Online ISSN 1848-3380, Print ISSN 0005-1144
ATKAFF 55(4), 405–415(2014)

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Adaptive Wavelet Neural Network Backstepping Sliding Mode Tracking Control for PMSM Drive System

DOI 10.7305/automatika.2014.12.456
UDK 681.513.3-55:621.313.8; 004.032.26
IFAC 3.2; 4.7

Original scientific paper

This paper presents a wavelet neural network backstepping sliding mode controller (WNNBSSM) for permanent-magnet synchronous motor (PMSM) position servo control system. Backstepping sliding mode (BSSM) is utilized to guarantee favorable tracking performance and stability of the whole system, meanwhile, wavelet neural network (WNN) is used for approximating nonlinear uncertainties. The designed controller combined the merits of the backstepping sliding mode control with robust characteristics and the WNN owning the capability of artificial neural networks for online learning and the capability of wavelet decomposition for identification. An observed error compensator is developed to compensate the estimated error of the WNN and the adaptive law is derived according to Lyapunov theorem. The effectiveness of the proposed controller is investigated in simulation under different operating conditions. The simulation results demonstrate the proposed WNNBSSM controller can provide precise tracking performance and robust characteristics despite unknown parameter uncertainties and load disturbance. Moreover, an implemental wavemaker system is established to verify the effectiveness of the proposed control algorithm.

Key words: Backstepping, PMSM position servo system, Robustness, Sliding mode control, Wavelet neural network

Adaptivno slijedno upravljanje u kliznom režimu rada s valićnom neuronskom mrežom za sustav s PMSM elektromotornim pogonom. U radu je predstavljen povratnokoračni regulator u kliznom režimu rada za servo sustav pozicioniranja s motorom s permanentnim magnetima (PMSM) zasnovan na valićnim neuronskim mrežama (WNNBSSM). Povratnokoračni klizni režim rada (BSSM) korišten je za jamčenje željenih performansi slijeđenja i stabilnost cijelog sustava, dok je valićna neuronska mreža (WNN) korištena za aproksimaciju nelinearnih nesigurnosti. Sintetizirani regulator povezuje prednosti i robusne karakteristike povratnokoračnog kliznog režima upravljanja te WNN sa sposobnostima umjetnih neuronskih mreža za online učenje i mogućnost valićne dekompozicije za identifikaciju. Razvijen je kompenzator pogreške estimacije WNN i adaptivni upravljački zakon prema Ljapunovljevom teoremu. Djelotvornost predloženog regulatora istražena je kroz simulacije u različitim uvjetima rada. Rezultati simulacija pokazuju da predloženi WNNBSSM može osigurati precizna svojstva slijeđenja i robusne karakteristike unatoč nepoznatim nesigurnostima parametara i poremećaja tereta. Uz to, razvijen je izvedbeni sustav generatora valova za provjeru djelotvornosti predloženog upravljačkog algoritma.

Ključne riječi: povratnokoračni regulator, PMSM servo sustav za pozicioniranje, robusnost, upravljanje s kliznim režimom rada, valićna neuronska mreža

1 INTRODUCTION

As the development magnetic materials, semiconductor technology and modern control theories, PMSMs show excellent features such as high efficiency, high power density, lower noise, low loss and compact size in industrial application, and are more and more widely used in motion servo control. It is required in high precision servo control that systems should have good static and dynamics performance in order to obtain steady state accuracy and fast dynamical response. Unfortunately, the control performance is hard to be satisfied on account of the intrinsic nonlinear characters like multi-variables, strong couple and so on. As a result, classical control scheme such as PID combined with vector control method is no longer suitable for accurate servo motion control and searching new and more effective control techniques becomes a focus in PMSM study field. The development of control theory provides a foundation to solve these problems and especially the nonlinear control is introduced that brings novel ideas and solutions.

State feedback linearization, sliding mode variable structure, backstepping control are applied in PMSM control systems which can achieve precise servo control by decouple the current and speed nonlinear terms in PMSM's mathematical model [1-3]. The backstepping control is popular in nonlinear system design since the derived system control law and parameters adaptive law are able to make controlled system be global stable and robust. The backstepping control principle is to simplify controlled system state space to many reduced order subsystem by designing virtual control function and actual control law can be derived through selecting appropriate Lyapunov candidate function. Accordingly, combined with adaptive technique, backstepping is a useful design method for nonlinear system with uncertainties. However, there are disadvantages in backstepping design including complexity increasing, over-parametrization, pre-knowing model information and so on which need to be overcome in practical application [4]. Sliding mode control is another effective tool to solve nonlinear problems in motion control [5-6]. It designs sliding manifold by using a lower order model to achieve the control objective that state variable trajectories are forced to reach a sliding manifold in finite time and stay on the manifold for all future time and motion on the manifold is independent of matched uncertainties in the same time [7]. Therefore it represents good characteristics such as insensitivity to parameter variations, external load disturbance rejection and fast dynamic response [8]. Yet sliding mode control suffers from drawbacks that a high-frequency oscillatory output phenomenon called chattering caused by high-speed switching used in sliding mode, and it is vulnerable to measurement noise [9]. In addition, the sliding mode control effect is still subject to parameters uncertainties. In recent years, intelligent control has been comprehensively known as an important and potential way to treat complex control problems; especially it shows excellent performance as is integrated with other control technology. On this situation, researchers make their efforts to utilize the advantages to meet all kinds of requirements in application and many new available fusional ways are created.

Fuzzy control is used to approach sliding mode variable structure control scheme in [10], good dynamics is achieved, but the influence of parameters variety and external load changes of the plant is not considered. In [11], fuzzy method is adopted to approximate a complex nonlinear term in backstepping design, but similarly disturbances have not been taken into account and using and choosing fuzzy rule increase the complexity of the algorithm. Wavelet neural network possesses both abilities of artificial neural networks and wavelet decomposition which can not only approximate a wide range of non-linear functions to any desired degree of accuracy under certain conditions but also realize wavelet decomposition for identification and control of dynamic systems. It is proved that WNN has more superiority over the other neural networks in rapid learning and system identification which are suitable for handling nonlinearity and uncertainties of real-time servo control system [12-14]. Papers [8, 15-16] present that sliding mode control combined with WNN which can grants robust performance and obtain precise response. However, the accurate current regulation has not been considered which is important for high precision PMSM drive control. In this paper, a new adaptive WNN based backstepping sliding mode controller is proposed to achieve favorable tracking performance under all parameter uncertainties and unknown load torque disturbance. Backstepping design technology is adopted to assure the stability of whole PMSM closed loop drive system and appropriate Lyapunov function is selected to derive control and adaptation laws. By introducing the sliding manifold, the tracking error can converge to zero asymptotically and a WNN is combined to approximate the nonlinear uncertainty terms caused by parameter perturbation and load torque disturbance. The position trajectory, rotor speed, stator currents are all designed to precisely track the required reference. Accordingly, not only precise steady-state accuracy and fast dynamic response can be attained but also robustness is guaranteed despite parameter uncertainties and load torque disturbance.

2 FORMULATED MATHEMATICAL MODEL OF PMSM DRIVE

Based on the assumption that the magnetic circuit is unsaturated, impact of hysteresis and eddy current losses is neglect, space magnetic field is sinusoidal distribution and the three phase windings are balanced, the mathematical model of a PMSM dynamics can be expressed by the following equations in a synchronously rotating rotor d–q reference frame:

$$\dot{\theta} = \omega, \tag{1}$$

$$\dot{\omega} = \frac{3p\psi}{2J} i_q - \frac{B}{J}\omega - \frac{T_L}{J}, \tag{2}$$

$$\dot{i}_q = -\frac{R}{L} i_q - p\omega i_d - \frac{p\psi}{L}\omega + \frac{u_q}{L}, \tag{3}$$

$$\dot{i}_d = -\frac{R}{L} i_d + p\omega i_q + \frac{u_d}{L}, \tag{4}$$

where θ is rotor position; ω is rotor mechanical angular velocity; p is the number of pole pairs; ψ is the rotor magnetic flux linking the stator; u_d, u_q are d and q axes stator voltages respectively; i_d, i_q are d and q axes stator currents respectively; L is the stator inductance, R is the stator resistance; J is the moment of inertia of the motor and load; B is the viscous friction coefficient of the motor; T_L is the load torque.

While PMSM is on operation, the parameters is varying as a result of load torque disturbance, changing temperature and saturation effects, thus, the PMSM dynamical equations (2-4) could be rewritten as follows:

$$\dot{\omega} = \frac{1}{(J+\Delta J)}\left[\frac{3}{2}p\left(\psi+\Delta\psi\right)i_q - \left(B+\Delta B\right)\omega - \right. \\ \left. -\left(T_L+\Delta T_L\right)\right] = \frac{3p\psi}{2J}i_q - \frac{B}{J}\omega + L_1, \tag{5}$$

$$\begin{aligned}\dot{i}_q &= -\frac{1}{L+\Delta L}\left[\left(R+\Delta R\right)i_q + \left(L+\Delta L\right)p\omega i_d + \right. \\ &\quad \left. p\left(\psi+\Delta\psi\right)\omega - u_q\right] \\ &= -\frac{R}{L}i_q - p\omega i_d - \frac{p\psi}{L}\omega + \frac{u_q}{L} + L_2,\end{aligned} \tag{6}$$

$$\begin{aligned}\dot{i}_d &= -\frac{(R+\Delta R)}{(L+\Delta L)}i_d + p\omega i_q + \frac{u_d}{(L+\Delta L)} \\ &= -\frac{R}{L}i_d + p\omega i_q + \frac{u_d}{L} + L_3,\end{aligned} \tag{7}$$

where L_1, L_2, L_3 is the lumped uncertainty of each state equation. It can be seen from the PMSM dynamics formulated with all uncertainties that the nonlinearity exist due to the multiplied rotor angular velocity and stator currents terms, and it is a significant reason for high precision control of PMSM is difficult. Moreover, the varying parameters also increase the system nonlinearity and degrade controller performance, or even destroy the stability of the system. Hence, the nonlinearities, parameter uncertainties and external disturbances should be eliminated or limited in an attenuate level as designing a high performance PMSM drive controller.

3 DESIGN OF ADAPTIVE WNN BACKSTEPPING SLIDING MODE CONTROLLER

3.1 Backstepping sliding mode control of PMSM

The control aim of PMSM position servo drive system is to design a controller that is able to make the system can track given trajectory in a sense of asymptotical stability. In order to achieve this goal, backstepping design procedure is adopted in this paper. The core idea of backstepping is to choose a state variable needing to be stabilized as a new state variable to construct a new subsystem. Generally, the chosen variables are error of state variables which are expected to attenuate to zero. Then an appropriate Lyapunov candidate function is selected to derive a virtual control law. In this way, step by step, a real control law could be derived at last and the whole system stability can be assured. According to this rule, to asymptotically track the position signal, position tracking error is defined:

$$e_\theta = \theta - \theta_m, \tag{8}$$

where θ_m is desired rotor position and assumed to be twice differentiable. e_θ can be seen as a state variable of new subsystem, and is differentiated respect to time:

$$\dot{e}_\theta = \dot{\theta} - \dot{\theta}_m = \omega - \dot{\theta}_m. \tag{9}$$

Define a virtual function $\alpha_1 = -k_1 e_\theta + \dot{\theta}_m$, $k_1 > 0$, and a new state variable of subsystem can be denoted as $e_\omega = \omega - \alpha_1$. For equation (6), $V_1 = \frac{1}{2}e_\theta^2$ is selected as a Lyapunov candidate function and differentiated respect to time as follow:

$$\dot{V}_1 = e_\theta\dot{e}_\theta = e_\theta\left(-k_1 e_\theta + e_\omega\right) = -k_1 e_\theta^2 + e_\theta e_\omega. \tag{10}$$

In order to realize the robust tracking capability of the PMSM drive system, sliding mode is introduced here. Define Lyapunov candidate function,

$$V_2 = V_1 + \frac{1}{2}s^2, \tag{11}$$

where s is the sliding manifold(or surface) and it can be expressed:

$$s = ae_\theta + e_\omega, \tag{12}$$

where $a > 0$. The movement of e_θ and e_ω are supposed to be forced to reach the sliding manifold in finite time and constrained on the manifold. The reaching time can be controlled by choice of a and the motion of state variables on the manifold is independent of the PMSM dynamics equations. Then, differential of V_2 is derived:

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + s\dot{s} = -k_1 e_\theta^2 + e_\theta e_\omega + s\left[a\dot{e}_\theta + \dot{e}_\omega\right] \\ &= -k_1 e_\theta^2 + e_\theta e_\omega + s\left[a\left(\omega - \dot{\theta}_m\right) + \dot{\omega} - \dot{\alpha}_1\right] \\ &= -k_1 e_\theta^2 + e_\theta e_\omega + s\left(a\omega - a\dot{\theta}_m + \frac{3p\psi}{2J}i_q - \right. \\ &\quad \left. -\frac{B}{J}\omega + \bar{L}_1\right),\end{aligned} \tag{13}$$

where $\bar{L}_1$ is a sum of nonlinear term $-\dot{\alpha}_1$ and lumped uncertainty L_1.

According to (13), a backstepping sliding mode control law is designed as:

$$i_q^* = \frac{2J}{3p\psi}\left[\left(\frac{B}{J} - a\right)\omega + a\dot{\theta}_m - b\left(s + c\,\mathrm{sgn}(s)\right) - \hat{L}_1\right], \tag{14}$$

where $b > 0$, $\hat{L}_1$ is estimated value of $\bar{L}_1$ which is approximated by WNN in the next section in this paper.

Substitute (14) into (13) yield:

$$\begin{aligned}\dot{V}_2 &= -k_1 e_\theta^2 + e_\theta e_\omega - bs^2 - bc\left|s\right| + s\left(\bar{L}_1 - \hat{L}_1\right) \\ &= -k_1 e_\theta^2 + e_\theta e_\omega - b(ae_\theta + e_\omega)^2 - bc\left|s\right| + \\ &\quad + s\left(\bar{L}_1 - \hat{L}_1\right).\end{aligned} \tag{15}$$

To make the time derivative of the Lyapunov function candidate negative semi-definite, the following inequality should be established:

$$e_\theta e_\omega - b(ae_\theta + e_\omega)^2 \le 0. \tag{16}$$

Inequality (16) can be rewritten as:

$$a^2be_\theta^2 + (2ab-1)\, e_\theta e_\omega + be_\omega^2 \ge 0, \tag{17}$$

then the following inequality is obtained such that (17) is satisfied,

$$(2ab-1)^2 - 4a^2b^2 < 0. \tag{18}$$

Thus the positive constants a and b should meet the following requirement:

$$ab > \frac{1}{4}. \tag{19}$$

Based on field-oriented vector control, the terms of rotor speed and d and q axes stator currents can be decoupled so as to maximum control efficiency. Choose (14) and (20) as reference currents of current control loop for the PMSM drive system:

$$i_d^* = 0. \tag{20}$$

To maintain asymptotical tracking for PMSM currents, define current tracking error as:

$$e_q = i_q - i_q^*, \tag{21}$$

$$e_d = i_d - i_d^*, \tag{22}$$

which are selected as state variables of new subsystem. Considering parameter uncertainty in (6) and (7), the derivative of (21) and (22) are derived by substituting (14) and (20) as follows:

$$\begin{aligned}\dot e_q &= \dot i_q - \dot i_q^* = -\frac{R}{L}i_q - p\omega i_d - \frac{p\psi}{L}\omega + \frac{u_q}{L} + L_2 - \\ &\quad - \frac{2J}{3p\psi}\left[\left(\frac{B}{J}-a\right)\dot\omega + a\ddot\theta_m - b\dot s\right] \\ &= \left(-\frac{R}{L} - \frac{B}{J} + a\right)i_q - p\omega i_d + \left[\frac{2B}{3p\psi}\left(\frac{B}{J}-a\right) - \right. \\ &\quad \left. - \frac{p\psi}{L}\right]\omega + \frac{u_q}{L} + \bar L_2 + \frac{2bJ}{3p\psi}\left(a\dot e_\theta + \dot e_\omega\right),\end{aligned} \tag{23}$$

$$\dot e_d = \dot i_d - \dot i_d^* = -\frac{R}{L}i_d + p\omega i_q + \frac{u_d}{L} + \bar L_3, \tag{24}$$

where $\bar L_2$ is a lumped linear combination of $\bar L_1$, L_2 and $\ddot\theta_m$, and $\bar L_3$ is a renotation of L_3 for uniform nomenclature.

Then the current tracking errors dynamics are stabilized by designing the following control inputs to achieve the tracking objective

$$\begin{aligned}u_q &= L\left[\left(\frac{R}{L} + \frac{B}{J} - a\right)i_q - p\omega i_d - \left(\frac{2B}{3p\psi}\left(\frac{B}{J} - \right.\right.\right. \\ &\quad \left.\left.\left. - a\right) - \frac{p\psi}{L}\right)\omega - \hat L_2 - \frac{2bJ}{3p\psi}\left(a\dot e_\theta + \dot e_\omega\right) - k_2 e_q\right],\end{aligned} \tag{25}$$

$$u_d = L\left(\frac{R}{L}i_d - p\omega i_q - \hat L_3 - k_3 e_d\right), \tag{26}$$

where $k_2, k_3 > 0$, and $\hat L_2$,$\hat L_3$ is estimated value of $\bar L_2$, $\bar L_3$.

3.2 Adaptive WNN uncertainty observer

Since the lumped uncertainty$\bar L_1$, $\bar L_2$, $\bar L_3$ may deteriorate the system performance and are difficult to determine, an adaptive WNN observer is proposed to predict the value of the lumped uncertainties. A four layers WNN is introduced as shown in Fig. 1, which is comprised of an input layer, a mother wavelet layer, a wavelet layer, and an output layer [17-19]. The basic principle and function of each layer are specified in the following section.

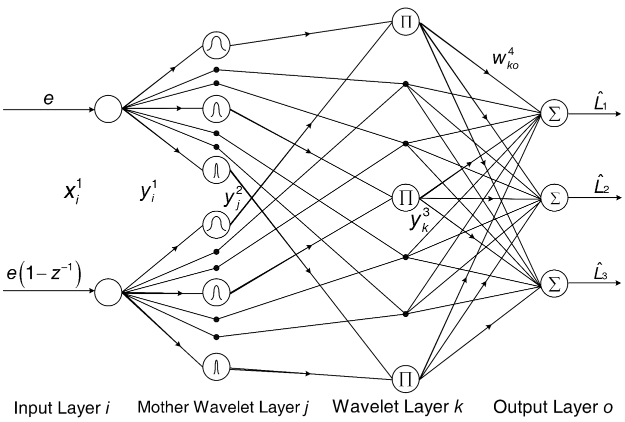

Fig. 1. Four-layer wavelet neural network

3.2.1 Specific WNN structure

Choose position tracking error e_θ and its difference $e_\theta\left(1-z^{-1}\right)$ as the input of WNN, in which z^{-1} is the time delay, and the input and output for each node of input layer is represented as:

$$\begin{aligned} x_1^1 = e_\theta, \qquad x_2^1 = e_\theta\left(1-z^{-1}\right), \\ y_i^1 = net_i^1 = x_i^1, \quad i = 1,2. \end{aligned} \tag{27}$$

The mother wavelet layer is composed of a family of wavelets constructed by translations and dilations performed on a single fixed function. This function is called mother wavelet, and first derivative of a Gaussian function, $\varphi_j = -x\exp\left(-x^2/2\right)$, is adopted as a mother wavelet in this paper. The j^{th} node of mother wavelet layer is expressed as:

$$y_j^2 = f_j^2\left(net_j^2\right) = \varphi_j\left(net_j^2\right) = \varphi_j\left(-\frac{x_i^2-\mu_{ij}}{\sigma_{ij}}\right), \quad j = 1,\cdots,n, \tag{28}$$

where μ_{ij} and σ_{ij} are the translation and dilation in the j^{th} term of the i^{th} input x_i^2 to the node of mother wavelet layer, and n is the total number of wavelets with respect to the input nodes.

In the wavelet layer, each node k multiplies the input signals and outputs the result of the product. For the k^{th} node:

$$y_k^3 = f_k^3\left(net_k^3\right) = net_k^3 = \prod_j x_j^3, \quad k = 1,\cdots,l, \tag{29}$$

where x_j^3 represents the j^{th} input to the node of wavelet layer, $l = n/i$ is the number of wavelets if each input node has the same mother wavelet nodes.

The outputs of the WNN are approximated lumped uncertainty $\hat{L}_1$, $\hat{L}_2$, $\hat{L}_3$ for backstepping sliding mode controller. The nodes in the output layer compute the overall output as the summation of all input signals:

$$y_o^4 = f_o^4\left(net_o^4\right) = net_o^4 = \sum_k w_{ko}^4 x_{ko}^4, \quad o = 1,2,3, \tag{30}$$

where the connecting weight w_{ko}^4 is the output action strength of the o^{th} output associated with the k^{th} wavelet; x_{ko}^4 represents the k^{th} input to the o^{th} node of output layer.

3.2.2 Online Adaptive WNN Training algorithm

A general supervised gradient descent backpropagation learning algorithm, in which gradient vectors is recursively calculated by deriving an energy function with respect to a parameter of the network, is adopted to online train parameters of WNN [8,19-22]. The energy function is defined as

$$E = \frac{1}{2}e_\theta^2. \tag{31}$$

The error term to be propagated in output layer is calculated as

$$\delta_o^4 = -\frac{\partial E}{\partial y_o^4} = -\frac{\partial E}{\partial e_\theta}\frac{\partial e_\theta}{\partial \theta}\frac{\partial \theta}{\partial net_o^4}, \tag{32}$$

then the weight updating amount is

$$\Delta w_{ko}^4 = -\eta_w\frac{\partial E}{\partial w_{ko}^4} = -\eta_w\frac{\partial E}{\partial net_o^4}\frac{\partial net_o^4}{\partial w_{ko}^4} = \eta_w\delta_o^4 x_{ko}^4, \tag{33}$$

where η_w is the weight learning rate. Similarly, error term to be propagated in wavelet layer and mother wavelet layer is calculated as follows:

$$\delta_k^3 = -\frac{\partial E}{\partial net_k^3} = -\frac{\partial E}{\partial net_o^4}\frac{\partial net_o^4}{\partial y_k^3}\frac{\partial y_k^3}{\partial net_k^3} = \delta_o^4\sum_o w_{ko}^4, \tag{34}$$

$$\delta_j^2 = -\frac{\partial E}{\partial net_j^2} = \left(-\frac{\partial E}{\partial y_o^4}\frac{\partial y_o^4}{\partial net_o^4}\frac{\partial net_o^4}{\partial y_k^3}\frac{\partial y_k^3}{\partial net_k^3}\right)\cdot \cdot\left(\frac{\partial net_k^3}{\partial y_j^2}\frac{\partial y_j^2}{\partial net_j^2}\right) = \sum_k \delta_k^3 y_k^3, \tag{35}$$

and the update law of $\mu_{ij}$and$\sigma_{ij}$are obtained as

$$\Delta\mu_{ij} = -\eta_\mu\frac{\partial E}{\partial \mu_{ij}} = -\eta_\mu\frac{\partial E}{\partial net_j^2}\frac{\partial net_j^2}{\partial \mu_{ij}} = -\frac{\eta_\mu\delta_j^2}{\sigma_{ij}}, \tag{36}$$

$$\Delta\sigma_{ij} = -\eta_\sigma\frac{\partial E}{\partial \sigma_{ij}} = -\eta_\sigma\frac{\partial E}{\partial net_j^2}\frac{\partial net_j^2}{\partial \eta_\sigma} = \eta_\sigma\delta_j^2\frac{\left(\mu_{ij}-x_i^2\right)}{\left(\sigma_{ij}\right)^2}, \tag{37}$$

where η_μ and η_σ are the learning rates of the translation and dilation of the mother wavelet. Thus, the weight of the output layer, translation and dilation of the mother wavelet are updated as follows:

$$w_{ko}^4(N+1) = w_{ko}^4(N) + \Delta w_{ko}^4, \tag{38}$$

$$\mu_{ij}(N+1) = \mu_{ij}(N) + \Delta\mu_{ij}, \tag{39}$$

$$\sigma_{ij}(N+1) = \sigma_{ij}(N) + \Delta\sigma_{ij}, \tag{40}$$

where N is the number of iterations.

To overcome the difficulty in calculation of $\partial\theta/\partial net_o^4$ in (32) which is caused by nonlinear uncertainties of PMSM drive system and to increase the online learning rate of the weights, (32) is replaced by an approximation law as follows:

$$\delta_o^4 = e_\theta + e_\theta\left(1 - z^{-1}\right). \tag{41}$$

The adaptive learning rates, which are modeled based on a discrete-type Lyapunov function to confirm the convergence of the speed tracking error, is adopted in order to train the WNN effectively [19]:

$$\eta_\omega = \lambda/R_u, \tag{42}$$

$$\eta_\mu = \eta_\sigma = \eta_\omega\left[\left|w_{ko}^4\right|_{\max}\left(2e^{-0.5}/\left|\sigma_{ij}\right|_{\min}\right)\right]^{-2}, \tag{43}$$

where λ is a positive constant gain, and R_u is the number of nodes in the wavelet layer.

3.3 Observed error compensator

According to the universal approximation theorem, there exists approximation error between the approximated results of WNN and the true value so that an observed error compensator is used to compensate the observed error induced by the WNN uncertainty observer and to further guarantee the stable characteristic of the whole PMSM control system.

Define $\bar{L} = (\bar{L}_1, \bar{L}_2, \bar{L}_3)$ and $\hat{L} = \left(\hat{L}_1, \hat{L}_2, \hat{L}_3\right)$, then the approximation error is obtained

$$E = L - \hat{L}, \tag{44}$$

where $E = (E_1, E_2, E_3)$ is a compact form of observed error. To develop the adaptation law of the observed error compensator, the Lyapunov candidate function is defined as

$$V = V_2 + \frac{1}{2}e_q^2 + \frac{1}{2}e_d^2 + \frac{1}{2k_4}\left(E - \hat{E}\right)\left(E - \hat{E}\right)^T, \tag{45}$$

where $\hat{E}$ is estimated value of E,$k_4 > 0$. Assume that controller can compute fast enough so that E could be treated as a constant. Considering the compensated error, control law (14), (25) and (26) can be rewritten as follows:

$$i_q^* = \frac{2J}{3p\psi}\Bigg[\left(\frac{B}{J} - a\right)\omega + a\dot{\theta}_m - b\left(s + c \cdot \mathrm{sgn}(s)\right) - \\ - \hat{L}_1 - \hat{E}_1\Bigg], \tag{46}$$

$$u_q = L\Bigg[\left(\frac{R}{L} + \frac{B}{J} - a\right)i_q - p\omega i_d - \left(\frac{2B}{3p\psi}\left(\frac{B}{J} - a\right) - \\ - \frac{p\psi}{L}\right)\omega - \hat{L}_2 - \hat{E}_2 - k_2 e_q\Bigg], \tag{47}$$

$$u_d = L\left(\frac{R}{L}i_d - p\omega i_q - \hat{L}_3 - \hat{E}_3 - k_3 e_d\right). \tag{48}$$

Taking derivative of (45) and substituting (23-24), (47-48) into the resulting equation yield

$$\begin{aligned}
\dot{V} &= \dot{V}_2 + e_q\dot{e}_q + e_d\dot{e}_d - \frac{1}{k_3}\left(E - \hat{E}\right)\dot{\hat{E}}^T \\
&= -k_1 e_\theta^2 - \left[a^2 b e_\theta^2 + (2ab - 1)\,e_\theta e_\omega + b e_\omega^2\right] - bc\,|s| \\
&\quad - k_2 e_q^2 - k_3 e_d^2 + s\left(\bar{L}_1 - \hat{L}_1 - \hat{E}_1\right) + e_q(\bar{L}_2 - \\
&\quad \hat{L}_2 - \hat{E}_2) + e_d\left(\bar{L}_3 - \hat{L}_3 - \hat{E}_3\right) - \frac{1}{k_4}\left(E - \hat{E}\right)\dot{\hat{E}}^T \\
&= -k_1 e_\theta^2 - \left[a^2 b e_\theta^2 + (2ab - 1)\,e_\theta e_\omega + b e_\omega^2\right] - \\
&\quad - bc\,|s| - k_2 e_q^2 - k_3 e_d^2 + e\left(E - \hat{E}\right)^T - \\
&\quad - \frac{1}{k_4}\left(E - \hat{E}\right)\dot{\hat{E}}^T,
\end{aligned} \tag{49}$$

where $e = (ae_\theta + e_\omega, e_q, e_d)$, then the adaptation laws for $\hat{E}$ are designed as follows:

$$\dot{\hat{E}} = k_4 e. \tag{50}$$

The schematic of the overall control system with proposed adaptive WNN backstepping sliding mode controller is shown in Fig. 2.

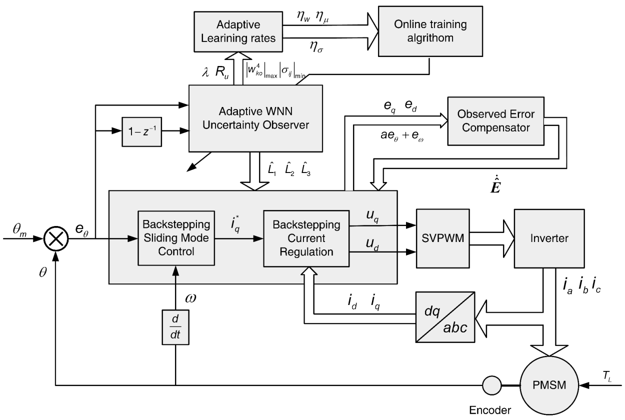

Fig. 2. Structure of proposed adaptive WNN backstepping sliding mode control system for PMSM drive

3.4 Stability analysis

Theorem 1 *For the PMSM position tracking drive system given in* (1)–(4)*, under the action of the proposed adaptive controller (46-48) and error compensator* (50) *with restriction* (19)*, the tracking errors in*(8)*,(21-22) will converge to zero and the whole closed system can be guaranteed to be asymptotically stable.*

Proof: To address the stability analysis of PMSM position tracking drive system, substituting (50) into (49) yield:

$$\dot{V} = -k_1 e_\theta^2 - \left[a^2 b e_\theta^2 + (2ab - 1)\,e_\theta e_\omega + b e_\omega^2\right] - bc\,|s| - \\ - k_2 e_q^2 - k_3 e_d^2. \tag{51}$$

If the constants a and b are supposed to hold the restrain (19), (17) will be satisfied, (51) will be negative semi-definite, namely

$$\dot{V} \leq 0, \tag{52}$$

which implies V is nonincreasing and therefore has a limit as $t \to \infty$ and $\dot{V}$ is uniformly continuous since $\ddot{V}$ is bounded. As a result, e_θ, e_ω, e_d and e_d are bounded which indicates that all the signals in the closed loop system remain bounded for all time. Define positive definite function Θ as:

$$\Theta\left(e_\theta, e_\omega, e_q, e_d\right) = k_1 e_\theta^2 + \left[a^2 b e_\theta^2 + (2ab-1)\, e_\theta e_\omega + \right. \\ \left. + b e_\omega^2\right] + bc\left|s\right| + k_2 e_q^2 + k_3 e_d^2. \quad (53)$$

Integrating both sides of (53) with respect to time, the following result is obtained

$$\int_0^t \Theta\left(e_\theta(t), e_\omega(t), e_q(t), e_d(t)\right) dt = -\int_0^t \dot{V}(t)\, dt \\ = V(0) - V(t) \geq 0. \quad (54)$$

Since $V(0)$ is bounded and $V(0)$ is nonincreasing and bounded, the result follows that

$$\lim_{t\to\infty} \int_0^t \Theta\left(e_\theta(t), e_\omega(t), e_q(t), e_d(t)\right) dt < \infty. \quad (55)$$

Furthermore, $\Theta(t)$ is uniformly continuous as $\dot{\Theta}(t)$ is bounded. From Barbalat's lemma [23], it can be conclude that

$$\lim_{t\to\infty} \Theta\left(e_\theta(t), e_\omega(t), e_q(t), e_d(t)\right) = 0, \quad (56)$$

which means that all of the tracking errors of the PMSM drive system will converge to zero as $t \to \infty$. In the other words, the asymptotical stability of the whole closed loop system can be guaranteed by employing proposed adaptive WNN backstepping sliding mode controller. ■

4 SIMULATION RESULTS ANALYSIS

The nominal parameters of the PMSM used in the simulation are given as: rated power: 0.5 kW; rated speed: 3000 rpm; DC voltage: 220 V; stator per phase resistance and inductance: $\bar{R} = 8.4\ \Omega$ and $\bar{L} = 18.7$ mH, respectively; viscous friction coefficient $\bar{B} = 0.0001$ Nms; rotor moment of inertia $\bar{J} = 4 \times 10^{-4}$ kg m^2; permanent magnet flux $\bar{\psi} = 0.14$ Wb; number of pole pairs: 3. The parameters of proposed controller are chosen as: $k_1 = 10$, $k_2 = 10000$, $k_3 = 100$, $k_4 = 10$, $a = 10$, $b = 1000$, and $c = 1$. Moreover, initial conditions of all the estimated parameters are assumed to be equal to zero for observing the exact adaptation performance. In order to investigate the performance of the proposed controller, two cases, nominal and the one all parameter uncertainties, are carried out. The two simulation cases with periodic sinusoidal command input are addressed as follows:

$$\text{Case1}: R = \bar{R}, L = \bar{L}, B = \bar{B}, J = \bar{J}, \psi = \bar{\psi}, T_L = 0,$$
$$\text{Case2}: R = 6\bar{R},\ L = 5\bar{L},\ B = 4\bar{B},\ J = 2\bar{J},\ \psi = 2\bar{\psi},$$

and $T_L = \{3, -3, 3\}$ Nm at $\{0.5, 1.5, 3.5\}$ s.

In addition, an optimally tuned PI controller and a backstepping sliding mode controller are comparatively implemented to show the better effectiveness of proposed WNNBSSM controller. The proportional and integral gains of PI controller are turned by Ziegler and Nichols criteria [24] and empirically optimized. The single backstepping sliding mode controller for comparison is designed as introduced above with the only difference there is no combined WNN observer and error compensator.

The reference position command input is chosen as $\theta_m = 10\sin(\pi t)$. The dynamic response of the PMSM drive system due to reference command input for the controllers including PI, BSSM and WNNBSSM in the first case is shown in Fig. 3 (a). The position tracking error is illustrated in Fig. 3 (b).

From the simulation results, it can be revealed that favorable tracking responses are obtained by all three control schemes on ideal condition without parameter perturbation and load disturbance. However, there are obviously differences among PI, BSSM and WNNBSSM in terms of comparisons on position tracking error. PI controller has the most error amplitude due to the nonlinear components existing in the PMSM dynamics. BSSM controller attenuates much effect caused by nonlinearity and attains better tracking performance. A more smooth position tracking trajectory is realized by WNNBSSM controller under the same condition.

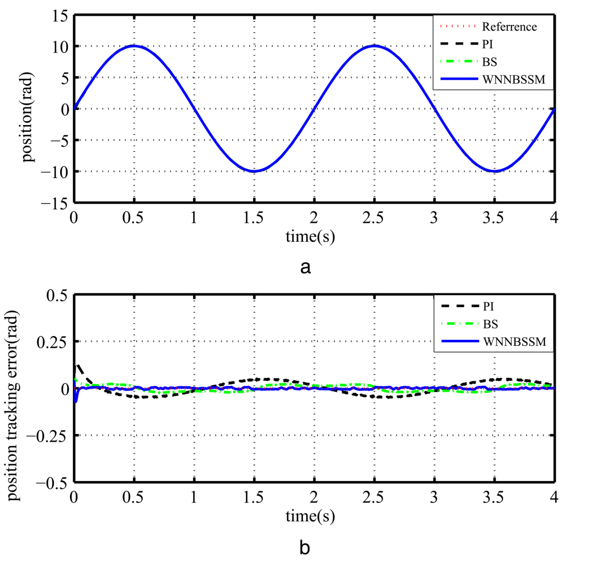

Fig. 3. Dynamic response of the PMSM drive system under nominal parameters: (a) position tracking performance for three controllers in Case1. (b) position tracking error for three controllers in Case1

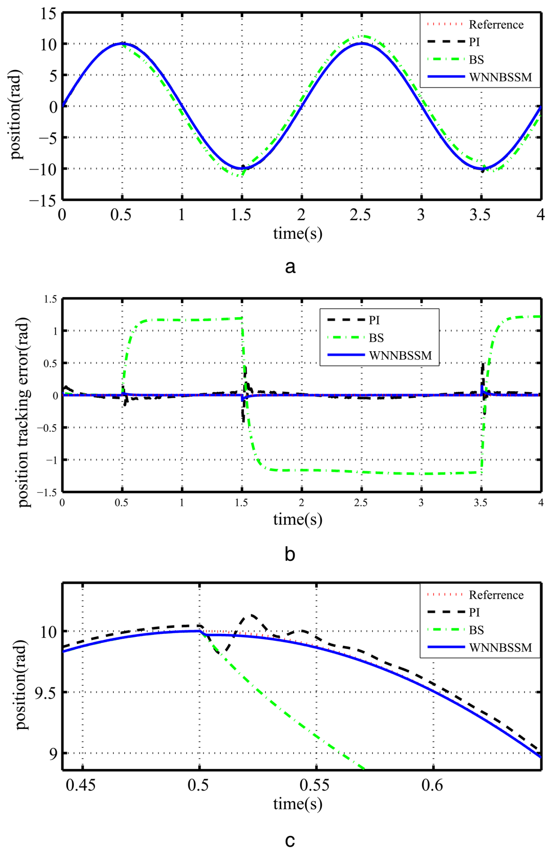

Fig. 4. Dynamic response of the PMSM drive system with uncertainties in Case2: (a) position tracking performance comparison of three controllers in Case2. (b) position tracking error of three controllers in Case2. (c) zoomed position tracking performance of three controllers at $t = 0.5$ *s*

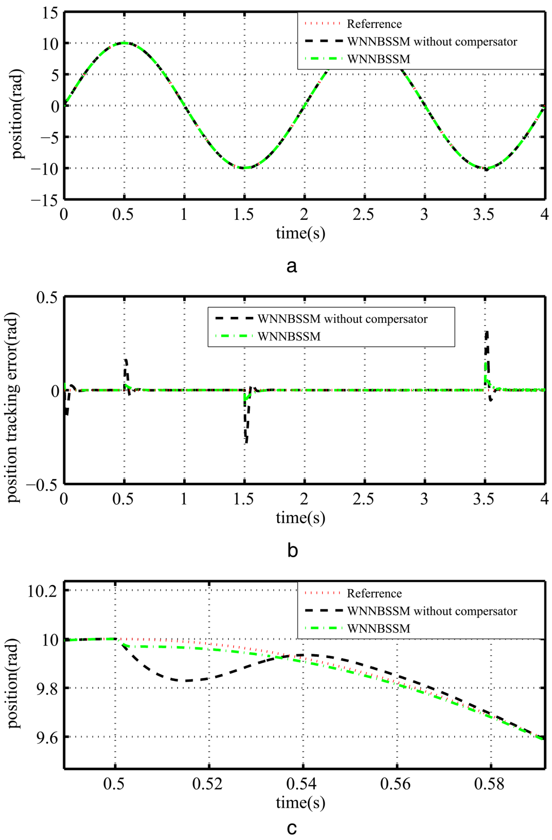

Fig. 5. Tracking response comparison for observed error compensator with uncertainties in Case2: (a) position tracking performance in Case2. (b) position tracking error in Case2. (c) zoomed position tracking performance at $t = 0.5$ *s*

To investigate parameter variation insensitiveness and disturbance rejection capabilities, simulations, three controllers are compared in simulation in Case2, and the tracking dynamics and tracking error are depicted in Fig. 4(a), Fig. 4(b) respectively. In order to observe the differences more clearly, a zoom display of position tracking comparison in detail at $t = 0.5$ s is shown in Fig. 4(c). It is evident that as the uncertainties of the PMSM parameters is existed, the response deviate significantly from the nominal case while the WNNBSSM controller assure the accurate operation and slightly influenced by load disturbance condition under the case of PMSM uncertainties. In addition, comparing to PI controller, the WNNBSSM controller provides a fast and accurate response, in other words, it quickly returns the rotor position to the reference command under parameter variation and load disturbance with a recovery time of 0.05 s and a maximum dip of 0.12 rad, whereas PI controller provides a slower response for the reference signal under the same operation condition with a long recovery time of about 0.08 s and a larger dipping in the position tracking of about of 0.51 rad. Moreover, the tracking error of the WNNBSSM controller can stably converge to zero after a small dip due to suddenly applied load, whereas there is steady error for PI controller.

Figure 5 illustrates performance of the proposed observed error compensator. Figure 5(a) gives the tracking response comparison between the WNNBSSM controller with observed error compensator and the one without, Fig. 5 (b) shows the relative tracking error and a zoom of position tracking at $t = 0.5$ s is shown in Fig. 5(c). From the simulation results, it is apparent that improvement in the control performance due to addition of the proposed observed error compensator can be observed in command tracking and parameters and load varying characteristics. It is obvious that the controller with compensator owns a maximum tracking dip as load applied is 0.14 rad, whereas the one without compensator reaches 0.33 rad. Therefore

Fig. 6. Experimental setup for the validation of WNNBSSM control

robust control characteristics under occurrence of uncertainties can be clearly granted by utilizing compensator proposed speed controllers.

It can be definitely conclude from the simulation results that favorable position tracking performance and strong robust characteristics are established by employing the proposed controller despite the presence of all the parameter uncertainties and load torque disturbance variation. Comparing to the other two controllers, the proposed WNNBSSM controller is more suitable to achieve high position precision robust tracking performance for the PMSM drive system under the occurrence of the parameter variation and external disturbance.

5 IMPLEMENTATION AND EXPERIMENTAL RESULTS

The designed algorithm has been implemented in a system called wavemaker. This equipment is used to produce particular types of waves in laboratory for marine engineering test. In wavemaker, host computer calculates the position command according to desired wave requirement and therefore the controller where the algorithm is applied can utilize the command to produce drive signals to drive PMSM by power unit. A wave paddle is coupled with the axis of the PMSM by screw which converts the rotary motion of the motor into linear motion such that the liquid in the flume is forced to move as expected. The wavemaker setup used in this work is shown in Fig. 6.

In experiments, sinusoidal and irregular waves command are tested to verify the wave paddle is able to follow reference position precisely for generating expected wave performance in the flume ultimately. All the parameters in the implemental wavemaker system are chosen to achieve the best control performance in experimentation considering the requirement for making waves. The experimental results are plotted and shown in Fig. 7 and Fig. 8, which plot the position tracking performance and position error on condition of giving regular wave and J spectral wave command respectively.

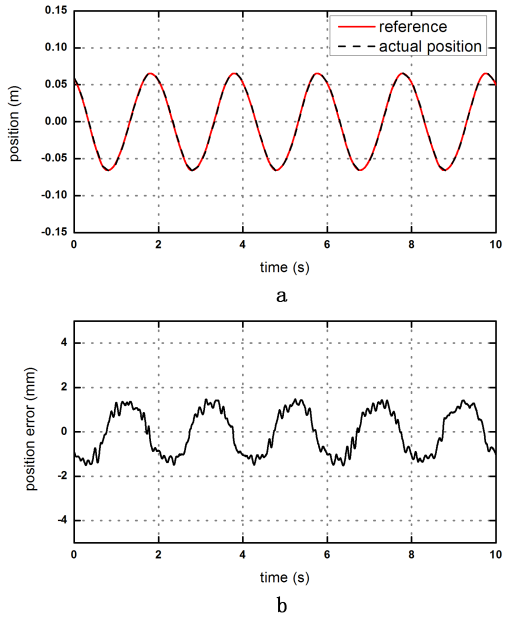

Fig. 7. Tracking performance and error on the command of the regular wave

It is evident from the results that the system implementing proposed control method has favorable position tracking performance that given position commands can be accurately followed and the tracking error is little. Thus the control purpose is achieved and performance is satisfied. As a result, we can conclude that the wavemaker system is capable of making required wave and the developed control scheme is suitable for this kind of systems which have higher tracking precision requirement.

6 CONCLUSION

A WNNBSSM control strategy for PMSM position servo system is proposed in this paper. Backstepping sliding mode technology is utilized to realize asymptotic tracking of position trajectory and stator currents and guarantee stability of whole the field-oriented PMSM drive system. A WNN is novelly used to online observe the uncertainties due to parameter variation and load disturbance in the PMSM dynamics to obtain high precision drive performance under all system uncertainties. Moreover, the observed error is compensated with an adaptive law derived in the sense of Lyapunov stability theory such that the tracking error can convergence to zero and stability

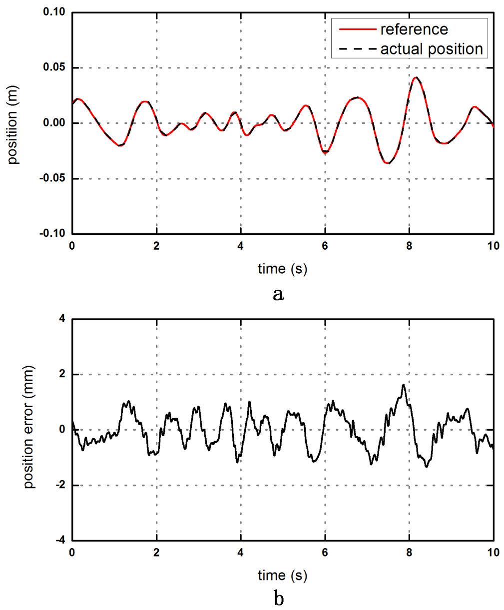

Fig. 8. Tracking performance and error on the command of the J spectral wave

is assured. Excellent performance is demonstrated by the successful application of the propose control scheme in the PMSM position servo system. The feasibility and effectiveness of the newly developed controller is evaluated by simulation and experimental results. It has been found that the WNNBSSM controller possesses favorable tracking and nonlinear uncertainty rejection capabilities and is suitable for PMSM position servo control.

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Received: 2013-01-08
Accepted: 2013-10-14