AN APPROXIMATE RESPONSE OF THE LARGE SYSTEM WITH LOCAL CUBIC NONLINEARITIES SUBJECTED TO HARMONIC EXCITATION

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ARTICLE INFO

***Article history*:**
Received: 23.10.2013.
Received in revised form: 25.03.2014.
Accepted: 26.03.2014.

Keywords:
Multi-DOFs
Nonlinear dynamic behavior
Local cubic nonlinearities
Model reduction
Harmonic resonance

Abstract:

This paper considers a multi-degree-of-freedom mechanical system with local cubic nonlinearities. A major concern is placed on nonlinear dynamic behaviors of the system subjected to a soft harmonic excitation under primary resonance condition. The classical model reduction method associated with the single modal resonance theory is employed to investigate the system and obtain a reduced dynamic model with only a single DOF (degree of freedom) under resonance condition. In the case of the soft excitation, the analytical expression of dynamic response and the frequency response characteristic equation can be derived from the reduced model of the system using the harmonic balance method. Some qualitative and quantitative results are then obtained. An example of ten-story nonlinear shear structure is included. Results from the reduction method of the system are in good agreements with those obtained from the numerical integration of the dynamic equation of the original system. This paper demonstrates an effective way in fast analysis of the multi-DOFs nonlinear system qualitatively and quantitatively, especially large scale multi-DOFs system at primary resonance state.

1 Introduction

Dynamic systems with local cubic nonlinearities are employed as models of various physical and engineering situations, such as Josephson junctions, a rotating flexible blade, buckled beams, ship dynamics, moored structures in the ocean, electrical circuits, fluid-film bearings in rotating machinery, dry friction and backlash phenomena in certain connections of mechanical systems, non-linear spring and damper supports in piping or vehicle systems, etc. Even though nonlinearities may constitute only a small part of the system, the dynamic behavior of the system is wholly nonlinear. Great interests are focused on nonlinear dynamic

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phenomena of these systems, such as bifurcations of nonlinear resonances (primary, subharmonic, superharmonic, and narrow band random resonances), and even transition to chaos, when forced by harmonic excitations. Research papers on these topics are in Refs. [1-5].

Brennan used the harmonic balance method (HBM) to determine the jump-up and jump-down frequencies of a softening and hardening lightly damped Duffing oscillator with linear viscous damping [1]. The analytical results are validated for a range of parameters by comparing the predictions with calculations from direct numerical integration of the equation of motion. In Ref. [2], the harmonic balance method was employed to analyze super or sub harmonic response of torsional system with two DOFs. Elnaggar studied harmonic, subharmonic, superharmonic and combination resonances of the additive type of self-excited two coupled-second order systems subjected to multi-frequency excitation. The theoretical results are obtained by the multiple-scales method [3]. Rajan studied the response of a Duffing oscillator to narrow-band random excitation by both multiple time scaling and stochastic averaging approaches [4]. Random jumps were observed to occur if the excitation bandwidth is sufficiently small. The random vibration was also involved in Ref. [5]. These analytical methods mentioned above have the advantage of producing a steady state solution without using artificial damping and by providing an analytical (as opposed to numerical) representation. However, they confined their performances to the system with low degree of freedom.

Quite often characteristic dynamic model for complex mechanical systems contains many degrees of freedom. For example, finite element analysis often results in the discrete models of continuous structural system, usually with hundreds of DOF. A direct analytical analysis of such multiple degrees of freedom nonlinear systems is generally quite difficult. In this case, the numerical methods are available and can give their solutions with enough numerical precision. However, only quantitative but not qualitative results are often obtained. Moreover, the time consumption of numerical computation in these problems is usually huge. Some serious numerical problems of stiff integration may/ might occur, especially for large scale multi-DOF nonlinear system. Additional criteria or algorithms in computation are required.

The aim of this paper is to fast investigate the nonlinear behavior of multi-DOF system with local cubic nonlinearities under harmonic excitation, both quantitatively and qualitatively. The major concern of this paper is focused on the nonlinear frequency-response dynamics of such a system under primary resonance condition. For the system with cubic nonlinearity, there are frequencies at which the vibration suddenly jumps up or down when it is excited harmonically with slowly changing excitation frequency. The frequencies at which these jumps occur depend upon whether the frequency is increasing or decreasing and whether the nonlinearity is hardening or softening. Between these frequencies, multiple solutions exist for a given frequency of excitation, and the initial conditions determine which of these solutions represents the response of the system. According to the author’s knowledge, little previous studies are involved in this problem. The difficulty of this problem lies on solving high dimensional differential equations in quality and quantity. In this paper, the classical model reduction method is first employed to reduce the size of the equations and an approximate reduced order equation is then obtained [6, 7]. Unfortunately, this reduced equation is not often analytically solved. It means that only quantitative but qualitative solutions are often obtained from the reduced model. To obtain the quantitative and qualitative solution of the reduced equation, the single mode resonance theory proposed by Zheng is applied in this paper [8]. The outline of this paper is as follows. The application of the reduction method associated with the single natural mode resonance theory in investigating nonlinear dynamic system with large DOFs is briefly introduced in section two. An illustratable example is examined and some significant results are obtained in the subsequent section. The results obtained from the presented method will be verified by numerically solving the original system and they are all in good agreement. Some important conclusions are drawn in the last section.

2 Mathematic fundamentals and formulations

The motion equations of the *n*-DOFs system with local cubic nonlinearities can be expressed in matrix form as:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}+\{F_N(x)\}=\{F_d(t)\}, \quad (1)$$

where $\{x\}$ is the n-vector of physical coordinates, [M], [C] and [K] are the($n\times n$) mass, damping and stiffness matrices respectively, $\{F_N\ (x)\}$ is the n-vector of cubic nonlinear applied force, and $\{F_d\ (t)\}$ is the vector of time dependent external excitations.
In this paper, the mass matrix [M] and the stiffness [K] are symmetric, the external excitations $\{F_d\ (t)\}$ are harmonic, and the damping matrix [C] in Eq. (1) is proportional to the mass and/or stiffness matrices for system, that is,

$$[C]=a[M]+b[K], \quad (2)$$

where the parameters a and b are constants.
The homogeneous undamped equation

$$[M]\{\ddot{x}\}+[K]\{x\}=0 \quad (3)$$

leads to eigenvalue or spectral matrix [Λ] and eigenvector matrix [Φ].
The spectral matrix is expressed by

$$[\Lambda]=diag\left(\begin{bmatrix}\omega_1 & \omega_2 & \cdots & \omega_n\end{bmatrix}\right) \quad (4)$$

and the eigenvector matrix is defined by

$$[\Phi]=\begin{bmatrix}\phi_1 & \phi_2 & \cdots & \phi_n\end{bmatrix}. \quad (5)$$

The eigenpairs (ω_i,ϕ_i) denote the i-th modal frequency and modal shape, i = 1, 2, …, n.
Introduce the transformation

$$\{x\}=[\Phi]\{q\}, \quad (6)$$

where $\{q\}$ is the n-vector of normal mode coordinates.
Substituting Eq. (6) into Eq. (1) yields

$$\begin{aligned}&[M][\Phi]\{\ddot{q}\}+[C][\Phi]\{\dot{q}\}+[K][\Phi]\{q\}\\&\quad+\{F_N([\Phi]\{q\})\}=\{F_d(t)\}\end{aligned} \quad (7)$$

Multiplying Eq. (7) by the transpose [Φ^T], one obtains

$$\begin{aligned}&[\Phi^T][M][\Phi]\{\ddot{q}\}+[\Phi^T][C][\Phi]\{\dot{q}\}+[\Phi^T][K][\Phi]\{q\}\\&\quad+[\Phi^T]\{F_N([\Phi]\{q\})\}=[\Phi^T]\{F_d(t)\}\end{aligned} \quad (8)$$

The eigenvector matrix satisfies the following orthogonality properites with respect to the mass and stiffness matrix

$$[\Phi^T][M][\Phi]=[I],\ [\Phi^T][K][\Phi]=[\Lambda], \quad (9)$$

where [I] is unit matrix.
In the case of Rayleigh damping, the damping term [Φ^T], [C], [Φ] in Eq. (8) can be written as follows

$$[\bar{C}]=[\Phi^T][C][\Phi]=diag\left(\begin{bmatrix}2\xi_1\omega_1 & 2\xi_2\omega_2 & \cdots & 2\xi_n\omega_n\end{bmatrix}\right) \quad (10)$$

where ξ_i is the i-th modal damping coefficient.
Using Eq. (9) and Eq. (10), Eq. (8) is then transformed into

$$\{\ddot{q}\}+[\bar{C}]\{\dot{q}\}+[\Lambda]\{q\}+[\Phi^T]\{F_N(q)\}=[\Phi^T]\{F_d(t)\} \quad (11)$$

or

$$\begin{aligned}&\ddot{q}_i+2\xi_i\omega_i\dot{q}_i+\omega_i^2q_i+\sum_{j=1}^{n}\phi_{ji}F_{Nj}\left(q_1\quad q_2\quad\cdots\quad q_n\right)\\&=\sum_{j=1}^{n}\phi_{ji}F_{dj}(t)\end{aligned} \quad (12)$$

Compared Eq. (1) with Eq. (11) or Eq. (12), one will find that they are equivalent in mathematics. Unlike Eq. (1), the terms in Eq. (11) or Eq. (12) expect the nonlinear force are uncouple.
The system with multiple degrees of freedom often has multiple eigenmodes. As forced by the excitation, the total response of the system is a sum of the response of full modes. In these modes, some modes take up a large proportion of the total time response, whereas other modes contribute much less toward the total response. Generally, the higher modes have a small portion in the total response of engineering system. It implies that the total response can be expressed by retaining only a few leading modes while ignoring the contribution of higher modes. That is, Eq. (6) can be written to

$$\{x\} \approx \sum_{i=1}^{l} \phi_i q_i \,, \tag{13}$$

where l denotes the number of leading modes.
Usually, the number l is no more than to the number n – the degree of freedom of system. Using Eq. (12) and Eq. (13), one can obtain the solutions with less effort. This is the key of the classical model reduction method. More details are found in Refs. [6, 7].
Quite often analytical solutions of the reduced equations (seen in Eq. (12) and Eq. (13)) are not available. In many situations, one will care for the system under resonance state. For example, the vibration amplitude of the rotor system will reach the maximum as the spinning speed of its shaft passes through the critical natural frequency. Now, the case of primary resonance is discussed. When the system enters into the resonance state, for example the primary resonance of the j-th mode, the response of the j-th mode is observed to take up large proportion of total response and other modes contribute less toward the total response. Hence, Eq. (13) can be further reduced to

$$\{x\} \approx \{\phi_j\} q_j \,. \tag{14}$$

This phenomenon is called "single natural mode resonance theory".
Substituting Eq. (14) into Eq. (11) or Eq. (12), one can obtain the dynamic equation of the resonance mode q_j

$$\ddot{q}_j + 2\xi_j \omega_j \dot{q}_j + \omega_j^2 q_j + \sum_{k=1}^{n} \phi_{kj} F_{Nk}\left(q_j\right) = \sum_{k=1}^{n} \phi_{kj} F_{dk}\left(t\right) \tag{15}$$

and the dynamic equations of the non-resonance modes (for $i \neq j$)

$$\ddot{q}_i + 2\xi_i \omega_i \dot{q}_i + \omega_i^2 q_i + \sum_{k=1}^{n} \phi_{ki} F_{Nk}\left(q_j\right) = \sum_{k=1}^{n} \phi_{ki} F_{dk}\left(t\right) . \tag{16}$$

Considering cubic nonlinearities and harmonic excitation (for instance cosine excitation), Eq. (15) can be expressed compactly as

$$\ddot{q}_j + 2\mu \dot{q}_j + \omega_j^2 q_j + \beta q_j^3 = F_0 \cos(\Omega t) \,, \tag{17}$$

where the parameters μ, β, F_0 and Ω are constants.

Noting that Eq. (17) is the normal Duffing oscillator with a single DOF, its solution can be easily obtained by analytic methods [1-5] or numerical methods. Then, the approximate response of the system Eq. (1) can be determined in terms of Eq. (14) and Eq. (17). The qualitative and quantitative conclusions can also be drawn from Eq. (14) and Eq. (17).
In this paper, the excitation is assumed to be soft. In a weak vibration, the HBM method is employed to derive the analytical solution of Eq. (17). An approximate solution of first order can be expressed as

$$q_j = a\cos(\Omega t + \varphi) \,, \tag{18}$$

where a is the amplitude of response, Ω and φ are frequency and phase.
Substituting Eq. (18) into Eq. (17) yields

$$\left(\omega_j^2 - \Omega^2\right) a + \frac{3}{4} \beta a^3 = F_0 \cos\varphi \,, \tag{19}$$

$$2\mu a \Omega = F_0 \sin\varphi \,. \tag{20}$$

Using Eq. (19) and Eq. (20), the approximate frequency response curve for system Eq. (1) is determined by

$$\left[\left(\omega_j^2 - \Omega^2\right) a + \frac{3}{4} \beta a^3\right]^2 + \left[2\mu a \Omega\right]^2 = F_0^2 \,. \tag{21}$$

The method based on the single mode resonance theory provides a new way to investigate the nonlinear system with multi-DOFs. However, this method is valid under some conditions. Here, we discuss and summarize these conditions.
Generally, Eq. (14) is usually satisfied under the following conditions:
1) The response of resonance mode is leading in the total response, while the non resonance modes contribute less toward the total response.
2) The resonance mode does not interact with other non-resonance modes. It implies the phenomenon internal resonance does not occur.
3) For multi-frequency excitation, combination resonance may/might arise, but the response of combination resonance is not preponderant compared with that resonance mode of interest.

3 Numerical Examples

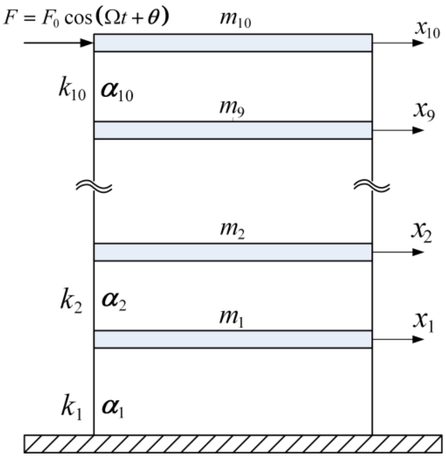

Figure 1. Ten-story building model.

The ten-story shear building is shown in Fig. 1. The structure has cubic stiffness. The stiffness of the *j*-th story in building is described by linear component with coefficient k_j and nonlinear component with coefficient α_j in which $j = 1, 2, 3, \ldots, 10$. The mass of the corresponding story is m_j. x_j denotes the transverse displacement of the *j*-th story of the building. The harmonic excitation induced by wind load is assumed to be applied at the top story of the building. F_0 is the amplitude of excitation. The parameters Ω and θ are the frequency and phase of excitation, respectively.

The dynamic equations of motion for the building can be written in matrix form as

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=\{F(t)\}+\{F_N(X)\}. \quad (22)$$

In Eq. (22), $\{X\}$, $\{\dot{X}\}$ and $\{\ddot{X}\}$ denote displacement, velocity and acceleration vector of the structure and are expressed as

$$\{X\}=\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \{\dot{X}\}=\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix}, \{\ddot{X}\}=\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \vdots \\ \ddot{x}_n \end{bmatrix} \quad (23)$$

and the mass $[M]$, stiffness $[K]$ matrices are equal to

$$[M]=diag\left(\begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_9 \\ m_{10} \end{bmatrix}\right),$$

$$[K]=\begin{bmatrix} k_1+k_2 & -k_2 & & & \\ -k_2 & k_2+k_3 & -k_3 & & \\ & \ddots & \ddots & \ddots & \\ & & -k_9 & k_9+k_{10} & -k_{10} \\ & & & -k_{10} & k_{10} \end{bmatrix}, \quad (24)$$

and the damping $[C]$ is in proportion to the mass $[M]$ and the stiffness $[K]$, and then defined by

$$[C]=a[M]+b[K], \quad (25)$$

where the parameters *a* and *b* are constant.

The excitation force $\{F(t)\}$ and the nonlinear component of the restoring force $\{F_N(X)\}$ are equal to

$$\{F(t)\}=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ F_0\cos(\Omega t) \end{bmatrix},$$

$$\{F_N(X)\}=\begin{bmatrix} \alpha_2(x_2-x_1)^3-\alpha_1 x_1^3 \\ \alpha_3(x_3-x_2)^3-\alpha_2(x_2-x_1)^3 \\ \vdots \\ \alpha_{10}(x_{10}-x_9)^3-\alpha_9(x_9-x_8)^3 \\ -\alpha_{10}(x_{10}-x_9)^3 \end{bmatrix}. \quad (26)$$

In this section, the parameters are:

$k = 100000$,	$m_i = 1000$ kg,
$\alpha_i = 20\ k^2$,	$i = 1, 2, \ldots, 10$,
$k_1 = k_2 = 5k$ N/m,	$k_3 = k_4 = 4k$ N/m,
$k_5 = k_6 = 3k$ N/m,	$k_7 = k_8 = 2k$ N/m,
$k_9 = k_{10} = k$ N/m,	$F_0 = 1.5$ N, $\theta = 0$,
$a = 0.0171$,	$b = 0.0016$.

3.1 Linear system analysis

Let the coefficients α_i in Eq. (22) be equal to zeros for $i = 1, 2, \ldots, 10$, the dynamic equation of the linear system for the structure shown in Fig. 1 is then obtained.

A computer program based on the Householder QR method was used to calculate the natural frequencies and modes of vibration for the linear system. Table 1 shows the first five natural frequencies in rad/s, and modal damping for the structure shown in Fig. 1. The modes of vibration normalized with respect to the mass matrix are listed in Table 2.

Table 1. The First Five Natural Frequencies and Modal damping.

Mode	Freq. [rad/s]	Modal Damping [ξ_i]
1	2.7502	5.3‰
2	6.6523	6.6‰
3	10.8492	9.5‰
4	15.0825	12.6‰
5	17.5917	14.6‰

The frequency response characteristic curve for the transverse displacement of the tenth story in building is plotted in Fig. 2.

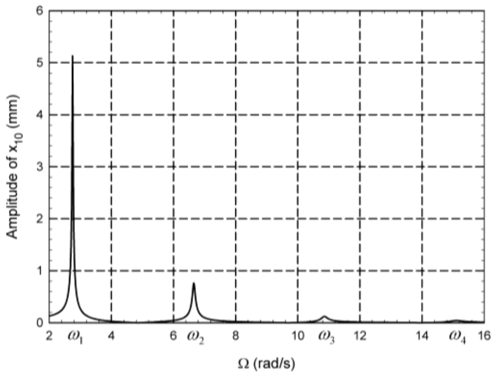

Figure 2. The frequency response curve of coordinate x_{10}.

Table 2. Normal Modes of Vibration.

Level	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
10	0.0166	-0.0173	0.0134	0.0128	0.0090
9	0.0153	-0.0096	-0.0024	-0.0163	-0.0189
8	0.0129	0.0023	-0.0153	-0.0083	0.0117
7	0.0112	0.0077	-0.0128	0.0051	0.0089
6	0.0091	0.0115	-0.0027	0.0127	-0.0076
5	0.0075	0.0123	0.0051	0.0081	-0.0108
4	0.0056	0.0113	0.0109	-0.0026	-0.0028
3	0.0042	0.0093	0.0120	-0.0092	0.0054
2	0.0026	0.0062	0.0096	-0.0105	0.0094
1	0.0013	0.0033	0.0055	-0.0068	0.0068

As shown in Fig. 2, the first peak amplitude appears at frequency Ω equaling to 2.76 rad/s. When the excitation frequency is turned to approach the second natural frequency, the other peak amplitude appears. The value of the first peak amplitude is 5.1 mm, and the second peak amplitude is about one eighth of the first peak amplitude. Fig. 2 also shows that the vibration of higher frequencies is very weak.

3.2 Nonlinear system analysis

According to the results in linear system analysis, we focus on the dynamic behavior of the system at excitation frequency $\Omega \approx \omega_1$ and $\Omega \approx \omega_2$. In this section, the excitation is assumed to be soft.

3.2.1 The case of excitation frequency $\Omega \approx \omega_1$

First, Eq. (22) is written to the equivalent equations in form of mode coordinates like Eq. (12) by transforming Eq. (6). At given excitation frequency and initial integral condition $q_{i0} = \dot{q}_{i0} = 0$ for i = 1, 2, …, 10, the dynamical response for mode coordinates can be obtained from the numerical integration of the mode equations. In this paper, the RKF45 method with adaptive step size was employed to calculate integral problems. Numerical results are plotted in Fig. 3. Fig. 3 illustrates the response of the first four modes with regard to time. Fig. 3a, Fig. 3b, Fig. 3c and Fig. 3d are for excitation frequency 2.76 rad/s, 2.8 rad/s, 2.84 rad/s and 2.9 rad/s, respectively. From the data curve plotted in Fig. 3, the response of the first mode is leading compared with that of other modes.

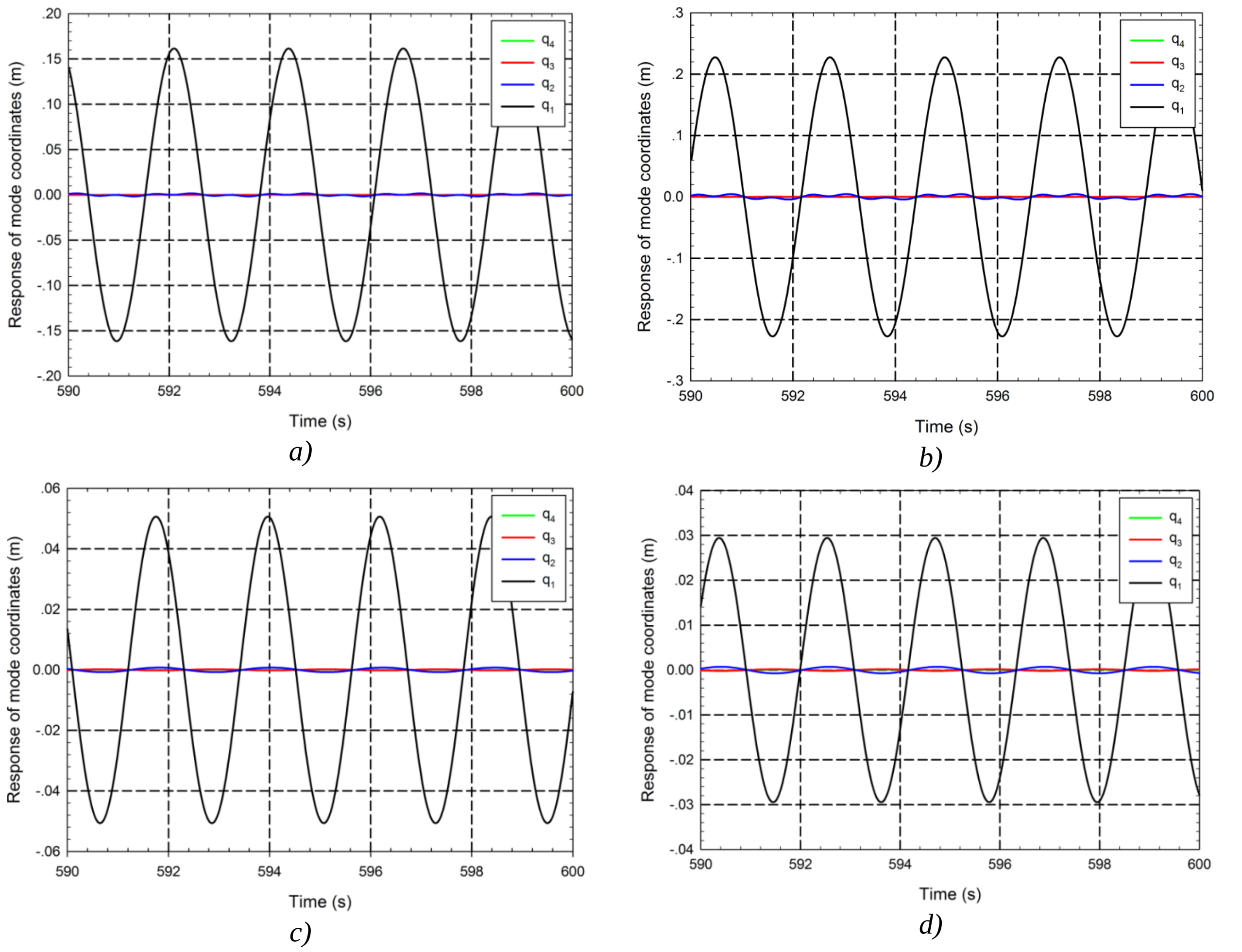

Figure 3. the time response of the first four modes for excitation frequency $\Omega \approx \omega_1$: a) Ω = 2.76, b) Ω = 2.8, c) Ω = 2.84, d) Ω = 2.9, unit: rad/s.

Secondly, we examine the dynamical response of different reduction models at different given excitation frequencies. Two interested reduced models include the ones governed by the leading mode q_1, and the leading mode q_1 with the minor mode q_2, respectively. By comparison, the original model with full modes is also examined. All dynamic models are solved numerically by the RKF45 method. The results are shown in Fig. 4.

In Fig. 4, the solid line plus circle denotes the results from the model for one mode; the dashed line is for the results from the model for two modes; the dash point line is for the results from the original model for full modes. Fig. 4 shows that the total response of system is approximately controlled by the leading mode q_1 at different frequencies in range of the first natural frequency. Fig. 4b and Fig. 4c are for the same excitation frequency but different initial conditions. The plot in Fig. 4b and Fig. 4c implies that there are multiple steady period solutions in the region of excitation frequency 2.82 rad/s.

In order to determine the frequency region for coexistence of multiple steady state solutions, the approximate frequency response characteristic equation was derived based on the reduced model controlled by the leading mode q_1.

This reduced model for the leading mode q_1 is expressed as

$$\ddot{q}_1 + 2\mu_1\dot{q}_1 + \omega_1^2 q_1 + \beta_1 q_1^3 = F_1 \cos(\Omega t)\,. \qquad (27)$$

In Eq. (27), the parameters are: μ_1 = 0.0146, β_1 = 0.0146 and F_1 = 0.0249. Using these parameters, the frequency response curve is depicted by the dash line in Fig. 5. As shown in Fig. 5, the vibration suddenly jumps down at frequency 2.8685 rad/s when the excitation slowly increases.

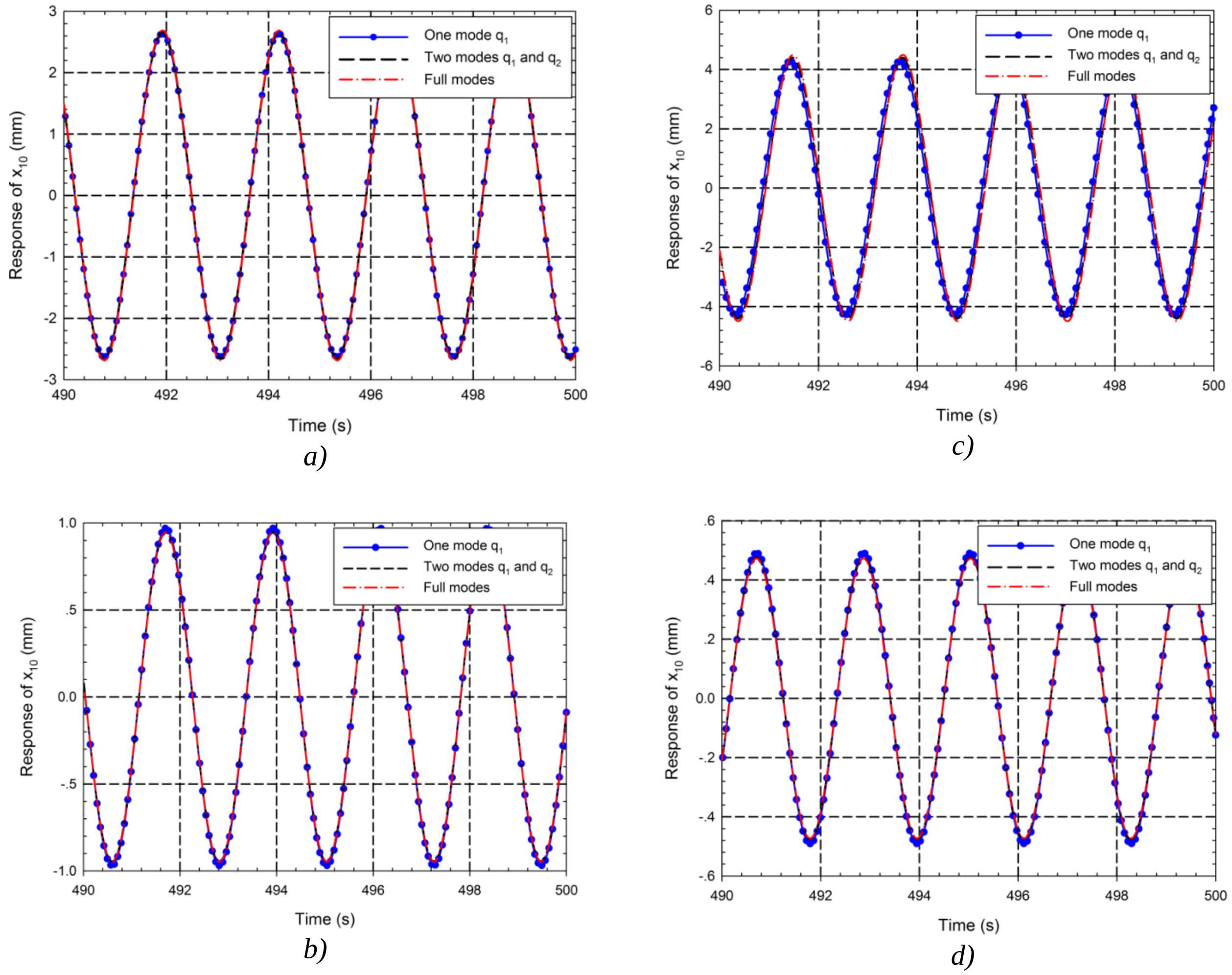

Figure 4. Comparison of response of coordinate x_{10} from variable types of models at different given excitation frequency Ω: solid line plus circle for one mode; dashed line for two modes; dash point line for full modes: a) Ω = 2.76, b) Ω = 2.82, c) Ω = 2.82, d) Ω = 2.9, unit: rad/s.

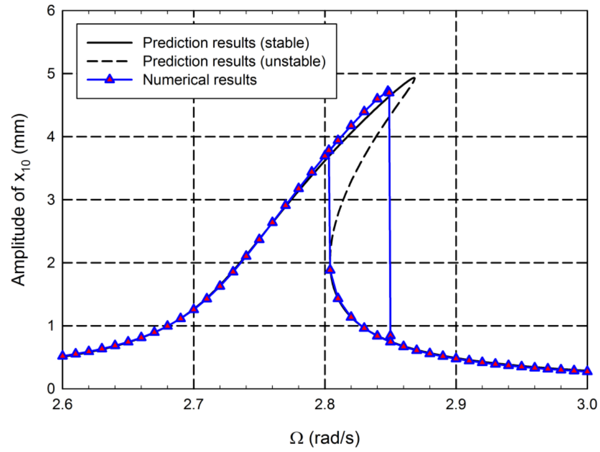

Figure 5. Frequency response of x_{10} for the first primary resonance.

While the excitation frequency slowly decreases, the vibration suddenly jumps down at frequency 2.803 rad/s. There are two steady period solutions in the frequency range from 2.803 rad/s to 2.8685 rad/s. The information gives a qualitative prediction for nonlinear dynamic behavior of the system. To validate the prediction results, the frequency response relation is also obtained from the response of system Eq. (22) solved numerically. The corresponding results are plotted by the solid line plus triangle in Fig. 5. Obviously, the prediction results match well with numerical results.

3.2.2 The case of excitation frequency $\Omega \approx \omega_2$

Similarly, we examined the dynamic behavior of the structure as excitation frequency approaching the second natural frequency in the same way as used in the previous section. Fig. 6 shows the first six mode responses at different given excitation frequencies. Fig. 6a–d are for Ω equaling to 6.7 rad/s, 6.79 rad/s, 6.82 rad/s and 6.9 rad/s, respectively. In Fig. 6, the black line denotes the time response of the second mode at given conditions. Clearly, the second mode is leading as excitation frequency being in the domain of the second mode resonance.

Fig. 7 shows the response of the vibration transverse displacement of the tenth story of the building for different excitation frequencies in the domain of the second natural frequency. The results are obtained from three types of equivalent models: the reduction model governed by the leading mode q_2, the reduction model controlled by the two modes q_1 and q_2, the original model. The plots in Fig. 7 indicate both and the two reduction dynamic models generate a satisfied solution for the original dynamic system. There is only little difference between them being mainly determined by the second mode in the domain of second resonance. In Fig. 7, Fig. 7a, Fig. 7b– c, and Fig. 7d are for Ω = 6.7 rad/s, Ω = 6.82 rad/s and Ω = 6.94 rad/s, respectively. Fig. 7(b) and Fig. 7 c) are both for Ω = 6.82 rad/s but under different initial conditions.

An approximate dynamic equation for the building shown in Fig. 1 can be defined by the leading mode q_2 under second primary resonance as:

$$\begin{cases} \ddot{q}_2 + 2\mu_2\dot{q}_2 + \omega_1^2 q_2 + \beta_2 q_2^3 = F_2\cos(\Omega t) \\ \{x\} \approx \{\phi_2\} q_2 \end{cases} \tag{28}$$

The parameters in Eq. (28) are: μ_2 = 0.044, β_2 = 2488.1 and F_2 = 0.0259. The frequency response curve obtained from Eq. (28) is plotted in Fig. 8.

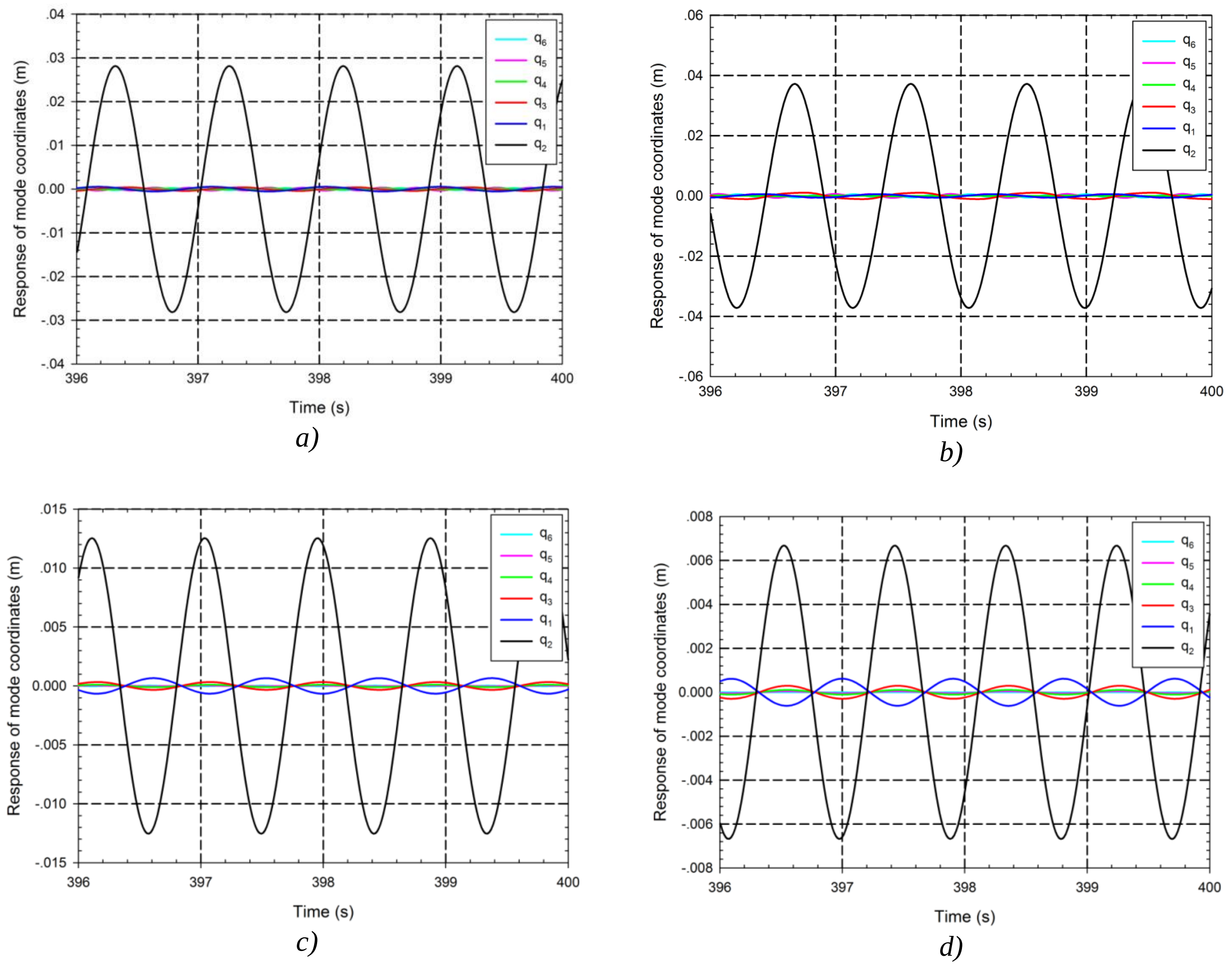

Figure 6. The time response of the first six modes for excitation frequency $\Omega \approx \omega_2$: a) Ω = 6.7, b) Ω = 6.79, c) Ω = 6.82, d) Ω = 6.94, unit: rad/s.

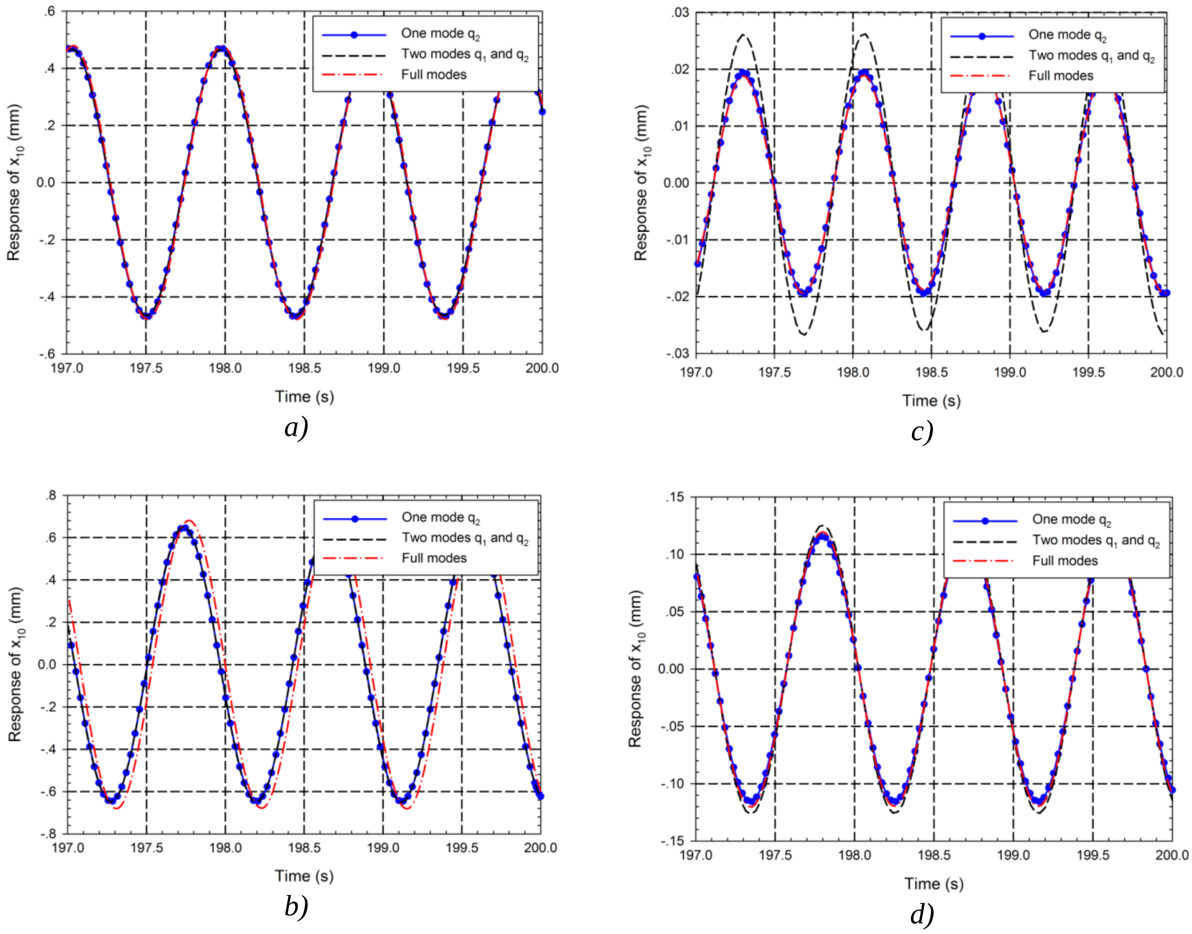

Figure 7. Comparison of response of coordinate x_{10} *from variable types of dynamic models at different given excitation frequency Ω: solid line plus circle for one mode; dashed line for two modes; dash point line for full modes: a) Ω* = 6.7*, b) and c) Ω* = 6.82*, d) Ω* = 6.94*, unit: rad/s.*

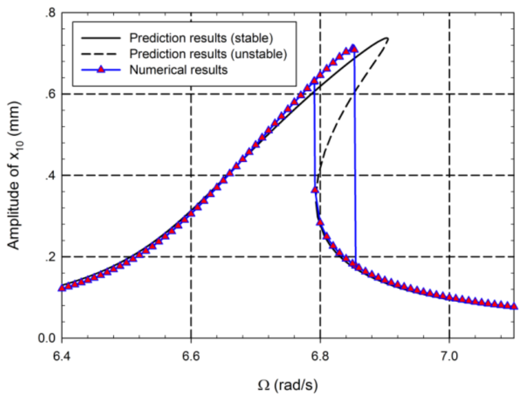

Figure 8. Frequency response curve for $\Omega \approx \omega_2$.

The solid line in frequency response curve denotes the stable solutions, while the dashed line is for the unstable solutions. By comparison, the frequency response relation obtained from the numerical response is also plotted in Fig. 8, seen for the solid line plus circle.

As shown in Fig. 8, the jump in frequency response curve appears at *Ω* = 6.79 rad/s or *Ω* = 6.83 rad/s according to numerical results. The presented method in this paper predicts that the jump frequencies are 6.79 rad/s and 6.852 rad/s. Clearly, there is only a slight difference between the numerical results and the prediction results.

4 Conclusion

In this paper an application of the classical model reduction method associated with the single modal resonance theory is introduced to investigate the MDOF system with local cubic nonlinearities under soft harmonic excitation. An illustrative example of a ten-story nonlinear shear structure is also considered. The analytical solutions of nonlinear response and frequency-response behaviors of the system are derived from the reduction model with only a single DOF at primary resonance, which generates the qualitative and quantitative results for the original system. These results are validated by comparing with calculations from direct numerical integration of the equation of motion for original system and good agreements are obtained. This paper gives a new way for making a fast analysis in quality and quantity of the nonlinear dynamic behaviors of the MDOF system, especially large scale multi-DOF system.

Acknowledgment

This research was supported by the Key Research Fund of Xihua University (No. Z1020212); the Key Laboratory for Manufacturing and Automation of Provincial Universities (No. SZjj2011-016); the Project of Education Department of Sichuan (No. 11ZB014) and Doctoral Fund Project of Chengdu Technological University (No. KYBS12006).
The first author would like to thank C.J. Yang for his help in this work.

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