent response of material is quite similar to the response of material in the simplest uniaxial state of stress. Contribution of this research is limited to the stress level below the material yield limit and behaviour of the rock matrix. Notwithstanding such limitations, the presented experimental data suggest that the results of uniaxial creep tests can be used, in terms of practical application, for the analysis of time-dependent deformations around the tunnel opening.

The experimental evidence for certain rock behaviour phenomena considered in this paper, such as the effect of relationship between the size of opening in the model and the actual size of the tunnel opening, requires some additional tests to quantify all material parameters or functions that are needed for a complete description of the stress-strain response of the rock mass type concerned, under actual working conditions. Above all, this refers to the description and incorporation of the effects of transverse isotropy, effects of discontinuity, and effects of relation between the size of opening in the model and the actual size of the tunnel opening.

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