GLASNIK MATEMATIČKI
Vol. 42(62)(2007), 109 – 116

A COHOMOLOGICAL CHARACTERIZATION OF SHAPE DIMENSION FOR SOME CLASS OF SPACES

JACK SEGAL AND STANISŁAW SPIEŻ
University of Washington, USA and Polish Academy of Sciences, Poland

Dedicated to Professor Sibe Mardešić on his eightieth birthday

ABSTRACT. It is known that if X is a metric compact space (compactum) with finite shape dimension $\mathrm{sd}(X) \neq 2$, then $\mathrm{sd}(X)$ is equal to the generalized coefficient of cyclicity $c[X]$, equivalently $\mathrm{sd}(X \times S^1) = \mathrm{sd}(X) + 1$. In general, these equalities do not hold in the case of compacta with $\mathrm{sd}(X) = 2$. In this paper we prove that if X is a regularly 1-movable connected pointed space with $\mathrm{sd}(X) = 2$, then $c[X] = 2$.

1. INTRODUCTION

The *shape dimension* of compact metric spaces was first defined (under the name of *fundamental dimension*) by K. Borsuk [B]. J. Dydak [D] generalized this notion by defining a shape invariant for topological spaces called *deformation dimension* ddim as follows: *for a (topological) space X,* $\mathrm{ddim} X \leq n$ *if any (continuous) map f from X to a polyhedron P is deformable into the n-skeleton $P^{(n)}$ of P, i.e., there is a homotopy $H : X \times [0,1] \to P$ such that $H(x,0) = x$ and $H(x,1) \in P^{(n)}$ for each $x \in X$.* Deformation dimension agrees with the notion of *shape dimension* sd for topological spaces introduced by S. Mardešić and J. Segal [M-S]. It is known that if $(X, *)$ is a pointed space then $\mathrm{sd}(X, *) = \mathrm{sd}(X)$.

S. Nowak [N] has proved that if X is a compact metric space such that $\mathrm{sd}(X) < \infty$ and $\mathrm{sd}(X) \neq 2$ then $\mathrm{sd}(X) = c[X]$, where $c[X]$ (called the *generalized coefficient of cyclicity of* X) is the maximum (finite or infinite) of all integers n such that $H^n(X, \mathfrak{L}) \neq 0$ for some generalized local system $\mathfrak{L}$

2000 *Mathematics Subject Classification.* 54F45, 55P55.
Key words and phrases. Shape dimension, regularly movable, cohomological dimension, Stallings-Swan theorem.

of Abelian groups on X (see [N, N-S1, N-S2]). This result was generalized to topological spaces in [N-S1]. It is known [N] that for any closed n-manifold M^n, $n \geq 1$, and any compact metric space X with $\mathrm{sd}(X) < \infty$, we have $\mathrm{sd}(X \times M^n) = c[X] + n$, so in particular $\mathrm{sd}(X \times S^1) = c[X] + 1$. The equality $\mathrm{sd}(X) = c[X]$ fails, in general, if X is a compactum with $\mathrm{sd}(X) = 2$. There is a 2-dimensional connected compact metric space X with $c[X] < \mathrm{sd}(X) = 2$, i.e., such that $\mathrm{sd}(X \times S^1) = \mathrm{sd}(X) = 2$ (see [Sp]). In [Sp] an obstruction theory based on cohomologies with local coefficients was used to prove some of the required properties of the example. In a subsequent paper we will prove, in a geometric way, a new theorem concerning maps between 2-polyhedra which can be applied to show these properties. The following question is open.

PROBLEM ([Sp]). *Is it true that* $c[X] = 2$ *for each movable (or pointed movable) connected compact metric space* X *with* $\mathrm{sd}(X) = 2$*?*

We say that an inverse system of groups $\mathbf{G} = (G_\gamma, q_{\gamma\gamma'}, \Gamma)$ is *regularly movable* if

for each $\gamma \in \Gamma$ *there exists* $\gamma' \in \Gamma$, $\gamma' \geq \gamma$, *such that for any* $\gamma_1 \in \Gamma$ *there exists* $\gamma'' \in \Gamma$, $\gamma'' \geq \gamma', \gamma_1$, *such that* $q_{\gamma'\gamma''}$ *admits a right inverse.*

We say that a connected pointed space X is *regularly 1-movable* if pro-$\pi_1(X)$ is isomorphic to a regularly movable inverse system of groups. This notion is shape invariant. We prove (Theorem 2.1) that if a connected pointed space X with $\mathrm{sd}(X) = 2$ is regularly 1-movable then $c[X] = 2$. In the proof we apply the following theorem of J. R. Stallings and R. Swan: *groups of cohomological dimension 1 are free* ([St, Sw]). Since a regularly movable continuum is regularly 1-movable, we also obtain that $\mathrm{sd}(X) = c[X]$ for every regularly movable continuum X.

In this paper by a space we understand a topological space, and by a map a continuous map. To simplify notation, for a pointed space we use X instead of $(X, *)$. We also always assume, without noting, that the maps and homotopies between pointed spaces preserve the base point. For notions and results of pro-homotopy theory and shape theory we refer to [M-S].

2. A cohomological characterization of shape dimension of regularly 1-movable connected pointed spaces

The main result of the paper is the following

THEOREM 2.1. *If* X *is a regularly 1-movable connected pointed space with* $\mathrm{sd}(X) = 2$ *then* $c[X] = 2$.

The proof of the theorem is a consequence of Lemma 2.2 and Lemma 2.4 below.

LEMMA 2.2. *Let* $f : P \to Q$ *and* $g : Q \to R$ *be maps of* 2*-dimensional connected pointed* CW*-complexes such that*

a) *the homomorphism* $\pi_1(f) : \pi_1(P) \to \pi_1(Q)$ *can be factored by a free group, and*
b) *the homomorphism* $\pi_2(g) : \pi_2(Q) \to \pi_2(R)$ *is trivial.*

Then the composition $g \circ f : P \to R$ *is deformable to the* 1*-skeleton of* R.

PROOF. Let $\widehat{P}$ and $\widehat{Q}$ be pointed Eilenberg-McLane spaces with $\widehat{P}^{(2)} = P$, $\widehat{Q}^{(2)} = Q$ and $\pi_n(\widehat{P}) = \pi_n(\widehat{Q}) = 0$ for every $n > 1$. By $i : P \to \widehat{P}$ and $j : Q \to \widehat{Q}$ we denote the inclusions and by $\widehat{f} : \widehat{P} \to \widehat{Q}$ an extension of the map $j \circ f : P \to \widehat{Q}$. Note that $\pi_1(i)$ and $\pi_1(j)$ are isomorphisms.

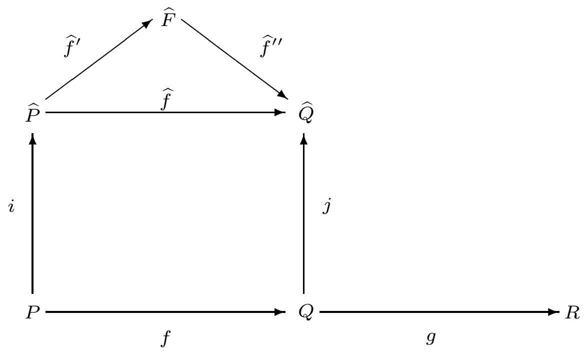
By a) there exist a free group F and homomorphisms $f' : \pi_1(P) \to F$ and $f'' : F \to \pi_1(Q)$ such that $\pi_1(f) = f'' \circ f'$. Let $\widehat{F}$ be a 1-dimensional connected pointed CW-complex with $\pi_1(\widehat{F}) = F$. It is well known that there exist maps $\widehat{f'} : \widehat{P} \to \widehat{F}$ and $\widehat{f''} : \widehat{F} \to \widehat{Q}$ such that

$$\pi_1(\widehat{f'}) = f' \circ (\pi_1(i))^{-1} \ \ \textit{and} \ \ \pi_1(\widehat{f''}) = \pi_1(j) \circ f''.$$

Observe that

$$\pi_1(\widehat{f''}) \circ \pi_1(\widehat{f'}) = \pi_1(j) \circ \pi_1(f) \circ (\pi_1(i))^{-1} = \pi_1(\widehat{f}).$$

It follows that the maps $\widehat{f}$ and $\widehat{f''} \circ \widehat{f'}$ are homotopic. So in the following diagram



the square commutes and the triangle commutes up to homotopy.

Since $j \circ f$ is homotopic to $\widehat{f''} \circ \widehat{f'} \circ i$ and $\widehat{F}$ is a 1-dimensional CW-complex the map $j \circ f$ is deformable in $\widehat{Q}$ to the 1-skeleton $\widehat{Q}^{(1)}$ of $\widehat{Q}$. Let

$$H : P \times [0,1] \to \widehat{Q}$$

be a homotopy such that $H(x,0) = j \circ f(x)$ and $H(x,1) \in \widehat{Q}^{(1)}$ for every $x \in P$. Since $\dim P \leq 2$, we may assume that $H(P \times [0,1]) \subset \widehat{Q}^{(3)}$, i.e., $j \circ f$ is deformable to the 1-skeleton $\widehat{Q}^{(1)}$ in $\widehat{Q}^{(3)}$.

Since $g : Q \to R$ induces the trivial homomorphism $\pi_2(g)$, there is an extension $\widetilde{g} : \widehat{Q}^{(3)} \to R$ of g. Note that $g \circ f = \widetilde{g} \circ j' \circ f$, where $j' : Q \to \widehat{Q}^{(3)}$ is the inclusion map. It follows that $g \circ f$ is homotopic to $\widetilde{g} \circ h$, where $h : P \to Q^{(3)}$ is defined by $h(x) = H(x, 1)$ for each $x \in P$. But $h(P) \subset Q^{(1)}$ and so $\widetilde{g} \circ h(P) \subset R^{(1)}$ (without loss of generality we may assume that $g(Q^{(1)}) \subset R^{(1)}$ which implies $\widetilde{g}(Q^{(1)}) \subset R^{(1)}$). This finishes the proof of Lemma 2.2. ∎

In the sequel, we will use the following notation. Let $f : X \to Y$ be a map of spaces and let $\mathfrak{L}$ be a local system of Abelian groups on Y. By $\mathfrak{L}_f$ we denote the local system of Abelian groups on X induced by $\mathfrak{L}$ and f. If B is a subspace of Y and $j : B \to Y$ is the inclusion map, we denote $\mathfrak{L}_j$ by $\mathfrak{L}|B$. Note that if A is a subspace of X then $\mathfrak{L}_f|A = \mathfrak{L}_{f|A}$.

In the proof of Lemma 2.4 we need

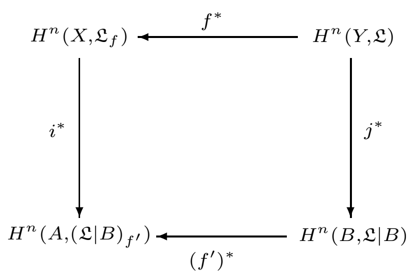
LEMMA 2.3. *Let $f : X \to Y$ be a map of CW-complexes, let A and B be subcomplexes of X and Y, respectively, such that $f(A) \subset B$ and $A^{(n)} = X^{(n)}$ for some integer n, and let $\mathfrak{L}$ be a local system of Abelian groups $\mathfrak{L}$ on Y. If the map $f' : A \to B$, defined by $f'(x) = f(x)$, induces the trivial homomorphism*

$$(f')^* : H^n(B, \mathfrak{L}|B) \to H^n(A, (\mathfrak{L}|B)_{f'})$$

then the map f induces the trivial homomorphism

$$f^* : H^n(Y, \mathfrak{L}) \to H^n(X, \mathfrak{L}_f).$$

PROOF. Consider the following commutative diagram



where $i : A \to X$ and $j : B \to Y$ are the inclusions. Note that $(\mathfrak{L}|B)_{f'} = \mathfrak{L}_f|A$. Observe that

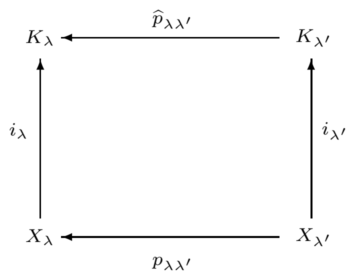
$$i^* : H^n(X, \mathfrak{L}_f) \to H^n(A, \mathfrak{L}_f|A)$$

is a monomorphism since A is a subcomplex of X such that $A^{(n)} = X^{(n)}$. By the assumption $(f')^*$ is trivial, thus $(f')^* \circ j^*$ and, consequently, $i^* \circ f^*$ are also trivial. It follows that f^* is trivial. ∎

LEMMA 2.4. *Let X be a regularly 1-movable connected pointed space with* $\mathrm{sd}(X) = 2$. *If $c[X] < 2$ then pro-$\pi_1(X)$ is isomorphic to an inverse system of free groups.*

Proof. By [M-S, Theorem 2, p. 96], the space X admits an HPol$_*$-expansion $X \to \mathbf{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$, where all X_λ are connected pointed polyhedra of dimension ≤ 2. We assume that pro-$\pi_1(X) = \pi_1(\mathbf{X})$, cf. [M-S, p. 130]. Observe that it suffices to prove that for each $\lambda \in \Lambda$ there is $\lambda' \in \Lambda$ such that the homomorphism $\pi_1(p_{\lambda\lambda'})$ can be factored by a free group.

For any $\lambda \in \Lambda$ let K_λ denote an Eilenberg-McLane space with $(K_\lambda)^{(2)} = X_\lambda$ and $\pi_n(K_\lambda) = 0$ for every $n > 2$. By $i_\lambda : X_\lambda \to K_\lambda$ we denote the inclusion. Then $\pi(i_\lambda)$ is an isomorphism for any $\lambda \in \Lambda$, and for any $\lambda, \lambda' \in \Lambda$, $\lambda \leq \lambda'$, the following diagram



is commutative, where $\widehat{p}_{\lambda,\lambda'} : K_{\lambda'} \to K_\lambda$ denotes an extension of the map $i_\lambda \circ p_{\lambda,\lambda'}$.

Let $\mathbf{G} = (G_\gamma, q_{\gamma\gamma'}, \Gamma)$ be a regularly movable inverse system of groups isomorphic to pro-$\pi_1(X)$. For each $\gamma \in \Gamma$, let $\widehat{G}_\gamma$ be a connected pointed Eilenberg-MacLane space such that $\pi_1(\widehat{G}_\gamma) = G_\gamma$ and $\pi_n(\widehat{G}_\gamma)$ is trivial for each $n > 1$. Let $\widehat{q}_{\gamma\gamma'} : \widehat{G}_{\gamma'} \to \widehat{G}_\gamma$, where $\gamma, \gamma' \in \Gamma$ and $\gamma \leq \gamma'$, be a map such that $\pi_1(\widehat{q}_{\gamma\gamma'}) = q_{\gamma\gamma'}$.

Since $\pi_1(\mathbf{X})$ and $\mathbf{G}$ are isomorphic, the inverse systems $\mathbf{K} = (K_\lambda, \widehat{p}_{\lambda,\lambda'}, \Lambda)$ and $\widehat{\mathbf{G}} = (\widehat{G}_\gamma, \widehat{q}_{\gamma\gamma'}, \Gamma)$ are isomorphic in the category pro-HPol$_*$. Let

$$\mathbf{f} = (f_\lambda, \Phi) : \widehat{\mathbf{G}} \to \mathbf{K} \quad \text{and} \quad \mathbf{g} = (g_\gamma, \Psi) : \mathbf{K} \to \widehat{\mathbf{G}},$$

where $\Phi : \Lambda \to \Gamma$, $f_\lambda : \widehat{G}_{\Phi(\lambda)} \to K_\lambda$ for each $\lambda \in \Lambda$, $\Psi : \Gamma \to \Lambda$ and $g_\gamma : K_{\Psi(\gamma)} \to \widehat{G}_\gamma$ for each $\gamma \in \Gamma$, be morphisms of inverse systems such that $\mathbf{g} \circ \mathbf{f} = \mathrm{id}_{\widehat{\mathbf{G}}}$ and $\mathbf{f} \circ \mathbf{g} = \mathrm{id}_{\mathbf{K}}$ in pro-HPol$_*$.

Let us fix $\lambda \in \Lambda$. Since $\mathbf{G}$ is regularly movable for $\gamma = \Phi(\lambda)$ there exist $\gamma' \in \Gamma$, $\gamma \leq \gamma'$, such that

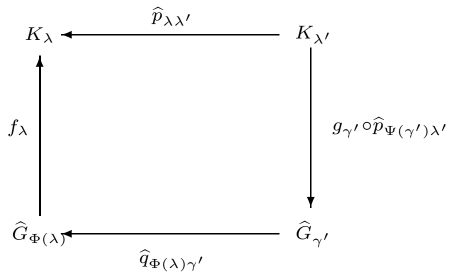
(a) for any $\gamma_1 \in \Gamma$ there exist $\gamma'' \in \Gamma$, $\gamma'' \geq \gamma', \gamma_1$, such that the map $\widehat{q}_{\gamma'\gamma''}$ admits a right inverse.

Since $\mathbf{g}$ is a morphism and $\mathbf{f} \circ \mathbf{g} = \mathrm{id}_{\mathbf{K}}$, there exists $\lambda' \in \Lambda$, $\lambda' \geq \lambda, \Psi(\gamma'), \Psi \circ \Phi(\lambda)$, such that

$$g_{\Phi(\lambda)} \circ \widehat{p}_{\Psi\circ\Phi(\lambda)\lambda'} = \widehat{q}_{\Phi(\lambda)\gamma'} \circ g_{\gamma'} \circ \widehat{p}_{\Psi(\gamma')\lambda'} \quad \text{and} \quad f_\lambda \circ g_{\Phi(\lambda)} \circ \widehat{p}_{\Psi\circ\Phi(\lambda)\lambda'} = \widehat{p}_{\lambda\lambda'}$$

in HPol$_*$. It follows that

(b) $f_\lambda \circ \widehat{q}_{\Phi(\lambda)\gamma'} \circ g_{\gamma'} \circ \widehat{p}_{\Psi(\gamma')\lambda'} = \widehat{p}_{\lambda\lambda'}$ in HPol$_*$.
Thus the following diagram



commutes in HPol$_*$.

Now, let $\widehat{\mathfrak{L}}$ be any local system of Abelian groups on $\widehat{G}_{\gamma'}$. The local system of Abelian groups on $K_{\lambda'}$ induced by $\widehat{\mathfrak{L}}$ and the map $g_{\gamma'} \circ \widehat{p}_{\Psi(\gamma')\lambda'}$ we denote by $\mathfrak{L}$. Since $c[X] < 2$, for λ' and the local system of Abelian groups $\mathfrak{L}|X_{\lambda'}$ on $X_{\lambda'}$ there is $\lambda'' \in \Lambda$, $\lambda'' \geq \lambda'$, such that $p_{\lambda'\lambda''}$ induces the trivial homomorphism of the second cohomology groups

$$(p_{\lambda'\lambda''})^* : H^2(X_{\lambda'}, \mathfrak{L}|X_{\lambda'}) \to H^2(X_{\lambda''}, (\mathfrak{L}|X_{\lambda'})_{p_{\lambda'\lambda''}}).$$

Thus by Lemma 2.3, the map $\widehat{p}_{\lambda'\lambda''}$ induces the trivial homomorphism of the second cohomology groups

$$(\widehat{p}_{\lambda'\lambda''})^* : H^2(K_{\lambda'}, \mathfrak{L}) \to H^2(K_{\lambda''}, \mathfrak{L}_{p_{\lambda'\lambda''}}).$$

Since $\mathbf{f}$ is a morphism and $\mathbf{g} \circ \mathbf{f} = \mathrm{id}_{\widehat{\mathbf{G}}}$ in pro-HPol$_*$, there exists $\gamma_1 \in \Gamma$, $\gamma_1 \geq \gamma', \Phi(\lambda''), \Phi \circ \Psi(\gamma')$, such that

$$f_{\Psi(\gamma')} \circ \widehat{q}_{\Phi\circ\Psi(\gamma')\gamma_1} = \widehat{p}_{\Psi(\gamma')\lambda''} \circ f_{\lambda''} \circ \widehat{q}_{\Phi(\lambda'')\gamma_1}$$

and

$$g_{\gamma'} \circ f_{\Psi(\gamma')} \circ \widehat{q}_{\Phi\circ\Psi(\gamma')\gamma_1} = \widehat{q}_{\gamma'\gamma_1}$$

in HPol$_*$. It follows that

(c) $g_{\gamma'} \circ \widehat{p}_{\Psi(\gamma')\lambda''} \circ f_{\lambda''} \circ \widehat{q}_{\Phi(\lambda'')\gamma_1} = \widehat{q}_{\gamma'\gamma_1}$ in HPol$_*$.

By (a) there exist $\gamma'' \in \Gamma$, $\gamma'' \geq \gamma', \gamma_1$, and a map $h : \widehat{G}_{\gamma'} \to \widehat{G}_{\gamma''}$ such that

(d) $\widehat{q}_{\gamma'\gamma''} \circ h = \mathrm{id}_{\widehat{G}_{\gamma'}}$ in HPol$_*$.

By (c), the following diagram

$$\begin{array}{ccc} K_{\lambda'} & \xleftarrow{\widehat{p}_{\lambda'\lambda''}} & K_{\lambda''} \\ {\scriptstyle g_{\gamma'}\circ\widehat{p}_{\Psi(\gamma')\lambda'}}\downarrow & & \uparrow{\scriptstyle f_{\lambda''}\circ\widehat{q}_{\Phi(\lambda'')\gamma''}} \\ \widehat{G}_{\gamma'} & \xleftarrow[\widehat{q}_{\gamma'\gamma''}]{} & \widehat{G}_{\gamma''} \end{array}$$

commutes in HPol_*. Therefore, the homomorphism

$$(\widehat{q}_{\gamma'\gamma''})^* : H^2(\widehat{G}_{\gamma'}, \widehat{\mathfrak{L}}) \to H^2(\widehat{G}_{\gamma''}, \widehat{\mathfrak{L}}_{\widehat{q}_{\gamma'\gamma''}})$$

is trivial, because the homomorphism $(\widehat{p}_{\lambda'\lambda''})^*$ is trivial. Thus, by (d), the homomorphism

$$(id_{\widehat{G}_{\gamma'}})^* : H^2(\widehat{G}_{\gamma'}, \widehat{\mathfrak{L}}) \to H^2(\widehat{G}_{\gamma'}, \widehat{\mathfrak{L}})$$

induced by the identity map on $\widehat{G}_{\gamma'}$ is trivial. So the group $H^2(\widehat{G}_{\gamma'}, \widehat{\mathfrak{L}})$ is trivial for any local system of Abelian groups $\widehat{\mathfrak{L}}$. Thus cohomological dimension $\mathrm{cd}(\pi_1(\widehat{G}_{\gamma'})) \le 1$. By Stallings-Swan theorem [St, Sw], the group $\pi_1(\widehat{G}_{\gamma'})$ is free.

Finally, by (b), the map $\widehat{p}_{\lambda\lambda'} : K_{\lambda'} \to K_\lambda$ is factored by $\widehat{G}_{\gamma'}$ in HPol_*. It follows that the homomorphism $\pi_1(\widehat{p}_{\lambda\lambda'})$, and thus the homomorphism $\pi_1(p_{\lambda\lambda'})$, is factored by the free group $\pi_1(\widehat{G}_{\gamma'})$. This finishes the proof of the lemma. ∎

3. Proof of Theorem 2.1

Let X be a regularly 1-movable connected pointed space with $\mathrm{sd}(X) = 2$. Let $X \to \mathbf{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ be an HPol_*-expansion of the space X, where all X_λ are pointed polyhedra of dimension ≤ 2. Suppose $c[X] < 2$.

If X is not approximatively 2-connected space with $\mathrm{sd}(X) = 2$ then $c[X] = 2$ (cf. [N, Theorem 8.3, p. 35]). Thus we may assume that X is approximatively 2-connected space. It follows that for any $\lambda \in \Lambda$ there exist $\lambda' \in \Lambda$, $\lambda' \ge \lambda$, such that the homomorphism

$$\pi_2(p_{\lambda\lambda'}) : \pi_2(X_{\lambda'}) \to \pi_2(X_\lambda)$$

is trivial.

By Lemma 2.4 there exist $\lambda'' \in \Lambda$, $\lambda'' \ge \lambda'$, such that the homomorphism

$$\pi_1(p_{\lambda'\lambda''}) : \pi_1(X_{\lambda''}) \to \pi_1(X_{\lambda'})$$

can be factored by a free group.

By Lemma 2.2, the composition $p_{\lambda\lambda''} = p_{\lambda\lambda'} \circ p_{\lambda'\lambda''}$ is deformable to the 1-skeleton of X_λ. It follows that $\mathrm{sd}(X) = 1$, which contradicts the assumption that $\mathrm{sd}(X) = 2$. Thus the proof of Theorem 2.1 is complete.

References

[B] K. Borsuk, Theory of Shape, PWN–Polish Scientific Publishers, Warszawa, 1975.

[D] J. Dydak, *The Whitehead and Smale theorems in shape theory*, Dissertationes Math. **156** (1979), 1–51.

[M-S] S. Mardešić and J. Segal, Shape Theory, North-Holland Publishing Co., Amsterdam-New York-Oxford 1982.

[N] S. Nowak, *Algebraic theory of fundamental dimension*, Dissertationes Math. **187** (1981), 1–59.

[N-S1] S. Nowak and S. Spież, *Some properties of deformation dimension*, in: Shape theory and geometric topology (Dubrovnik, 1981), Lecture Notes in Math. **870**, Springer, Berlin-New York, 1981, 93–104.

[N-S2] S. Nowak and S. Spież, *Remarks on deformability*, Fund. Math. **125** (1985), 95–103.

[Sp] S. Spież, *An example of a continuum X with* $Fd(X \times S^1) = Fd(X) = 2$, Bull. Acad. Polon. Sci. Sér. Sci. Math. **27** (1979), 923–927.

[St] J. R. Stallings, *On torsion-free groups with infinitely many ends*, Ann. of Math. (2) **88** (1968), 312–344.

[Sw] R. G. Swan, *Groups of cohomological dimension one*, J. Algebra **12** (1969), 585–610.

J. Segal
Department of Mathematics
University of Washington
Seattle, WA 98195
U.S.A.
E-mail: segal@math.washington.edu

S. Spież
Institute of Mathematics
Polish Academy of Sciences
Sniadeckich 8, P.O.B. 137, 00–950 Warszawa
Poland
E-mail: s.spiez@impan.gov.pl

Received: 22.9.2006.