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NUMERICAL INTEGRATION IN VOLUME CALCULATION OF IRREGULAR ANTICLINES

NUMERIČKO INTEGRIRANJE KOD IZRAČUNA VOLUMENA NEPRAVILNIH ANTIKLINALA

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Ključne riječi: nepravilna antiklinala, ležište ugljikovodika, planina Fuji, volumen, trapezno pravilo, Simpsonovo pravilo.

Sažetak

Pri izračunavanju volumena geoloških struktura često se koristi određeni integral. Iako se u nekim slučajevima integral može riješiti analitički, u praksi se njegova vrijednost obično procjenjuje koristeći tehnike numeričke integracije. Primjena određenog integrala u izračunavanju volumena ilustrirana je dvama primjerima. Volumen planine Fuji, koja je svjetski poznati geomorfološki primjer "stožaste" strukture, izračunat je analitičkom integracijom. Dvije temeljne metode numeričkog integriranja, tj. trapezno i Simpsonovo pravilo, primijenjene su na izračun volumena ležišta ugljikovodika, gdje je struktura nepravilne antiklinale aproksimirana pravilnim krnjim stošcem

Key words: irregular anticline, hydrocarbon reservoir, Fuji Mt., volume, trapezoidal rule, Simpson's rule

Abstract

The volume of geological structures is often calculated by using the definite integral. Though in some cases the integral can be solved analitically, in practice we usually approximate its value by numerical integration techniques. The application of definite integral in volume calculation is illustrated by two examples. The volume of Mount Fuji, the world-known "conic" geomorphological structure, is calculated by analytical integration. Two basic numerical integration methods, that is, the trapezoidal and Simpson's rule are applied to subsurface hydrocarbon reservoir volume calculation, where irregular anticline is approximated by a frustum of a right circular cone.

1. Introduction

In practical problems, one is working with measured data of limited accuracy, so the accuracy of the results cannot be expected to exceed that of the initial data. In such cases, an approximation method may give an answer that is as accurate as we need. The basic problem in numerical integration is to compute an approximate solution to a definite integral $\int_a^b f(x)dx$ to a given degree of accuracy. There are two main reasons for carrying out numerical integration: analytical integration may be difficult or impossible, or the integrand f is given by a table of values. Polynomial approximation like the Lagrange interpolating polynomial method serves as the basis for the two integration methods: the trapezoidal rule and Simpson's rule, by means of which the approximations to the integral are obtained by using only values of the integrand $f(x)$ at a finite number of points x. In this paper we illustrate the application of these methods in calculating geological structures' volume. Volumes are often calculated by integrating the area functions with respect to distance.

Geological structures are mostly irregular and only sometimes apparently symmetrical (e.g., **Malvić & Novak Zelenika, 2014**), so in most cases their volumes cannot be calculated by analytical integration. Having values of the area of each of several equally spaced cross-sections (measured, for example, by a mechanical device called planimeter), the volume is calculated by using the numerical integration techniques.

2. Application of integration in mountain volume calculation

The definite integral can be applied for calculating geometrical quantities such as volumes. Suppose that a solid object (**Figure 1**) has boundaries extending from $x = a$ to $x = b$, and that its cross-section by a plane passing through the point $(x, 0, 0)$, and parallel to the yz-plane has the area $A(x)$. Let us suppose that the function $x \mapsto A(x)$ is continuous on $[a, b]$. To find a volume of the object, let us take a regular partition $\Delta = \{x_0, x_1, \dots, x_n\}$ of the interval $[a, b]$. The planes that are perpendicular to the x-axis at the partition points will divide the objects into n slices. The volume ΔV_i of the i −th slice between the planes $x = x_{i-1}$ and $x = x_i$ is approximated by the volume of the cylinder with cross-section area $A(t_i)$ and height $\Delta x_i = x_i - x_{i-1}$, where $t_i \in [x_{i-1}, x_i]$, $i = 1, \dots, n$. Thus,

$$\Delta V_i \approx A(t_i)\Delta x_{i,} \quad i = 1, \dots, n.$$

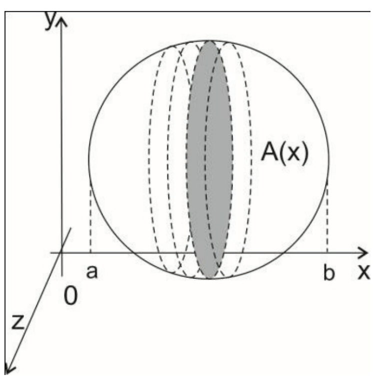

Figure 1: A solid object with boundaries extending from $x = a$ to $x = b$.

Slika 1: Tijelo s granicama $x = a$ i $x = b$.

The sum $\sum_{i=1}^{n} A(t_i)\Delta x_i$ is an approximation to the volume V of our solid object. We can expect the approximations to get better and better as $n \to \infty$. The volume of a solid object with known cross-section is defined as

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} A(t_i)\Delta x_i = \int_a^b A(x)dx \quad \textbf{(Eq. 1)}$$

We shall now apply this formula for measuring the volume of a mountain of relatively simple geomorphological shape and symmetric volcanic structure, such as Mount Fuji in Japan. The mountain is divided into a series of thin slices. As the shape is relatively regular (as the frustum of a cone), the slices are well approximated by small cylinders (**Figure 2**), which can be easily presented with almost regular isohypses (**Figure 3**).

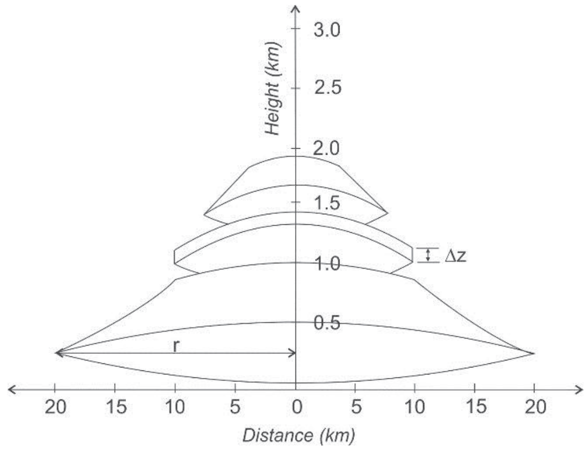

*Figure 2: Estimation of the volume of Mount Fuji by summing the volume of several small cylinders (modified after **Waltham, 2000**).*

Slika 2: Procjena volumena planine Fuji zbrajanjem volumena niza malih valjaka (modificirano prema **Waltham, 2000**).

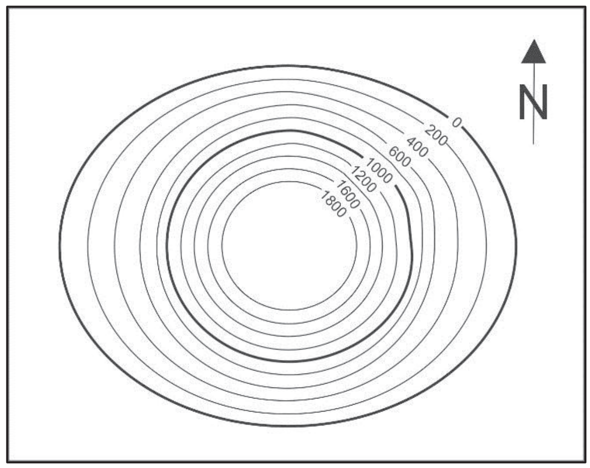

*Figure 3: Approximate isohypses map corresponding to the structure of Mount Fuji in the part shown in **Figure 2** and made from the cylinders. The height of solid (altitude) is approximately 1950 m, and of Mount Fuji 3776 m.*

Slika 3: Približna karta izohipsi koja bi odgovarala strukturi planine Fuji u dijelu koji je na **slici 2** aproksimiran valjcima. Visina prikazana kartom je približno 1950 m, a vrha planine Fuji 3776.

The base of i-th cylinder is a circle of radius r_i, so the area P_i of the base is equal to $P_i = \pi r_i^2$. If Δz_i denotes the height of i-th cylinder, then its volume V_i is equal to

$$V_i = P_i \Delta z_i = \pi r_i^2 \Delta z_i.$$

The volume of the entire mountain is then approximately equal to the sum of the volumes of n cylinders, that is,

$$\sum_{i=1}^{n} V_i = \pi r_i^2 \Delta z_i.$$

The logical question is how many cylinders (slices) are needed to reach a reliable volume calculation. In the case of a small number of slices, the mountain would be poorly approximated, with the flanks of the "staircase" structure. However, the calculation would be fast. On the contrary, with the large number of slices the result becomes more accurate, but the calculation could be time-consuming. The exact result is obtained by using **Eq. 1**, which gives

$$= \int_{z_{min}}^{z_{max}} \pi r(z)^2 \, dz, \qquad \textbf{(Eq. 2)}$$

where z_{min} is the altitude of the mountain base, and z_{max} the altitude of the mountain top. To calculate the volume by using **Eq. 2**, one must know how the radius r depends on the height z. Each "symmetrical" mountain has its own relation between the radius and the height. For Mount Fuji, it can be shown that, to a good approximation,

$$r(z)^2 = \left(\frac{400z}{3} - \frac{800\sqrt{z}}{\sqrt{3}} + 400\right) \text{ km}^2 \qquad \textbf{(Eq. 3)}$$

where $z_{min} = 0$ km and $z_{max} = 3$ km. It implies that the radius is 20 km at the mountain base, and 0 km at the mountain top (**Figure 4**).

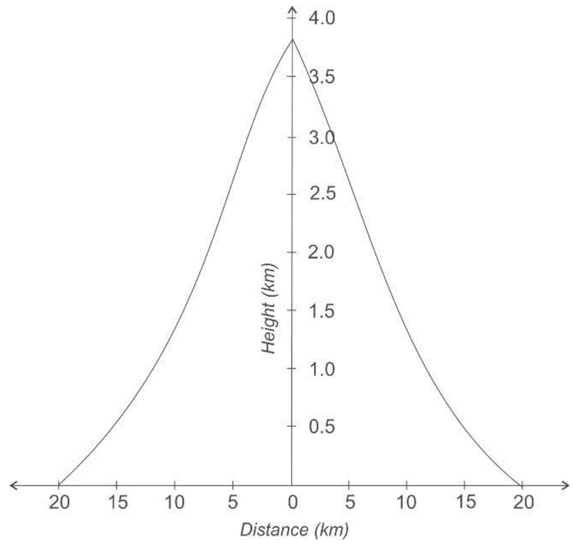

***Figure 4:** Model of radius versus altitude for Mount Fuji (modified after **Waltham, 2000**).*

Slika 4: Model odnosa radijusa i visine za planinu Fuji (modificirano prema **Waltham, 2000**).

Substituting **Eq. 3** into **Eq. 2,** we get

$$\begin{aligned} V &= \int_0^3 \pi \left[\frac{400z}{3} - \frac{800\sqrt{z}}{\sqrt{3}} + 400\right] dz \\ &= \pi \int_0^3 \frac{400z}{3} dz - \pi \int_0^3 \frac{800\sqrt{z}}{\sqrt{3}} dz + \int_0^3 400 dz \\ &= \pi \left[\frac{400z^2}{6} - \frac{800z^{1,5}}{1,5\sqrt{3}} + 400z\right] \Bigg|_0^3 \\ &= \pi(600 - 1600 + 1200) \\ &= 200\pi \approx 628 \text{ km}^3. \end{aligned}$$

The result is a good approximation of the volume of Mount Fuji.

3. Numerical integration: the trapezoidal and Simpson's rule

To get the exact value of the volume V of a solid object whose boundaries extend from $x = a$ to $x = b$ by using the definite integral $V = \int_a^b A(x)dx$, the area $A(x)$ of the cross-section by a plane parallel to the yz-plane must be known at every point $x \in [a,b]$. However, in practical work the integrand A is usually defined by a table of values. In such cases, an approximate value of the definite integral can be obtained by certain numerical formulas, and by the use of a planimeter - a mechanical device for measuring irregular areas. In this section we shall describe two basic numerical integration methods: the trapezoidal and Simpson's rule (see e.g. **Atkinson, 1989**, **Kevo, 1986,** or **Quarteroni et al., 2000**).

3.1. The trapezoidal rule

We would like to evaluate the integral $\int_a^b f(x)dx$. Recall that in the case of a positive continuous function f, this integral represents the area bounded by the curve $y = f(x)$ and the lines $y = 0,\ x = a,\ x = b$ (**Figure 5**).

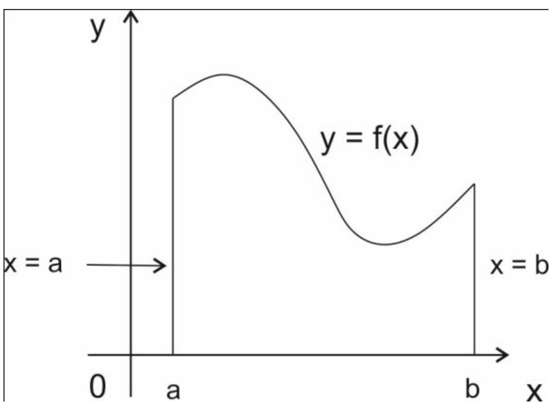

Figure 5: *The area bounded by* $y = f(x),\ y = 0,\ x = a,\ x = b$.

Slika 5: Površina ograničena s $y = f(x),\ y = 0,\ x = a,\ x = b$.

The first, and rather crude, approximation to the integral is obtained by replacing the curve $y = f(x)$ between $x = a$ and $x = b$ by a straight line segment; that is, a polynomial of degree 1. Then the approximation is the area of the trapezium with vertices at the points $(a,0)$, $(b,0)$, $(a,f(a))$, $(b,f(b))$, so we have

$$\int_a^b f(x)dx \approx \frac{1}{2}(b-a)\big(f(a)+f(b)\big).$$

To obtain better accuracy, we have to split $[a,b]$ into n subintervals and use the trapezoidal approximation (**Figure 6**) on each subinterval. If we take a uniform partition $a = x_0 < x_1 < \cdots < x_{n-1} < x_n = b$ with the step $h = \frac{b-a}{n}$, we get the approximation

$$\int_a^b f(x)dx = \sum_{i=1}^{n}\int_{x_{i-1}}^{x_i} f(x)dx$$
$$\approx \frac{1}{2}\sum_{i=1}^{n}(x_i - x_{i-1})\big(f(x_{i-1}) + f(x_i)\big),$$

that is,

$$I = \int_a^b f(x)dx \approx I_{trap}.$$
$$= \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(x_i)\right] \qquad \textbf{(Eq. 4)}$$

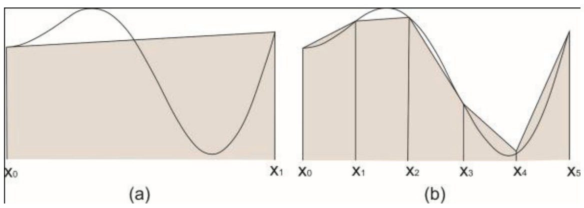

Figure 6: *The trapezoidal rule with one (sub)interval (a) and five subintervals (b) (web 4).*

Slika 6: Trapezno pravilo s jednim (pod)intervalom (a) i s pet podintervala (b) (poveznica 4).

If f'' (the second derivative of f) is continuous on $[a,b]$, then the error is bounded by

$$\left|I - I_{trap}\right| \le \frac{1}{12}(b-a)h^2 \max_{x\in[a,b]}|f''(x)|. \qquad \textbf{(Eq. 5)}$$

Since the error term for the trapezoidal rule involves f'', this rule gives the exact result for polynomials of degree 1.

3.2. Simpson's rule

This method of evaluating the integral $\int_a^b f(x)dx$ is based on approximating the curve $y = f(x)$ by a parabola; that is, a polynomial of degree 2 passing through the points $(a,f(a))$, $(b,f(b))$, $(c,f(c))$, where $c = \frac{1}{2}(a+b)$. Then we get

$$\int_a^b f(x)dx \approx \frac{1}{6}(b-a)\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]. \qquad \textbf{(Eq. 6)}$$

In practice, we usually divide (**Figure 7**) the interval $[a,b]$ into $2n$ subintervals

$a = x_0 < x_1 < \cdots < x_{2n-1} < x_{2n} = b$ of the same length $h = \frac{b-a}{2n}$. Applying Simpson's rule to the intervals $[x_{2i-2}, x_{2i}]$ for $i = 1, \ldots, n$ we get

$$\int_{x_{2i-2}}^{x_{2i}} f(x)dx \approx \frac{1}{6}(x_{2i} - x_{2i-2})[f(x_{2i-2}) + 4f(x_{2i-1}) + f(x_{2i})],$$

so that

$$\int_a^b f(x)dx = \sum_{i=1}^{n} \int_{x_{2i-2}}^{x_{2i}} f(x)dx \approx \frac{1}{6}\sum_{i=1}^{n}(x_{2i} - x_{2i-2})[f(x_{2i-2}) + 4f(x_{2i-1}) + f(x_{2i})]$$

and eventually,

$$I = \int_a^b f(x)dx \approx I_{Simp} = \frac{h}{3}[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(x_{2i}) + 4\sum_{1=1}^{n} f(x_{2i-1})] \qquad \textbf{(Eq. 7)}$$

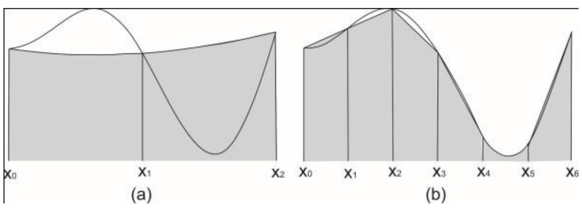

Figure 7: *Simpson's rule with two subintervals (a) and with* $2n = 6$ *subintervals (b) (web 4).*

Slika 7: Simpsonovo pravilo s dva podintervala (a) i s 2n = 6 podintervala (b) (poveznica 4).

If $f^{(iv)}$ (the fourth derivative of f) is continuous on the interval $[a, b]$, then the error is bounded by

$$|I - I_{Simp}| \leq \frac{1}{2880}(b-a)h^4 \max_{x\in[a,b]}|f^{(iv)}(x)| \qquad \textbf{(Eq. 8)}$$

For the trapezoid rule the error depends on h^2 (see **Eq. 5**), whereas the error for Simpson's rule depends on h^4 (see **Eq. 8**). It shows that the error in Simpson's rule goes to zero much more quickly than for the trapezoidal rule when h is reduced. As we can see from **Eq. 8**, the error term for Simpson's rule involves $f^{(iv)}$, so the rule gives the exact result when applied to polynomials of degree less than or equal to 3, since the fourth derivative of such polynomials is identically zero.

4. Prismoidal formula and its applications

A prismatoid is a polyhedron whose vertices all lie in one or the other of the two parallel planes. If both planes have the same number of vertices, and the lateral faces are either parallelograms or trapezoids, it is called a prismoid. The faces that lie in the parallel planes are called the bases of the prismatoid. The midsection is the polygon formed by cutting the prismatoid by a plane parallel to the bases halfway between them. The perpendicular distance between the bases is called the altitude or the height of the prismatoid. Families of prismatoids include pyramids, wedges, prisms etc. (e.g., **Nelson, ed., 1998**). The volume V of a prismatoid (**Eq. 9**) is given by the prismoidal formula

$$V = \frac{1}{6}h(A_1 + A_2 + 4M), \qquad \textbf{(Eq. 9)}$$

where h is the altitude, A_1 and A_2 are areas of the bases and M is the area of the midsection. This formula follows immediately by integrating the area parallel to the two planes of vertices by Simpson's rule (**Eq. 6**), since that rule is exact for integration of polynomials of degree up to 3, and in this case the area is a quadratic function in the height. The proof of the prismoidal formula obtained by using the solid geometry methods can be found in e.g. **Day Bradley, 1979** or **Halsted, 1907**.

Frustum of a right circular cone is a portion of right circular cone included between the base and a section parallel to the base not passing through the vertex (**Figure 8**).

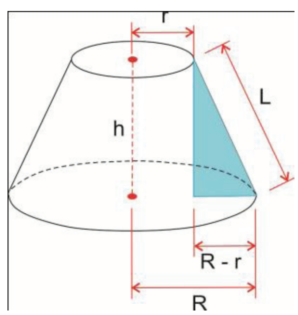

Figure 8: *Frustum of a right circular cone (web 1).*

Slika 8: Pravilni krnji stožac (poveznica 1).

Since in this case the area of cross-section by a plane parallel to the bases is also a quadratic function in the height, the volume of a frustum of a right circular cone can be calculated by applying prismoidal formula. If R is lower base radius, r is upper base radius, and h is the perpendicular distance between the two bases, then the area A_1 of lower base, the area A_2 of upper base and the area M of midsection are given in **Eqs. 10** as:

$$A_1 = R^2\pi,\ A_2 = r^2\pi,\ M = \left(\frac{R+r}{2}\right)^2\pi \qquad \textbf{(Eq. 10)}$$

Substituting the above expressions in **Eq. 9**, we get a formula **Eq. 11** for calculating the volume of a frustum of a right circular cone:

$$V = \frac{1}{3}h(R^2 + r^2 + Rr)\pi. \quad \textbf{(Eq. 11)}$$

As we have already mentioned, the prismoidal formula gives exact volume of any solid whose boundaries are two parallel planes, and the area of the cross-section in any intermediate plane parallel to the end planes is a polynomial function of degree up to 3. Most regular convex symmetrical geological structures could be approximated with a frustum of a right circular cone. However, the prismoidal formula can be used with a fair degree of accuracy for solids that do not satisfy the specified conditions, due to irregularities (e.g., **Figure 9**). The highly irregular surfaces may be divided into portions for which the formula holds.

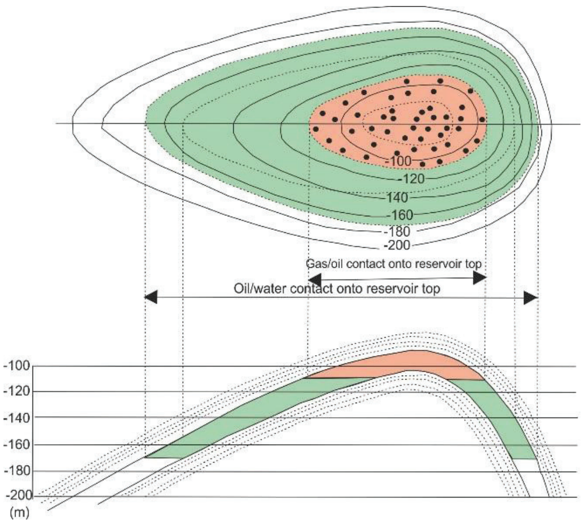

Figure 9: *Irregular brachianticline with different margin dips (from Malvić and Novak Zelenika, 2014, taken from Malvić and Velić, 2008; Brod and Jeremenko, 1957).*

Slika 9: Nepravilna brahiantiklinala s različitim nagibima (iz Malvić i Novak Zelenika, 2014, preuzeto iz Malvić i Velić, 2008; Brod i Jeremenko, 1957).

5. Application of the trapezoidal and Simpson's rule in hydrocarbon reservoir volume calculation

The Simpson's rule is regularly applied for hydrocarbon reservoir volume calculation, when the structure is close to a regular anticline. Such approximation is weaker in case of very irregular or faulted anticline, folded monocline and uplifted part of recumbent fold. It is why these are simultaneously calculated by both the trapezoidal and Simpson's rule and their difference (**Eq. 12**) is a criterion of method's applicability. If

$$\left|I_{trap} - I_{Simp}\right| \leq 0.2 I_{Simp}, \quad \textbf{(Eq. 12)}$$

then the volume calculated by Simpson's rule can be accepted. An example of successful reservoir volume calculation using the trapezoidal and Simpson's rule for irregular anticline, uplifted over the larger monocline, is shown at **Figures 10, 11** and **12**. The reservoir is delimited by structural top, roof plane and fluid's contact. Accuracy depends on equidistance, i.e., the number of isopachs, the areas of which have been measured by using a planimeter.

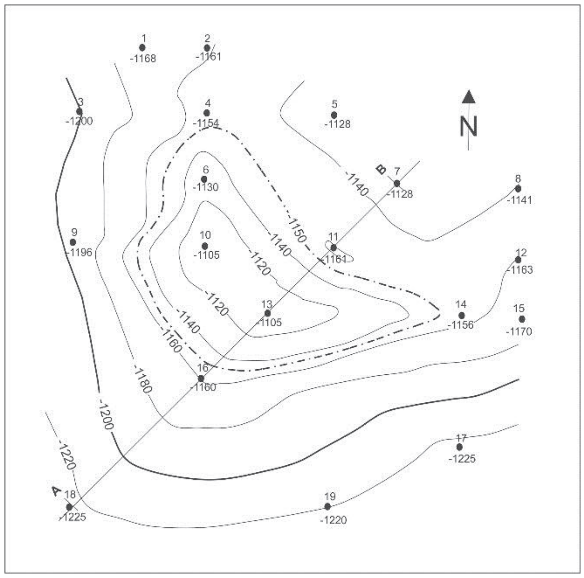

Figure 10: *Reservoir top structural map with oil-water contact (equidistance is 20 m).*

Slika 10: Strukturna karta po krovini ležišta te ucrtanim kontaktom nafte i vode (ekvidistancija je 20 m).

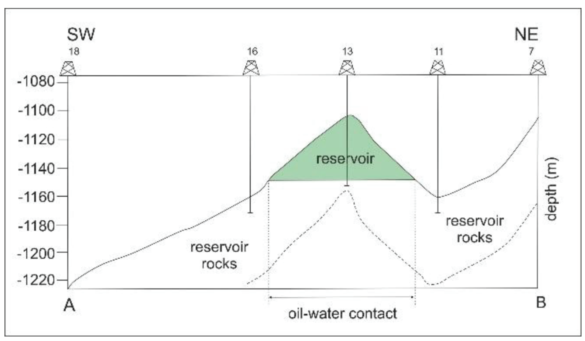

Figure 11: *Geological section across structural map on* ***Figure 10****.*

Slika 11: Geološki profil duž strukturne karte na **slici 10**.

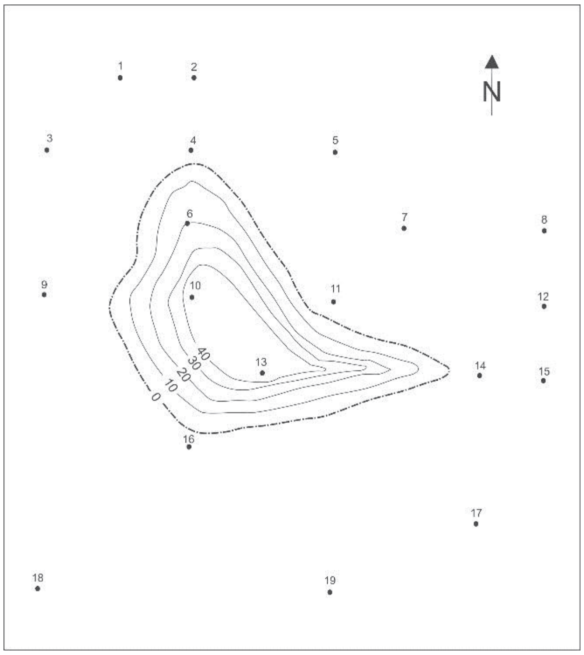

Figure 12: *Reservoir isopach map (equidistance is 10 m).*

Slika 12: Karta izopaha ležišta (ekvidistancija je 10 m).

There are five isopachs (**Figure 12**) that cut the reservoir, the structure of which is close to a frustum of a right circular cone. Isopach areas are, retrospectively:

$a_0 = 1143200 \text{ m}^2$,
$a_1 = 238000 \text{ m}^2$,
$a_2 = 157200 \text{ m}^2$,
$a_3 = 95600 \text{ m}^2$,
$a_4 = 58800 \text{ m}^2$.

The equidistance is $h = 10$ m. The application of Simpson's rule (**Eq. 7**) gives the volume

$$V_{Simp} = \frac{h}{3}(a_0 + 4a_1 + 2a_2 + 4a_3 + a_4) = 9502667 \text{ m}^3$$

As the structure is not real frustum, the volume V_{top} of its top with height $h_4 = 5$ m (less than equidistance h) is calculated as the average of volumes:

$$V_{top1} = \frac{h_4 a_4}{3} = 98000 \text{ m}^3,$$
$$V_{top2} = \frac{h_4^3 \pi}{6} + \frac{a_4 h_4}{2} = 147065 \text{ m}^3$$

Thus,

$$V_{top} = \frac{1}{2}(V_{top1} + V_{top2}) = 122533 \text{ m}^3.$$

The total volume is

$$V = V_{Simp} + V_{top} = 9625200 \text{ m}^3$$

The volume of a structural top usually adds only a few percent in total volume; here it is only 1.3%.

By the trapezoidal rule (**Eq. 4**), we get

$$V_{trap} = \frac{h}{2}(a_0 + 2a_1 + 2a_2 + 2a_3 + a_4) = 10918000 \text{ m}^3$$

The difference between volumes obtained by the trapezoidal and Simpson's rule is about 15% of V_{Simp}, which is significant, but acceptable.

6. Conclusions

The calculation of geological structures' volumes appears in many modelling tasks of subsurface as well as surface areas. Most of them are based on advanced software packages where the basic theory of applied methods is deeply hidden in the code and sometimes partially explained in handbooks. However, there is a strong need for students, scientists and engineers to be well educated in rules that lead to results in volume calculation. With this paper we have tried to improve their ability to apply numerical integration techniques to geological problems, such as hydrocarbon reservoir volume calculation. Understanding these rules is also crucial for avoiding significant calculation errors, since fieldwork and in-class tasks still include manual planimetring and the use of calculator in volume calculation, without any computer support.

7. Acknowledgment

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