

# Performance Analysis of Selecting Maximal Ratio Combining Hybrid Diversity System over Ricean Fading Channels

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Original scientific paper

In this paper an approach to the performance analysis of hybrid MRCS (Selection Maximal Ratio Combining) diversity system operating over Ricean fading channels is presented. Hybrid MRCS combining method has a simple implementation, where maximal-ratio combined signals are chosen on a selection combining basis. Closed form expressions are provided for standard first and second order statistical measures for the signal at the output of the combiner, i.e. PDF (probability density function), CDF (cumulative distribution function), LCR (level crossing rate). Capitalizing on them standard performance measures like ABEP (Average Bit Error Probability) over some modulation techniques and AFD (Average Fade Duration) are efficiently evaluated and discussed in the function of various system parameters.

**Key words:** Ricean fading, Hybrid diversity reception, Level crossing rate, Average bit error probability

**Analiza performansi selektirajućeg MRCS hibridnog raznolikog sustava u Riceovim kanalima.** U ovom radu predložen je pristup za analizu performansi hibridnog Selection Maximal Ratio Combining (MRCS) raznolikog sustava koji radi u Riceovim kanalima. Hibridna MRCS metoda je jednostavno primjenjiva, u kojoj se kombinirani signali maksimalnog omjera izabiru prema principu kombinirane selekcije. Predstavljani su eksplicitni izrazi za standardne statističke mjere signala prvog i drugog reda, odnosno za funkciju gustoće vjerojatnosti, kumulativnu funkciju distribucije i LCR (level crossing rate). Korištenjem izvedenih izraza učinkovito su evaluirane i raspravljene standardne mjere performansi kao što je ABEP (Average Bit Error Probability) nad pojedinim tehnikama modulacije i AFD (Average Fade Duration), u funkciji različitih parametara sustava.

**Ključne riječi:** Riceov kanal, hibridni raznoliki prijem, LCR, ABEP

## 1 INTRODUCTION

Few statistical models are used to describe fading occurrence in wireless environments in communications systems analysis. The most frequently used signal distributions are Nakagami- $m$ , Rice, Rayleigh,  $\alpha$ - $\mu$  and Weibull. The Rician fading distribution is often used to model propagation paths that consist of one strong direct line-of-sight (LoS) signal, and many randomly reflected and usually weaker signals. Such fading environments are typically encountered in microcellular and mobile satellite radio communication links. The Rician fading is also applicable for modeling the fading channels in frequency domain [1]. In particular, for mobile satellite communications, the Rician distribution is used to accurately model the mobile satellite channel for single [2], clear-state [3] channel conditions. An efficient method for mitigating fading effects by using multiple receiver antennas is called space diversity. The main goal of space diversity techniques is to improve trans-

mission reliability without increasing transmission power and bandwidth while increasing channel capacity. There are several principal types of space combining techniques that can be generally performed depending on the amount of channel state information available at the receiver.

The optimal combining technique is maximum ratio combining (MRC) [4]. This combining technique involves co-phasing of the useful signal in all branches, multiplication of the received signal in each branch by the estimated envelope of that particular signal and summing of the received signals from all antennas. By co-phasing, all the random phase fluctuations of the signal that emerged during transmission are eliminated. For this process it is necessary to estimate the phase of the received signal, so this technique requires all of the amount of the channel state information on received signal, and separate receiver chain for each branch of the diversity system, which increases the complexity of system. Unlike previous, selection combining (SC) technique processes only one of the diversity

branches. Generally, SC selects the branch with the highest signal-to-noise ratio (SNR), that is the branch with the strongest signal [5], assuming that noise power is equally distributed over branches.

Hybrid diversity reception where, first a group of signals is selected out of the total available, which are then maximal-ratio combined has been discussed in [6]. Similar approaches have been then explored in several other papers [7-8]. Another hybrid diversity scheme is MRCS (MRC Selection), where outputs of arbitrary number of multi branch maximal ratio combiners,  $i = 1, \dots, N$ , are combined into a  $N$ -branch selection combiner. Such a composite scheme finds applicability in practice and has been employed for some time (e.g., both in 2G and 3G networks).

In the Universal Mobile Telecommunications System (UMTS), the MRCS technique is already implemented, with the serving base stations providing the MRC of the signals received from a mobile terminal, and the serving switching center carrying out the SC of the MRC signals [9-10]. The combination of MRC and SC diversity has been also considered for the simultaneous mitigation of fading and shadowing in macro-diversity communication systems [11-12].

A general analysis of the PDF (reliability), first order moment, LCR (Level Crossing Rate), and AFD (Average Fade Duration) at the output of the MRCS combiner over the Nakagami- $m$  fading environment has been presented especially in the recent scientific and technical literature [13]. However, to the best of the authors' knowledge, performance analysis of this hybrid diversity system over Ricean fading channels has never been assessed in the literature.

In this paper we present the performance analysis of hybrid MRCS diversity system over Ricean fading channels. Closed form expressions are provided for standard first and second order statistical measures for the signal at the output of the combiner, i.e. PDF (probability density function), CDF (cumulative distribution function) and LCR (level crossing rate). Based on them ABEP (Average Bit Error Probability) over some modulation techniques is efficiently evaluated. Numerically obtained results are graphically presented and discussed as the functions of various system parameters.

Capitalizing on this analysis, in the process of designing a wireless communication system, one may determine optimal values of system parameters, in order to achieve requested values of LCR, AFD and ABER, over considered fading channel conditions.

## 2 SYSTEM MODEL

MRCS is hybrid diversity scheme where outputs of arbitrary number of multi branch MRC combiners,  $i =$

$1, \dots, N$ , are combined into a  $N$ -branch SC.

Assuming a perfect knowledge of the channel gains, the received signals by each MRC receiver are appropriately co-phased and weighted so that the output signal-to-noise ratio is optimal.

The selection combiner then outputs the MRC signal with the largest SNR.

Denoting the SNR at the  $j$ -th branch of the  $i$ -th selected MRC receiver as  $\gamma_{ij}$  ( $j = 1, \dots, L_i$  and  $i = 1, \dots, N$ ), the resultant output SNR  $\gamma$  of the MRCS combiner is given by:

$$\gamma = \max \left\{ \sum_{j=1}^{L_1} \gamma_{1j}, \sum_{j=1}^{L_2} \gamma_{2j}, \dots, \sum_{j=1}^{L_N} \gamma_{Nj}, \right\} \quad (1)$$

Fig. 1 shows the scheme of the analyzed system.

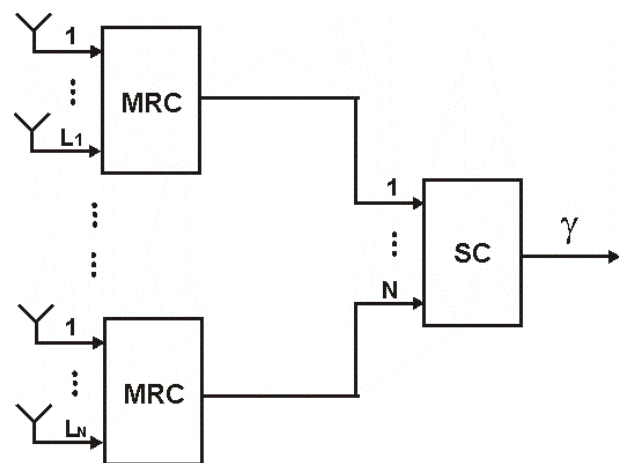


Fig. 1. Hybrid Selection Maximal Ratio Combining diversity system scheme

Let  $F_{MRCi}(\gamma_i)$  be the PDF of the output SNR  $\gamma_i$  of the  $i$ -th MRC combiner.

Assuming the selection branches to be independent, the distribution  $F_{sel}(\gamma)$  of the selection combiner output SNR  $\gamma$  is

$$F_{sel}(\gamma) = \prod_{i=1}^N F_{mrc_i}(\gamma). \quad (2)$$

Of course, in (2) the branches of the MRC receivers may not be independent, which must be considered for in  $F_{MRCi}(\gamma_i)$ .

The PDF  $f_{sel}(\gamma)$  for the SC output can be obtained by differentiating (2), which results in

$$f_{sel}(\gamma) = \sum_{i=1}^N f_{mrc_i}(\gamma) \prod_{\substack{j=1 \\ j \neq i}}^N F_{mrc_j}(\gamma). \quad (3)$$

The PDF of the instantaneous SNR conditioned on the average SNR value at the  $L$ -branch MRC output operating in Ricean fading environment can be written as following expression [14]:

$$f_{mrc_i}(\gamma/\bar{\gamma}_i) = \frac{K+1}{\bar{\gamma}_i} e^{-\frac{K+1}{\bar{\gamma}_i}\gamma - KL} \cdot \left(\frac{(K+1)\gamma}{KL\bar{\gamma}_i}\right)^{\frac{L-1}{2}} I_{L-1}\left(2\sqrt{\frac{KL(K+1)\gamma}{\bar{\gamma}_i}}\right) \quad (4)$$

with  $K$  denoting Ricean factor defined as the ratio of power in the specular and scattered components, while  $I_{L_i}(x)$  is the modified Bessel function of the first kind with order  $L$ .

Capitalizing on [15, Eq. (8.445)] and [15, Eq. (8.350.1)] it can be shown that the CDF of the instantaneous SNR at the MRC output operating in Ricean fading environment can be written in the form of:

$$F(\gamma/\bar{\gamma}) = \sum_{p=0}^{\infty} \frac{(KL)^p}{\Gamma(p+L)p!e^{KL}} \cdot \gamma \left(p+L, \frac{K+1}{\bar{\gamma}}\gamma\right), \quad (5)$$

with  $\Gamma(x)$  and  $\gamma(a, x)$  denoting Gamma and lower incomplete Gamma function, respectively [15].

### 3 PERFORMANCE ANALYSIS

Let us consider unbalanced, independent branches operating in an arbitrary fading scenario. As it was mentioned earlier MRCS output envelope  $R$  can be written as  $R = \max_i\{R_i\}$ , where  $R_i$  denotes the envelope at the output of the  $i$ -th maximal-ratio combiner.

Let  $R$  be the received signal envelope, and  $\dot{R}$  its derivative with respect to time, with joint PDF  $f_{R\dot{R}}(R, \dot{R})$ .

The LCR is defined as the average number of signal crossings at a given level  $R$ , in the negative or positive direction.

$$N_R(r) = \int_0^{\infty} \dot{r} f_{R\dot{R}}(\dot{r}, r) d\dot{r} \quad (6)$$

Considering independence of the signals received by the selection combiner, the level crossing rate  $N_R(r)$  of the MRCS output at level  $r$  is given by [16]

$$N_R(r) = \sum_{i=1}^N N_{R_i}(r) \prod_{\substack{j=1 \\ j \neq i}}^N F_{R_j}(r). \quad (7)$$

The expression (7) can be directly applied to fading environments with known LCR and envelope PDF at the output of each MRC receiver, denoted by  $N_{R_i}(\cdot)$  and  $F_{R_i}(\cdot)$ , respectively

$$N_R(\mu_i) = \sum_{p=0}^{\infty} \frac{\sqrt{2\pi} f_d (K_i L_i)^p (K_i + 1)^{p+L_i}}{\Gamma(p+L_i) p! e^{K_i L_i} \Omega_i^{p+L_i - \frac{1}{2}}} \mu_i^{2p+2L_i-1} e^{-\frac{(K_i+1)\mu_i^2}{\Omega_i}} \quad (8)$$

$$F_R(\mu_i) = \sum_{p=0}^{\infty} \frac{(K_i L_i)^p}{\Gamma(p+L_i) p! e^{K_i L_i}} \gamma\left(p+L_i, \frac{(K_i+1)}{\Omega_i} \mu_i^2\right). \quad (9)$$

Normalized LCR for various values of system's parameters is presented at Figs. 2 and 3. LCR is normalized by maximal Doppler shift frequency  $f_d$ . Numerical results for LCR are presented in the function of normalized signal level. Signal level is normalized with the square root of mean power of Ricean distributed signal  $\rho = \mu/\sqrt{\Omega}$ , where mean-square value of fading envelope is denoted with  $\Omega$ .

In practice it is usual that threshold level at the receiver  $-\mu$  is set to the value that is smaller than the root mean value  $\Omega$ , in order to obtain reasonable indication of outage [17-18].

Because of that, from practical point of view, more interesting are results for the values of normalized signal level that follows  $\rho < 0$  dB.

We can observe from Fig. 2, that for the normalized signal levels, which are  $\rho < 0$  dB, higher values of Ricean factor  $K_i$  provide smaller LCR values. Since presence of direct LOS component provides more reliable communication, when power of specular signal component is grows compared to the powers of scattered components, then the variation of signal at the reception is smaller, and smaller LCR values are obtained. Also we can observe that LCR has smaller values in observed range in the presence of MRCS with higher order of MRC applied (larger number of branches  $L_i$ ).

Another improvement is achieved with higher order of system (larger  $N$  values), which can be seen from Fig. 3. Comparing the presented values from Figs. 2 and 3 we also derive the conclusion that, slightly better improvement comes with the increase of  $L_i$  than it comes with increase of  $N$  (increasing the number of MRC branches is better than increasing the number of SC branches).

The AFD is defined as the mean time the received envelope remains below a given threshold  $r$  after crossing it in the negative direction.

$$T_R(r) = \frac{F_R(r \leq R)}{N_R(r)} \tag{10}$$

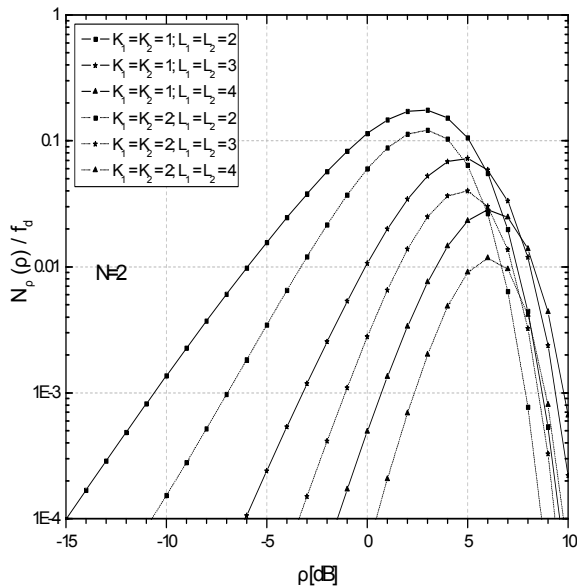


Fig. 2. Normalized average LCR of our MRCS structure versus normalized signal level for various values of Ricean factor and MRC order for the case of selection order  $N = 2$

Considering independent branches, the AFD  $T_R(r)$  at the output of the MRCS combiner can be calculated based on [19]

$$T_R^{-1}(r) = \sum_{i=1}^N T_{R_i}^{-1}(r), T_{R_i}(\mu) = \frac{F_R(\mu_i)}{N_R(\mu_i)} \tag{11}$$

Normalized AFD of observed system is presented at Figs. 4 and 5, normalized with maximal Doppler shift frequency  $f_d$ . We can observe from Fig. 4, that for higher values of Ricean factor  $K_i$  provide, better performances are reached (smaller AFD values).

Also we can see that AFD values decrease when higher order of system is used and when each MRC combining is performed with higher number of branches, as expected.

Average bit error probability (ABEP) is another useful performance criterion characteristic of wireless communication systems.

The ABEP at the MRCS output is derived for non-coherent and coherent binary signaling according to following expressions

$$P_e = \int_0^\infty f_R(t) \frac{1}{2} e^{-gt} dt \tag{12}$$

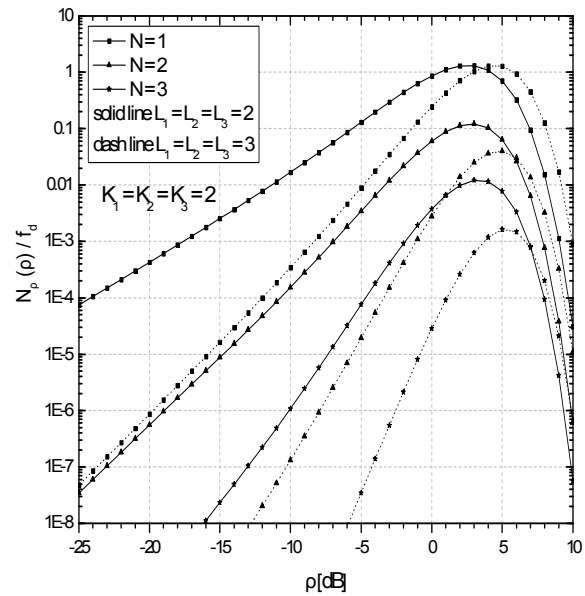


Fig. 3. Normalized average LCR of our MRCS structure versus normalized signal level for various values of MRC and selection order

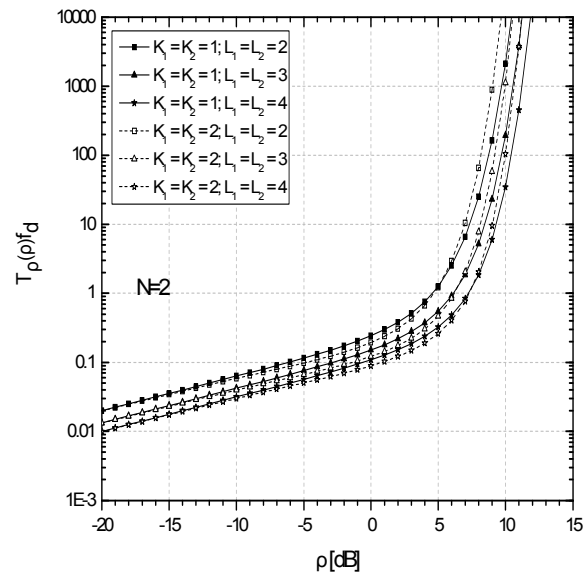


Fig. 4. Normalized average LCR of our MRCS structure versus normalized signal level for various values of Ricean factor and MRC order for the case of selection order  $N = 2$

where  $g$  denotes modulation constant, in non-coherent systems,  $g = 1$  for BDPSK (Binary Differentially Phase Shift Keying) and  $g = \frac{1}{2}$  for NCFSK (Non-coherent Frequency Shift Keying).

In a coherent system ABEP can be determined accord-

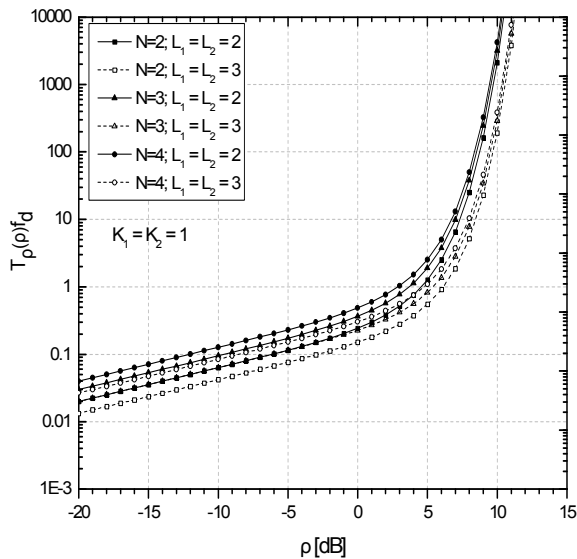


Fig. 5. Normalized average LCR of our MRCS structure versus normalized signal level for various values of MRC and selection order

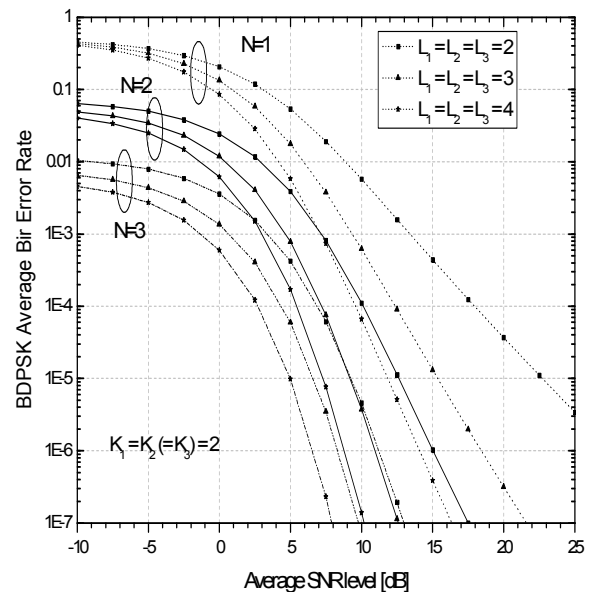


Fig. 6. ABEP versus average SNR in BDPSK for various values of Ricean factor, MRC and selection order

ing to [20-21].

$$P_e = \int_0^\infty f_R(t) \frac{1}{2} \operatorname{erfc}(\sqrt{\alpha\lambda}) \quad (13)$$

Substituting (3) in (12) ABEP is numerically obtained and shown on Figs. 6 and 7 for some values of Ricean factor  $K_i$  and MRC and SC diversity orders.

It can be concluded from the figures that system shows better error performances for higher order of diversities applied (larger number  $L_i$  and  $N$  branches) while slightly better improvement comes with the increase of  $L_i$  than with increase of  $N$  (i.e. comparing cases of  $L_i = 2, N = 3$  and  $N = 2, L_i = 3$ ).

Results from figures present influence of Ricean  $K_i$  factor on ABEP, too. Actually, ABEP decreases with the increase of Ricean factor.

Comparison of curves from Fig. 7 shows better error performance of BDPSK modulation technique, than NCFSK modulation technique, especially in the areas of higher average SNR.

#### 4 CONCLUSION

Performance analysis of hybrid MRCS over Ricean fading conditions has been done for the first time, and is of importance in the process of designing a wireless communication system.

Capitalizing on the closed form expressions for standard first statistical measures for the signal at the output of

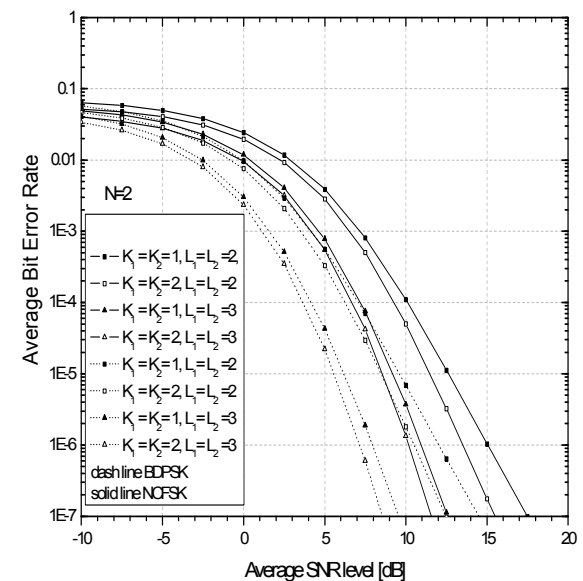


Fig. 7. ABEP versus average SNR in BDPSK and NCFSK in the our MRCS structure versus normalized signal level for various values of MRC and selection order  $N=2$

the combiner, i.e. PDF, CDF standard performance measures like LCR and ABEP over BDPSK and NCFSK were efficiently evaluated. Numerically obtained results were presented and discussed as the function of various system parameters, such as Ricean factor and MRC and selection diversity order.

Presented collection of system performances will help

researchers and system designers to perform trade-off studies among the various modulation type/diversity order/system parameters. in order to determine the optimal choice in the presence of their available constraints.

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## REFERENCES

- [1] K. Witrals, Y. H. Kim, R. Prasad, "A new method to measure parameters of frequency-selective radio channels using power measurements", *IEEE Transactions on Communications*, vol. 49, no. 10, pp. 1788 – 1800, 2001.
- [2] G. E. Corazza, F. Vatalaro, "A statistical model for land mobile satellite channels and its application to no geostationary orbit systems", *IEEE Transactions on Vehicular Technology*, vol. 43, no. 3, part 2, pp. 738–742, 1994.
- [3] H. Wakana, "A propagation model for land mobile satellite communications", *In Proceedings book of IEEE Antennas and Propagation Society International Symposium*, (placeCityLondon, StateOntario, country-regionCanada), vol. 3, pp. 1526–1529, June 1991.
- [4] W. C. Y. Lee, *Mobile communications engineering*, placeCityNew York, country-regionUSA: Mc-Graw-Hill, 2001.
- [5] M. K. Simon, M. S Alouini, *Digital communication over fading channels*, 2nd edn., placeCityNew York, country-regionUSA: Wiley-Interscience, 2005.
- [6] T. Eng, N. Kong, L. B. Milstein, "Comparison of diversity combining techniques for Rayleigh-fading channels", *IEEE Transactions on Communications*, vol. 44, no. 9, pp. 1117–1129, Sep. 1996.
- [7] C. R. C. M. da Silva, M. D. Yacoub, "A generalized solution for diversity combining techniques in fading channels", *IEEE Transaction on Microwave Theory and Technology*, vol. 50, no. 1, pp. 46–50, Jan. 2002.
- [8] S. Choi, M. S. Alouini, K. Qaraqe, H.C. Yang, "Finger assignment schemes for rake receivers with multiple-way soft handover", *IEEE Transactions on Wireless Communications*, vol. 7, no. 2, pp. 495–499, 2008.
- [9] H. Holma, A. Toskala, *WCDMA for UMTS*, 2nd edition, placeCityLondon, country-regionEngland: John Wiley & Sons Ltd, 2004.
- [10] J. Laiho, A. Wacker, T. Novosad, *Radio Network Planning and Optimization for UMTS*, 2nd edition, placeCityLondon, country-regionEngland: Wiley, Feb. 2006.
- [11] D. Stefanovic, S. R. Panic, P. Spalevic, "Second-order statistics of SC macrodiversity system operating over Gamma shadowed Nakagami-m fading channels", *AEU - International Journal of Electronics and Communications*, vol. 65, issue 5, pp. 413-418, 2011
- [12] S. R. Panić, D. Stefanovic, I. Petrović, M. Stefanović, J. Anastasov, D. Krstic "Second order statistics of selection macro-diversity system operating over Gamma shadowed  $k$ - $\mu$  fading channels", *EURASIP Journal on Wireless Communications and Networking*, Volume 2011, ID 151, pp 1-7, 2011.
- [13] F. P. Calmon, M. D. Yacoub, "MRCS: Selecting Maximal Ratio Combined Signals - A Practical Hybrid Diversity Combining Scheme", *IEEE Transactions on Wireless Communications*, vol. 8 issue 7, pp. 3425 – 3429, 2009.
- [14] J. Zhang, V. Aalo "Effect of Macrodiversity on Average Error Probabilities in a Rician fading channel with correlated lognormal shadowing", *IEEE Transaction on Communication*, vol 49, no.1, January 2001.
- [15] I. Gradshteyn, I. Ryzhik, *Tables of Integrals, Series, and products*, CityplaceNew York, country-regionUSA: Academic Press, 1980.
- [16] L. Yang, M. S. Alouini, "Level crossing rate over multiple independent random processes - an extension of the applicability of the Rice formula", *IEEE Transactions on Wireless Communications*, vol. 6, iss. 15, pp. 4280–4284, 2007.
- [17] X. placeCityDong, StateN.C. Beaulieu, "Average level crossing rate and average fade duration of selection diversity", *IEEE Communication Letters*, vol. 10, iss. 5, pp. 396–399, October 2001
- [18] A. Mitic, M. Jakovljevic, "Second-Order Statistics in Weibull- Lognormal Fading Channels", *In Proceedings book of TELSIS conference*, (placeCityNis, country-regionSerbia), vol.2, pp. 529-532, September 2007.
- [19] D. B. da Costa, J. C. S. S. Filho, M. D. Yacoub, G. Fraidenraich, "Crossing rates and fade durations for diversity combining schemes over  $\alpha$ - $\mu$  fading channels," *IEEE Transactions on Wireless Communications*, vol. 6, no. 11, pp 4263 – 4267, 2007.
- [20] V. Aalo, J. Zhang, "On the Effect of Cochannel Interference on Average Error Rates in Nakagami-Fading Channels", *IEEE Communication Letters*, vol. 3, Issue 5, pp. 136–138, 1999.
- [21] P. Spalević, M. Stefanović, S. Panić, B. Milošević, Z. Trajčevski "The co-channel interference effect on average error rates in Hoyt fading channels", *Revue Roumaine des Sciences Techniques. Ser. Electrotechnique et Energetique*, vol. 56, no. 3, pp. 305-313, 2011.



**Petar Spalević** was born in Kraljevo, Serbia in 1973. He received B.Sc. and M.Sc. degrees in electrical engineering from the Faculty of Electronic Engineering, University of Priština, Serbia, and Ph.D. degree from the Faculty of Electronic Engineering, University of Niš, Serbia. His research interests are statistical communication theory, optical and satellite communications and optimal receiver design. He has published several journal publications on the above subject.



**Mihajlo Stefanović** was born in Niš, Serbia in 1947. He received B.Sc., M.Sc. and Ph.D. degrees in electrical engineering from the Faculty of Electronic Engineering (Department of Telecommunications), University of Niš, Serbia, in 1971, 1976 and 1979, respectively. His primary research interests are statistical communication theory, optical and satellite communications. He has written or co-authored a great number of journal publications.



**Stefan R. Panić** was born in Pirot, Serbia, in 1983. He received M.Sc. and PhD degree in electrical engineering from Faculty of Electronic Engineering, Niš, Serbia, in 2007 and 2010 respectively. His research interests within mobile and multichannel communications, include statistical characterization and modeling of fading channels, performance analysis of diversity combining techniques, outage analysis of multi-user wireless systems subject to interference. Within digital communication his current research inter-

ests include the information theory, source and channel coding and signal processing. He has published over 40 SCI indexed papers. Currently he works as docent at the Department of Informatics, Faculty of Natural Science and Mathematics, University of Priština.



**Branimir Jaksic** was born in Kosovska Mitrovica, Serbia, in 1984. He received B.Sc. and M.Sc. degrees in electrical engineering from the Faculty of Technical Sciences in Kosovska Mitrovica, University of Pristina, Serbia. He is PhD candidate in the Faculty of Electronic Engineering, University of Nis, Serbia. Areas of research include statistical communication theory and optical telecommunications. He has authored several scientific papers on the above subject.



**Mile Petrovic** is full professor the Dpt. of Elec. and Comp. Engineering Faculty of Technical Sciences Kosovska Mitrovica, Serbia. Areas of interest include telecommunications - television techniques. He has authored over 50 scientific peer-reviewed papers and a large number of projects and patents. And a member of the technical program committee and reviewer for several international journals and symposia.

#### AUTHORS' ADDRESSES

**Petar Spalević, Ph.D.**

**Mihajlo Stefanović, Ph.D.**

**Asst. Prof. Stefan R. Panić, Ph.D.**

**Department of Informatics,**

**Faculty of Mathematics and Natural Science,**

**University of Pristina,**

**Ivo Lole Ribara 29, 40000 Kosovska Mitrovica, Serbia**

**email: {petarspalevic, mihajlo.stefanovic, stefanpnc}@yahoo.com**

**Branimir Jaksic, M.Sc.**

**Prof. Mile Petrovic, Ph.D.**

**Department of Electronic and Computing Engineering,**

**Faculty of Technical Sciences in Kosovska Mitrovica,**

**University of Pristina,**

**Kneza Milosa 7, 38220 Kosovska Mitrovica, Serbia**

**email: branimirjaksic@gmail.com,**

**petrovic.mile@yahoo.com**

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