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W-SETS AND APPROXIMATE LIMIT

W - SKUPOVI I APROKSIMATIVNI LIMES

ABSTRACT. The main purpose of the present paper is to prove the following:

THEOREM 3.7. Let $\underline{\mathbf{X}}=\{X_n,\epsilon_n,p_{mn},N\}$ be an approximate inverse sequence of the dendrites.Then $\mathbf{X}=lim\underline{\mathbf{X}}$ is hereditarily unicoherent.Moreover,if X is a Peano continuum,then X is a dendrite.

THEOREM 4.4. Let $\underline{\mathbf{X}}=\{X_n,\epsilon_n,p_{mn},N\}$ be an inverse sequence of metric compact spaces with onto bonding mappings.If each X_n is in Class W,then $lim\underline{\mathbf{X}}$ is in clas W.

THEOREM 5.2. Let $\underline{\mathbf{X}}=\{X_n,\epsilon_n,p_{mn},N\}$ be an approximate inverse sequence of metric continua X_n with $P(X_n) \leq n_0$.Then $P(X) \leq n_0$, $X=lim\underline{\mathbf{X}}$.

1 Introduction

All spaces considered in this paper are metric compact spaces.The boundary of a set A by Bd(A) is denoted.By ClA or Cl_X**A** the closure of a set $A \subseteq X$ is denoted.

The approximate inverse systems were introduced by S. Mardešić and L.R. Rubin [7] for compacta and by S. Mardešić and Watanabe [11] for general topological spaces.

DEFINITION 1.1 An *approximate inverse system* $\underline{\mathbf{X}}=\{X_a,\epsilon_a,p_{ab},A\}$ consists of the following data: A preordered set $(A,\leq)$ which is directed and has no maximal element; for each $a\in A$, a compact metric space X_a with metric d and a real number ε_a of X_a (called the mesh of X_a) and for each pair $a\leq b$ from A,a mapping $p_{ab}:X_b\rightarrow X_a$.Moreover the following three conditions must be satisfied:

(A1) The mappings $p_{ab}p_{bc}$ and p_{ac} are ϵ_a- near,$a\leq b\leq c$,i.e.

$$\mathbf{d(p_{ab},p_{bc},p_{ac})} \leq \varepsilon_a.$$

(A2) For each $a\in\mathbf{A}$ and each real number $\eta\geq 0$ there is $b\geq a$ such that $d(p_{ac}p_{cd},p_{ad})\leq\eta$,whenever $a\leq b\leq c\leq d$.

(A3) For each $a\in\mathbf{A}$ and each real number $\eta > 0$ there is $b\geq a$ such that for each $x,y\in\mathbf{X}_c$ $d(x,y) \leq \varepsilon_c \Rightarrow d(p_{ac}(x),p_{ac}(y))\leq \eta$ for each $c \geq b$.

DEFINITION 1.2 [7] Let $\underline{\mathbf{X}}=\{X_a,\epsilon_a p_{ab},A\}$ be an approximate system.A point $x=(x_a) \in \Pi\{X\_a :a\in A\}$ is called a *thread* of $\underline{\mathbf{X}}$ provided it satisfies the following condition:
(L) $(\forall\ a\in A)(\forall\ \eta > 0)(\exists\ b \geq a)(\forall\ c \geq b)\ d(p_{ac}(x_c),x_a)\leq\eta$.

Condition (**L**) is equivalent to the following condition:

(L)´ $(\forall\ a \in a)\ \lim\{p_{ac}(x_c):c\geq a\} = x_a$.

DEFINITION 1.3 [7] Let $\underline{\mathbf{X}}=\{X_a,\epsilon_a p_{ab},A\}$ be an approximate system. A point $x=(x_a)\in \Pi\{X_a : a\in A\}$ belongs to $X=\lim\underline{\mathbf{X}}$ iff x is a thread of $\underline{\mathbf{X}}$.

DEFINITION 1.4 *[7]*
Let $\underline{\mathbf{X}}=\{X_a,\epsilon_a p_{ab},A\}$ be an approximate system. A point $x=(x_a)\in \Pi\{X_a : a\in a\}$ is called a *prethread* of $\underline{\mathbf{X}}$ provided for every pair $a\leq b$ one has $d(x_a,p_{ab}x(_b))\leq \varepsilon_a$.

LEMMA 1.5 *[7, Lemma 2.]. If $x=(x_a)$ is a prethread,then* $\mathbf{y_a=\lim\{p_{ab}\ (x_a) : b \geq a\}}$ exists and $y=(y_a)$ is thread,i.e $y \in \lim \underline{\mathbf{X}}$.

In what follows we need the following properties.

THEOREM 1.6 *If in an approximate system* $\underline{\mathbf{X}}=\{X_a,\epsilon_a,p_{ab},A\}$ all $X_a \neq \emptyset$,then also $X = \lim\underline{\mathbf{X}} \neq \emptyset$.

Proof.See [7, Theorem 1.] ■

THEOREM 1.7 *The limit X of an approximate system of compact spaces is a compact Hausdorf space. [7. Theorem 2.].*

LEMMA 1.8 *Let $\underline{X}=\{X_a, \epsilon_a, p_{ab}, A\}$ be an approximate system of compacta. The collection of all sets of the form*
$p^{-1}_a(V_A)$,where $V_a \subseteq X_a$ is open,is a basis for the topology of $X = \lim\underline{\mathbf{X}}$.

Proof.See [7, Lemma 3.] ■

THEOREM 1.9 *Let $\underline{X}=\{X_a, \epsilon_a, p_{ab}, A\}$ be approximate system of metric compact spaces with limit X.Then the following statements hold [9, Proposition 6.]*

(B1) *Let $a \in A$ and let $U \subseteq X_a$ be an open set which contains $p_a(X)$. Then there exists a $b\in A$ such that $p_{ac}(X_c)\subseteq U$ for each $c\geq b$.*

(B2) *For every open covering **U** of **X** there exists an $a \in A$ such that for any $b\geq a$ there exists an open covering υ of X_b for which $p^{-1}_b(\upsilon)$ refines **U**.*

THEOREM 1.10 *The following statements hold for each approximate system $\underline{X}=\{X_a, \epsilon_a, p_{ab}, A\}$ with limit X [9, Proposition 7.] :*

(R1) *For every compact **A N R P**, $\eta>0$ and mapping $h:X \rightarrow P$,there is an $a \in A$ such that for any $b\geq a$ there is a mapping $f:X_b \rightarrow P$ for which $d(fp_b,h)\leq 2\eta$.*

(R2) *Let P be a compact **A N R** and $\eta>0$.Whenever $a \in A$ and $f,g:X_a \rightarrow P$ are mapping with the property $d(fp_a,gp_a)<\eta$, then there exists a $b\in A$ such that for any $c\geq b$ $d(fp_{ac},gp_{ac})<\eta$.*

LEMMA 1.11 *Let* $\underline{X}=\{X_a, \in_a, p_{ab}, A\}$ *be an approximate inverse system and let* $x_a \in X_a$ *be any point such that* $p^{-1}_{ab}(x_a)$ *is non-empty for each* $b \geq a$. *Then* $p^{-1}_a(x_a)$ *is non-empty.*

Proof. Suppose that $p^{-1}_a(x_a)$ is empty.Then $x_a \notin p_a(X)$,where$X=\lim\underline{\mathbf{X}}$. Thus, $U=X_a\setminus\{x_a\}$ is open set which contains $p_a(X)$.By the property (B1) [7, pp. 899.] we infer that there is $b \geq a$ such that for each $c \geq b$ one has $p_{ac}(X_c) \subseteq U$.It follows that $x_a \notin p_{ac}(X_c)$,i.e.,$p_{ac}^{-1}(x_a)$ is empty.This contradicts the assumption of Lemma. ■

2 Connectedness of approximate limit

We start with the following theorem.

THEOREM 2.1 *Let* $\underline{X}=\{X_a, \in_a, p_{ab}, A\}$ *be an approximate inverse system of compacta* X_a. *If all* X_a *are connected,then* $X=\lim\underline{\mathbf{X}}$ *is connected.*

Proof. Suppose that X is not connected.Then there exist a pair U,V of disjoint open sets such that $X=U \cup V$.By virtue of the property (B1) there exists an $a \in \mathbf{A}$ and an open cover $\mathcal{U}_a=\{U_b\}:b \in \mathbf{B}\}$ of X_a such that $p^{-1}_a(\mathcal{U}_a)$ refines $\{U,V\}$.Let $B_0=\{b:b \in B, p^{-1}_a(U_b) \subseteq U\}$ and $B_1=\{b:b \in B, p^{-1}_a(U_b) \subseteq V\}$. Clearly,$B_0$ and B_1 are disjoint and non-empty.Now we consider the sets $U_0=\cup\{U_b:b \in b_0\}$ and $U_1=\cup\{U_b:b \in b_1\}$.Clearly,$U_0$ and U_1 are disjoint.It is obvious that U_0 and U_1 are open and non-empty.Moreover,$p^{-1}_a(U_0)=U$ and $p^{-1}_a(U_1)=V$.This means that U_0 and U_1 are closed since p_a is closed.Thus,U_0 and U_1 are disjoint non-empty open-closed subset of X_a.This is impossible since X_a is connected.■

Alternate Proof. Now we use the property (R1).Suppose that X is not connected.Then there is a maping $f:X \to D=\{0,1\}$.We identify D with a subspace $\{0,1\}$ of the segment $I=[0,1]$.Consider a cover $\mathcal{U}$ of I containing the sets:$U=[0,1/2)$,$V=(1/4,3/4)$,$W=(1/2,1]$.By the property (R1) it follows that there exists an $a \in \mathbf{A}$ and a mapping $f_a:X_a \to I$ such that f and $f_a p_a$ are $\mathcal{U}$-near.This means that $f_a(X_a) \subseteq I-\{1/2\}$.We infer that $X_a=f^{-1}_a(U) \cup f^{-1}_a(W)$ and $f^{-1}_a(U) \cap f^{-1}_a(W)=\varnothing$ .This contradicts the connectedness of X_a.■

THEOREM 2.2 *Let* $\underline{X}=\{X_a, \in_a, p_{ab}, A\}$ *be an approximate inverse system of chainable compacta* X_a. *Then* $X=\lim\underline{\mathbf{X}}$ *is chainable.*

Proof. Suppose that $\mathcal{U}$ is an open finite cover of X.By (B1) there is an $a \in \mathbf{A}$ and an open cover υ A such that $p^{-1}_a(\mathcal{V}_a)$ refines $\mathcal{U}$.There is a chainable refinement $\mathcal{U}_a$ of $\mathcal{V}_a$ since X_a is chainable.Clearly,$p^{-1}_a(\mathcal{U}_a)$ is a chainable refinement of $\mathcal{U}$.■

We say that a metric continuum X is **circle-like** if for each $\varepsilon > 0$ there is an ε -mapping $f:X \to K$,where K is the circle (=simple closed curve).A metric continuum is circle-like iff it is inverse limit of usual inverse system of the simple closed curves [10].This means that X is circle-like iff for each open cover there is a finite refinement $\{U_1,...,U_n\}$ such that $U_i \cap U_j \neq \varnothing$ if abs $(i-j) \leq 1$ or $i,j \in \{1,n\}$.

THEOREM 2.3 *Let* $\underline{X}=\{X_a, \in_a, p_{ab}, A\}$ *be an approximate inverse system of circle-like compacta.Then* $X=\lim\underline{\mathbf{X}}$ *is circle-like.*

Proof. The proof is similar to the proof of Theorem 2.2.■

3 Exactly (n,1) mappings

We say that a mapping $f:X \to Y$ is *exactly (n,1)* if $f^{-1}(y)$ contains exactly n points,for each $y \in Y$ [13].

A *dendrite* is locally connected metrizable continuum which contains no simple closed curve.

A *Peano continuum* is a metric locally connected continuum [4, pp. 257].

In the sequel we us the following result from [13].

THEOREM 3.1

[13, Corollary 2.1.]. A Peano continuum Y is a dendrite if and only if for each n ($2 \leq n < \infty$) there is no exactly (n,1) mapping from any continuum onto **Y**.

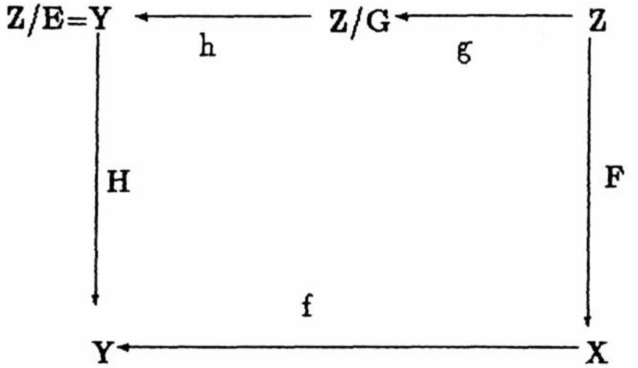
A mapping $f:X \to Y$ is said to be *exactly n-component-to-one* if $f^{-1}(y)$ has exactly n components for each $y \in Y$.

THEOREM 3.2 *[13, Corollary 2.2.].A Peano continuum Y is a dendrite if and only if for each n ($2 \leq n < \infty$) there is no exactly n-compopnents-to-one mapping from any continuum onto Y.*

Now we prove the following lemma.

LEMMA 3.3 Let $f:X \to Y$ be a mapping onto a dendrite Y such that the fibers $f^{-1}(y)$ are finite.If X is a Peano continuum,then X is a dendrite.

Proof. Suppose that X is not a dendrite.By virtue of Theorem above there exists a continuum Z and an exactly (n,1) mapping $\mathbf{F}:\mathbf{Z} \in \mathbf{X}$.Let E be be an equivalence relation induced by the mapping fF.A space **Z/E** is compact and thus homeomorphic to Y under the homeomorphism H.Clearly,the members of E are $(fF)^{-1}(y), y \in Y$.Now,we define an equivalence relation G as follows.For each $(fF)^{-1}(y)$ we have $(fF)^{-1}(y)=F^{-1}[f^{-1}(y)]$.Moreover,for each $x \in f^{-1}(y)$ there exist n points $z(y,x,1),...,z(y,x,n)$ such that $F^{-1}(x)=\{z(y,x,1),.., z(y,x,n)\}$.Let Z_i ,i=1,..,n,be a subset of Z defined by $Z_i(y)=\{z(y,x,i):x \in f^{\wedge -1}(y)\}$. It follows that the sets $Z_i(y)$,i=1,...,n,$y \in Y$,form an equivalence relation G on Z which is refinement of E.This means that there are the quotient mappings g and h such that g is induced by G and E is induced by hg (see the following diagram).A space X/G is a continuum and h is an exactly (n,1) mapping.This impossible since X/E is homeomorphic to Y and Y is a dendrite.



From the proof of Lemma 3.3. it follows

LEMMA 3.3.1. *Let f:X → Y be a mapping onto a dendrite Y such that,for each y ∈ Y,a fiber $f^{-1}(y)$ is finite.Then there is no exactly (n,1) mapping,2≤n < ∈ ∞,g:Z → X of a continuum Z onto X.*

LEMMA 3.4 Let f:X → Y be an exactly n-component-to-one mapping onto a dendrite Y.If X is a Peano continuum,then X is a dendrite.

Proof. By virtue of the Factorization theorem [20, pp. 141.] there exists a factorization $f=f_2 f_1$ such that f_2 is light and f_1 is monotone.If f is n-component-to-one mapping,then f_2 is exactly (n,1).Apply Lemma 3.3.■

LEMMA 3.4.1. *Let f:X → Y be a mapping onto a dendrite Y such that,for each y ∈ Y,a fiber $f^{-1}(y)$ is finite.Then there is no exactly n-component-to-one mapping,2≤n < ∈ ∞,g:Z → X of a continuum Z onto X.*

THEOREM 3.5 *The following is known:*

1. [13, Lemma.].Let Y be a continuum with an endpoint e and let n ∈ N,n ≥ 2.If there exists (n,1) mapping f from a continuum X onto Y,then there is a proper subcontinuum Y_1 of Y such that $f^{-1}(Y_1)$ is connected.

2. [13, Theorem 1.].Let Y be a continuum such that every nondegenerate subcontinuum of Y has an endpoint.If n ∈ N,n ≥ 2, then there is no exactly (n,1)-mapping from any continuum onto Y.

*3. [13, Corollary 1.1].*If n ≥ 2,*then there is no exactly (n,1) mapping onto a dendrite.*

4. [13, Theorem 2.].If Y is a continuum which contains a non-unicoherent subcontinuum and if n ∈ 2,then there is an exactly (n,1) mapping from some continuum X onto Y.

LEMMA 3.6 *Let f:X → Y be a mapping onto a dendrite Y.Then X is hereditarily unicoherente.*

Proof. Suppose that X is not hereditarily unicoherent.This means that there is a nonunicoherent subcontinuum of X.By 3.5.4. there is a continuum Z and an exactly (n,1) maping F:Z → X.In order to complete the proof it suffices to apply the proof of Lemma 3.3.■

THEOREM 3.7 *Let $\underline{X}=\{X_n, \epsilon_n, p_{mn}, N\}$ be an approximate inverse sequence of the dendrites.Then $X=\lim\underline{X}$ is hereditarily unicoherent.Moreover,if X is a Peano continuum,then X is a dendrite.*

Proof. Consider a mapping $p_n : X \to X_n$ and apply the above Lemma.It follows that X is locally connected and contains no a simple closed curve.■

THEOREM 3.8 *Let $\underline{X}=\{X_n, \epsilon_n, p_{mn}, N\}$ be an approximate inverse sequence of the simple closed curves.Then each proper subcontinuum of $X=\lim\underline{X}$ is hereditarily unicoherent.*

Proof. Let C be a proper subcontinuum of X.This means that there is a point x∈ X-C.By the definition of a base of X there exists an a∈**A** and an open set $U_a \subseteq X_a$ such that $p_a^{-1}(UA) \subseteq X-C$.It follows that pA(x)\not ∈ pA(C),i.e.,is a proper subcontinuum of X_a.Thus $p_a(C)$ is an arc.By Lemma 3.6. C is hereditarily unicoherent.■

4 Approximate inverse limit of continua in Class W

In this Section we use the hyperspace technique. We start with the following

THEOREM 4.1 *[6, Theorem 1.12.]*.Let $\underline{\mathbf{X}}=\{X_a,\epsilon_a,p_{ab},A\}$ be an approximate inverse system of metric compacta $X_a, a\in\mathbf{A}$.The spaces $2^{\lim\underline{X}}$ and $\lim 2^{\underline{X}}$ are homeomorphic.

By the same method of proof as in the proof of Theorem 2.1. we have

THEOREM 4.2 *Let (F_a) be a thread of $2^{\underline{X}}$ such that each F_a is connected. Then $F = \cap\{p^{-1}_a(FA):a\in\mathbf{A}\}$ is connected.*

Let C be a functor which assigns to a continuum X the hyperspace C(X) of all subcontinua of X.For each mapping $f:X\to Y$ there is a map $C(f):C(X)\to C(Y)$ defined by

$$C(f)(\mathbf{K})=f(\mathbf{K}),\mathbf{K}\in C(X) \tag{1}$$

For each mappings $f:X\to Y, g:Y\to Z$ we have

$$C(gf) = C(g)C(f) \tag{2}$$

For each inverse approximate system $\underline{\mathbf{X}}=\{X_n,\epsilon_n,p_{mn},N\}$ we have new system $C(\underline{\mathbf{X}})=\{C(X_n\}),\epsilon_n\},C(p_{nm}),\mathbf{A}\}$ [6] with the projections $\mathbf{P_n}:\lim C(\underline{\mathbf{X}})\to C(\mathbf{X_n})$. Moreover,we have the family of the mappings $C(p_n)$ which induces a homeomorphism $H:C(\lim\underline{\mathbf{X}})\to \lim C(\underline{\mathbf{X}})$ such that

$$C(p_n) = P_nH \quad \forall\mathbf{n}\in N \tag{3}$$

This means that for each $\mathbf{K}\in C(\lim \mathbf{X})$,i.e.,K is a subcontinuum of $\lim\underline{\mathbf{X}}$, we have

$$H(K) = \{p_n(K):n\in N\}\in \mathbf{\lim C(\underline{X})} \tag{4}$$

From 4.2. it follows that for each thread $\{K_n:n\in N\}$ there is a subcontinuum of $\lim\underline{\mathbf{X}}$ such that $p_n(K)=K_i$,i.e.,H is onto.Similarly it follows that H is 1-1.Thus,H is a homeomorphism.

Applying the functor C once more,we obtain the approximate inverse system $C^2(\underline{\mathbf{X}})=\{C^2(X_n),\epsilon_n,C^2(p_{nm}),A\}$ with the projections Q_n and the bonding mappings $C^2(p_{nm})$.Moreover,we have two families $\{C(P_n):n\in N\}$ and $\{C^2(p_n):n\in N\}$ which induce the homeomorphisms

$$H_1:C(\lim C(\underline{X}))\to\lim C^2(\underline{X}) \tag{5}$$

and

$$H_2:C^2(\lim\underline{X}) \to \lim C^2(\underline{X}) \tag{6}$$

We have also a homeomorphism

$$C(H):C^2(\lim\underline{X})\to C(\lim C(\underline{X})) \tag{7}$$

Moreover,we have the following relations of the commutative diagrams.

$$C(P_n) = Q_n H_1 \tag{8}$$

$$C^2(p_n) = Q_n H_2 \tag{9}$$

$$C^2(p_n) = C(P_n)C(H) \tag{10}$$

$$H_1 = H^2 C(H) \tag{11}$$

A mapping f:X → Y is said to be *weakly confluent* iff f ([18]:293) is onto and if any subcontinuum K of Y is the image of some component of f^{-1} (K).

A mapping f:X → Y is said to be *confluent* iff f ([18]:293) is onto and if any subcontinuum K of Y is the image of each component of f^{-1} (K).

If f is a map from a continuum X onto a continuum Y,then a subcontinuum K of Y is a w_f-set if there is a continuum K′ in X such that f(K′)=K [17].

A mapping f:X → Y *is n-partially confluent* if every subcontinuum of Y is the union of n or fewer w_f-sets [17].

A metric continum M is in *Class W* if and only if all mappings from metric contiua onto M are weakly confluent ([5] or [18]:293).

If f is a map from a continuum X onto a continuum Y,then a subcontinuum K of Y is a w_f-set if there is a continuum K′ in X such that f(K′)=K [17].

Define a function C* :C(X) → C(C(X)) by C* (A)=C(A) for each A in C(X),where C(X) is the hyperspace of all subcontinua of X (see [12]).It was proved that C* is upper semicontinuous [(15.2)] .A continuum X is said to be *C* -smooth at A*,A ∈ C(X),provided that C* is continuous at A.The continuum X is said to be *C* -smooth* [12, (15.5)] provided that it is C* -smooth at each A ∈ C(X).It is known that X is in Class W iff X is C* -smooth [2, 3.2. Theorem.].

We start with the following lemma.

LEMMA 4.3 *Let f:X → Y be a continuous mapping between C* -smooth continua.The the diagram*

$$\begin{array}{ccc} C(X) & \xrightarrow{C(f)} & C(Y) \\ \downarrow C^*_X & & \downarrow C^*_Y \\ C^2(X) & \xrightarrow{C^2(f)} & C^2(Y) \end{array}$$

commutes.

Proof. a) For each A ∈ C(X) C^2 (f)(C* (A)) is a collection of all f(K),where K is a subcontinuum of A.

b). For each A ∈ C(X) C* (C(f)(A)) is a collection of all subcontinua in f(A).

c) The continuum f(A) is in Class W since it is C* -smooth ([12,(15.6)] and [3.2. Theorem.]).This means that f/A is weakly confluent,i.e.,for each subcontinuum $L \subseteq f(A)$ there is a continuum $K \subseteq A$ such that f/A(K)=L.

d) From a),b) and c) it follows that C^2, (f)(C* (A))=C* (C(f)(A)).The proof is completed.■
The main theorem of this section

THEOREM 4.4 *Let $\underline{X}=\{X_n, \in_n, p_{mn}, N\}$ be an inverse sequence of metric compact spaces with onto bonding mappings.If each X_n is in Class W,then lim$\underline{\mathbf{X}}$ is in clas W.*

Proof.a) We have the following diagram

$$\begin{array}{ccccccc}
C(X_1) & \xleftarrow{C(p_{12})} & C(X_2) & \xleftarrow{C(p_{2n})} & C(X_n) & \cdots & \lim C(\underline{\mathbf{X}}) \\
\downarrow C_1^* & & \downarrow C_2^* & & \downarrow C_n^* & & \downarrow \lim C_n^* \\
C^2(X_1) & \xleftarrow{C^2(p_{12})} & C^2(X_2) & \xleftarrow{C^2(p_{2n})} & C^2(X_n) & \cdots & \lim C^2(\underline{\mathbf{X}})
\end{array}$$

For each thread $k=(K_n)$ in C($\underline{\mathbf{X}}$),i.e.,for each $k \in$ limC($\underline{\mathbf{X}}$),we have a collection C_n^* (K_n):n ∈ N}.Let us prove that (C_n^* (K_n):n ∈ N} is a thread in C^2, ($\underline{\mathbf{X}}$).It suffices to prove that the condition (L) is satisfied (see Section 1.).Let a n ∈ N be fixed.Let U be an open set about C_n^* (K_n).Then $(C_n^*)^{-1}$ (U)=V is an open set about K_n.From the condition (L) for $k=(K_n)$ it follows that there is a $m \geq n$ such that for each $c \geq m$ $C(p_{nc})(K_c) \in V$.Clearly, C_n^* $2^{P_{nc}}$ $(K_c) \in U$.By the commutativity of the diagram

$$\begin{array}{ccc}
C(X_n) & \xleftarrow{C(p_{nc})} & C(X_c) \\
\downarrow C_n^* & & \downarrow C_c^* \\
C^2(X_n) & \xleftarrow{C^2(p_{nc})} & C^2(X_c)
\end{array}$$

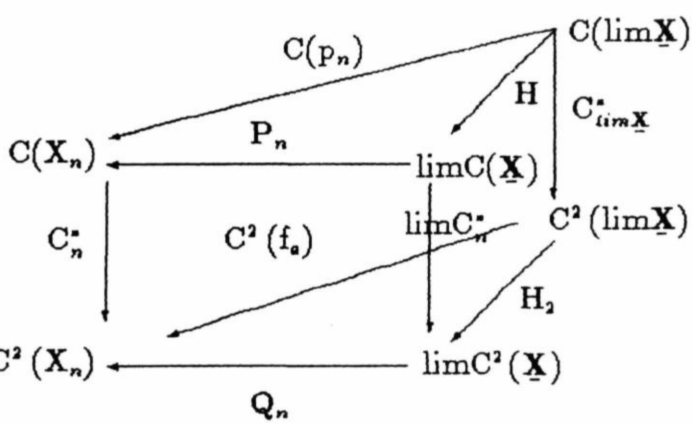
we infer that $C^2(p_{nc})$ C_c^* $(K_c) \in V$ for each $c \geq m$.This,means that the condition (L) is satisfied for the collection (C_n^* (K_n):n ∈ N).The continuity of $C^*_{\lim X}$ holds from the definition of a base in approximate limit (see Lemma 1.8.) and the comutativity of each diagram

$$\begin{array}{ccc}
C(X_n) & \xleftarrow{P_n} & \lim C(\underline{\mathbf{X}}) \\
\downarrow C_n^* & & \downarrow \lim C_n^* \\
C^2(X_n) & \xleftarrow{Q_n} & \lim C^2(\underline{\mathbf{X}})
\end{array}$$

where P_n are the projections. In order to complete the proof we prove that the diagram

$$\begin{array}{ccc} \lim C(\underline{X}) & \xleftarrow{H} & C(\lim \underline{X}) \\ \downarrow{\scriptstyle \lim C^*_n} & & \downarrow{\scriptstyle C^*_{\lim \underline{X}}} \\ \lim C^2(\underline{X}) & \xleftarrow{H_2} & C^2(\lim \underline{X}) \end{array}$$

commutes since then $C^*_{\lim X}$ is continuous.This follows from the next figure of the commutative diagrams.



The proof is completed.■

THEOREM 4.5 *A locally connected continuum is C* -smooth if and only if it is a dendrite.*

Proof. See [12,(15.11) Theorem.]■

THEOREM 4.6 *Let $\underline{X}=\{X_n, \in_n, p_{mn}, N\}$ be an aproximate sequence of the dendrites.If $X=\lim\underline{X}$ is locally connected,then X is a dendrite.*

Proof.X is C* -smoth and locally connected.Thus,X is a dendrite.■

5 Approximate inverse limit of continua with $P(X_n) \leq n_0$

For the continuum M let P(M) be the largest integer such that there is a map f from a continuum onto M that is not (P(M)-1)-partially confluent.This means that P(M) is the smallest integer such that for every map of a continuum onto M,every subcontinuum of M is the union of P(M) or fewer w_f-sets [17].For example,Class W is the clas of continua M for which P(M)=1.If M is a simple closed curve or a simple triod,P(M)=2.

Van C. Nall and Eldon J. Vought [17, Theorem 3.] proved the following theorem.

THEOREM 5.1 *Suppose n_0 is a positive integer ,and the continuum $X = \lim\{X_n, \in_n, p_{mn}, N\}$ where each X_n is a continuum such that $P(X_n) \leq n_0$, $n \in N$.Then $P(X) \leq n_0$.*

Now we prove the approximate version of Theorem above.

THEOREM 5.2 *Let $\underline{X}=\{X_n, \in_n, p_{mn}, N\}$ be an approximate inverse sequence of metric continua X_n with $P(X_n) \leq n_0$. Then $P(X) \leq n_0$, $X=\lim\underline{\mathbf{X}}$.*

Proof. Let $f:M \to X$ be a mapping onto X and let L be a subcontinuum of X.Since each $p_n f$ is n_0-partially confluent,for each positive integer n we have a collection $\{K_1^n,...,K^n_{n0}\}$ of subcontinua of M such that

$$\cup\{p_n f(K^n_j): j = 1,...,n_0\} = p_n(L) \tag{1}$$

For each n and for $j=1,...,n_0$,consider continua

$$L^n_j = f(K^n_j) \tag{2}$$

Choosing subsequences if necessary,assume that for each $j, 1 \leq j \leq n_0$,the sequence $\{L^n_j : n \in N\}$ converges to a continuum L_j in X.Likewise,the sequence $\{K^n_j : n \in N\}$ converges to a continuum K_j in M.From (2) it follows that

$$f(K_j) = L_j \quad \forall\ j \in [1,n_0] \tag{3}$$

In order to complete the proof we prove that

$$L = \cup\ \{L_j : j \in [1,n_0]\} \tag{4}$$

i.e.,we prove that L is the union of n w_f-sets since since each L_j is a w_f-set.Let x be any point in L.Then $\{p_n(x): n \in N\}$ is a thread and $p_n(x) \in p_n(L)$.From (1) it follows that there is a point $k^n_{j(n)} \in K^n_{j(n)}$ such that

$$p_n f(k^n_{j(n)}) = p_n(x) \tag{5}$$

Since $1 \leq j(n) \leq n_0$ for each $n \in N$,there is a cofinal subset N´ of N such that j(n) is constant function on N´.Thus,one can assume that j(n) is constant on N,say $j(n)=1, n \in N$.One can also assume that sequence$\{k^n_1 : n \in N\}$isconvergent.Let$k=\{k^n_1 : n \in N)$.Clearly,$k \in K_1$.Let$y=f(k) \in L_1$.Hence,$y=\lim \{f(k^n_1): n \in N\}$.

From (5) it follows that $f(k^n_1) \in p^{-1}_n(p_n(x))$.It follows that $\lim\{f(k^{nl}): n \in N\}=x$.Thus,x=y.The relation

$$L \subseteq \cup\ \{L_j : j \in [1,n_0]\} \tag{6}$$

is proved.Conversely,if $x \in \cup\ \{L_j : j \in [1,n_0]\}$,then x is in some L_j ,say L_1 .There is a point $k \in K_1$ such that x=f(k).We infer that $x=\lim\{f(k^n_1): n \in N\}$ since $k=\lim\{k_1^n : n \in N\}$.Each $p_n(k^n_1)$ is in $p_n(L)$.This means that each $f(k^n_1)$ is in $p_n^{-1}(p_n(L))$.It follows that $\lim\{f(k^n_1): n \in N\}$ is in L.Thus x is in $\cup\ \{L_j : j \in [1,n_0]\}$.The relation

$$L \supseteq \{L_j : j \in [1,n_0]\} \tag{7}$$

is proved.This means that (4) is proved.The proof of is completed.■

COROLLARY 5.3 *Let $\underline{X}=\{X_n, \in_n, p_{mn}, N\}$ be an inverse sequence of metric compact spaces with onto bonding mappings.If each X_n is in Class W,then $\lim\underline{\mathbf{X}}$ is in clas W.*

See also Theorem 4.4.

COROLLARY 5.4 *Let $\underline{X}=\{X_n, \in_n, p_{mn}, N\}$ be an approximate inverse system with onto bonding mappings.If each X_n contains n_0-od but no $(n_0 + 1)$-od for some $n_0 \geq 2$, then $P(\lim\underline{\mathbf{X}}) \leq n_0(n_0 - 1)$.*

Proof. For each X_n we have $P(X_n) \leq n_0(n_0 - 1)$ [15, Theorem II.2]. Apply Theorem 5.2.■

COROLLARY 5.5 *Let $\underline{X}=\{X_n, \epsilon_n, p_{mn}, N\}$ be an approximate inverse system with onto bonding mappings.If each X_n is a simple closed curve (or a triod),then $P(\lim\underline{\mathbf{X}}) \leq 2$.*

Proof. Apply Theorem 5.2. and the fact that if X is a simple closed curve or a triod,then P(X)=2.

6 Approximate limit of graphs

DEFINITION 6.1. *A continuum M is an **n-od**,where n is an integer greate than 1,if M contains a subcontinuum K,called the **core** of the n-od,such that M \K has n components.*

THEOREM 6.2 *[17, Theorem 4.] .If a continuum X contains an n-od,then there is a positive number ϵ such that if f is an ϵ-mapping from X onto Y,then Y contains an n-od.*

By 6.2. and (B2) it follows

THEOREM 6.3 *Let $\underline{X}=\{X_n, \epsilon_n, p_{mn}, N\}$ be an inverse sequence of metric compact spaces with onto bonding mappings.If $\lim\underline{\mathbf{X}}$ contains an n_0-od,then there is $n \in N$ such that each X_m ,$m \geq n$,contains an n_0-od.*

COROLLARY 6.4 *Let $\underline{X}=\{X_n, \epsilon_n, p_{mn}, N\}$ be an inverse sequence of metric continua with onto bonding mappings.If each X_n contains no an n_0-od,then $\lim\underline{\mathbf{X}}$ contains no an n_0-od.*

A space X is *semi-aposyndetic* if for each pair of points in X there is a continuum in X that contains one of the points in its interior and does not contain the other point.

A subcontinuum A of a continuum X is a *free arc* in X if A is an arc such that the boundary of A is contained in the set of endpoints of A.A continuum is a *graph* if it is the union of a finite number of free arcs [17].

THEOREM 6.5 *[17,Theorem 5.].A continuum is a graph iff it is semi-aposyndetic and does not contain an infinite-od.*

THEOREM 6.6 *Let $\underline{X}=\{X_n, \epsilon_n, p_{mn}, N\}$ be an inverse sequence of metric continua with onto bonding mappings such that each X_n contains no an n_0-od.If $\lim\underline{\mathbf{X}}$ is semi-aposyndetic,then $\lim\underline{\mathbf{X}}$ is a graph.*

Proof. If we suppose that a space X=$\lim\underline{\mathbf{X}}$ contains an infinite-od,then we infer that X contains n_0-od.This contradicts Theorem 6.4.Thus,X does not contain an infinite-od.By Theorem 6.5. we complete the proof.■

REMARK 6.7 Let $\underline{X}=\{X_n, \epsilon_n, p_{mn}, N\}$ be an inverse sequence of graphs X_n with onto bonding mappings such that each X_n contains no an n_0-od.If X=$\lim\underline{\mathbf{X}}$ is semi-aposyndetic,then X is a graph.Moreover,X is the approximate inverse limit of a single graph.This follows from the fact that there are only finitely many graphs X(1),...,X(k(n_0)) that do not contain n_0-od.Let N_j be a set of all $n \in N$ such that X_n =X(j),$1 \leq j \leq k(n_0)$).Clearly,N is the union of all N_k .This means that some N_j is infinite,i.e.,cofinal in N.Consider a system $\underline{\mathbf{X}}$**(j)**=$\{X(j), \epsilon_n, p_{mn}, N_j\}$.By virtue of Proposition 2. [8] we have X=lim $\underline{\mathbf{X}}$**(j).**

THEOREM 6.8 *Let $\underline{X}=\{X_n, \in_n, p_{mn}, N\}$ be an inverse sequence of a single graph G with onto bonding mappings.If $X=\lim\underline{X}$ is semi-aposyndetic,then X is a graph.*

THEOREM 6.9 *Let $\underline{X}=\{X_n, \in_n, p_{mn}, N\}$ be an inverse sequence of the arcs X_n with onto bonding mappings.If $X=\lim\underline{X}$ is semi-aposyndetic,then X is an arc.*

Proof. By Theorems 2.2. and 6.8. X is chainable graph.Thus,X is an arc.■

THEOREM 6.10 *Let $\underline{X}=\{X_n, \in_n, p_{mn}, N\}$ be an inverse sequence of the simple closed curves X_n with onto bonding mappings.If $X = \lim\underline{X}$ is semi-aposyndetic,then X is a simple closed curve.*

Proof. By Theorems 2.3. and 6.8. X is circle-like graph.Thus,X is a simple closed curve.■

COROLLARY 6.11 *The arc is the only non-degenerate metric semi-aposyndetic chainable continuum.*

Proof. Let X be a non-degenerate metric semi-aposyndetic chainable continuum.By Theorem 1. of [10] it follows that there exists an usual inverse system $\underline{\mathbf{I}}=\{I_n, p_{mn}, N\}$ of the arcs I_n such that X is homeomorphic to $\underline{\mathbf{I}}$.From Remark 2. of [7] it follows that one can define numbers $\in_n$ such that $\underline{\mathbf{X}}=\{I_n, \in_n, p_{mn}, N\}$ is an approximate inverse system whose approximate limit coincide with usual inverse limit of $\underline{\mathbf{I}}$. Thus,X is approximate limit of $\underline{\mathbf{X}}$.By Theorem 6.9. we infer that X is an arc.Conversely,each arc is non-degenerate metric semi-aposyndetic chainable continua.■

By the same method of proof we have

COROLLARY 6.12 *The simple closed curve is the only non-degenerate metric semi-aposyndetic circle-like continuum.*

We close this section with theorem whose usual version was proved in [14, IV.1. Theorem.].

THEOREM 6.13 *Let $\underline{X}=\{X_n, \in_n, p_{mn}, N\}$ be an approximate inverse system such that each X_n is an acyclic graph G with exactly s edges.Then each subcontinuum of $X=\lim\underline{X}$ is the union of n or fewer W-sets.*

Proof. Suppose that K is any subcontinuum of X.Each projection $p_n : X \to X_n$, $n \in N$,is weakly confluent with respect to $p_n(K)\cap E_i$ for each edge $E_1, ..., E_s$ of G [16,Lemma 3.].We define K_{ni} as a suncontinuum of K which projects onto $p_n(K)\cap E_i$ .We consider,for each $i \in \{1,2,...,s\}$ a sequence $\{K_{ni} : n \in N\}$.We may assume that this sequence converges to a subcontinuum K_i of K.Let us prove that $K = \cup\{K_i : i=1,...,s\}$ It is clear that $K \subseteq \cup \{K_i : i=1,...,s\}$.Conversely,if $x\in K$,then $x_n = p_n(x)$ is contained in $p_n(K)\cup E_i$ for some i=1,2,...,s.There is a point $x_{ni}\in K_{ni}$ such that $p_n(x_{ni})=x_n$ .Clearly,a sequence $\{x_{ni} : n \in N\}$ converges to x.It follows that $x\in\cup \{K_i : i=1,...,s\}$.Thus $K= \cup \{K_i : i=1,...,s\}$.In order to complete the proof it suffices to prove that each K_i ,i=1,...,s is a W-set.To see this,let $f:Y \to X$ be any mapping of some continuum Y onto X.For each integer n and i=1,...,s,$p_n f$ is weakly confluent with respect to $p_n(K)\cap E_i$ [16,Lemma 3.].Let C_{ni} be a subcontinuum of Y such that $p_n(C_{ni} = p_n(K)\cap E_i$ .We may asumme that a sequence $\{K_{ni} : n \in N\}$ converges to a subcontinuum C_i .Let x be any point of K_i ,and let $\{x_{ni}\}$ be a sequence of points converging to x such that $x_{ni}\in K_{ni}$ .For each n there is a point c_{ni} in C_{ni} such that $p_n(f(c_{ni})=p_n(x_{ni}$ .The sequence $\{f(c_{ni})\}$ converges to x.We have $K_i\subseteq f(C_i)$.On the other hand,if f(c) is in $f(C_i)$,then there is a sequence $\{c_{ni}\}$ converging to c such that $c_{ni}\in C_{ni}$ .There exists a sequence $\{x_{ni}\}$ such that $x_{ni}\in K_{ni}$ and $p_n(x_{ni})=p_n(f(c_{ni}))$.We may assume that $\{x_{ni}\}$ converge to $x\in K_i$ and $\{f(c_{ni})\}$ converge to x.Since

$f(c)=x$,$f(c)$ is in K_i .Thus,$f(C_i)\subseteq K_i$ .We infer that each $f:Y \rightarrow X$ is weakly confluent with respect to K_i .Thus K is the union of n or fewer W-sets.The proof is completed.■

QUESTION. *Is every map from a continuum onto a limit of an approximate inverse sequence of a graph G partially confluent? For usual inverse sequence see [16,VI. Question]*

7. Confluent mappings

We start with following theorem.

THEOREM 7.1. *Let $\underline{X}=\{X_a,\varepsilon_a,p_{ab},A\}$ be an approximate inverse system of compacta with onto bonding mappings.The projections $p_a:\lim\underline{X} \rightarrow X_a, a\in A$,are weakly confluent if the mappings p_{ab} are weakly confluent.*

Proof. Let $C(\underline{X})=\{C(X_a),\in_a,C(p_{ab}),A\}$ be an approximate inverse system corresponding to approximate system $\underline{X}=\{X_a,\in_a,p_{ab},A\}$ (see 4.4.).The mapping $C(p_{ab})$ are onto since p_{ab} are onto and weakly confluent.Moreover,$C(p_{ab})$ are onto if and only if p_{ab} are onto and weakly confluent.By Lemma 1.11 we infer that the projections $P_a:\lim C(\underline{X}) \rightarrow C(X_a)$ are onto if $C(p_{ab})$ are onto.Since $C(\lim\underline{X})$ and $\lim C(\underline{X})$ are homeomorphic,for each continuum $K_a\subseteq X_a$,i.e., $K_a\in C(X_a)$,there is a point K in $C(\lim\underline{X})$ such that $C(p_a)(K)=K_a$.The proof is completed since K is contained in some component of $p^{-1}_a(K_a)$.

A mapping $f:X \rightarrow Y$ is said to be weakly confluent at a point $y \in Y$,if for each subcontinuum K of Y such that $y \in K$ there exist a component of $f^{-1}(K)$ which is mapped onto the whole K under f [1]

The usual version of the following theorem was proved in [1].

THEOREM 7.2 *Let $\underline{X}=\{X_a,\in_a,p_{ab},A\}$ be an approximate inverse system.If there is a point $x_a\in X_a$ such that for each $b\geq a$ a mapping p_{ab} is weakly confluent at a point x_a,then a projection p_a is weakly confluent at a point x_a.*

Proof. The proof is similar to the proof of Theorem 1.11.■

A mapping $f:X \rightarrow Y$ is said to be *confluent relative to a point* $x\in X$ [1] if for each subcontinuum $K\subseteq Y$ such that $f(x)\in K$ the component of $f^{-1}(K)$ containing the point x is mapped onto the whole K under f.

REMARK 7.3 *The author is not able to answer to the following questions:*

1. QUESTION.Let $\underline{X}=\{X_a,\in_a,p_{ab},A\}$ be an approximate inverse system with confluent bonding mappings.Does it follow that the projections are confluent?

2. QUESTION.Let $\underline{X}=\{X_a,\in_a,p_{ab},A\}$ be an approximate inverse system with n-partially confluent bonding mappings.Does it follow that the projections are n-partially confluent?

3. QUESTION.Let $\underline{X}=\{X_a,\in_a,p_{ab},A\}$ be an aproximate inverse system and let $x=(x_a)$ be any point of $X=\lim\underline{X}$ such that p_{ab} are confluent relative to a points x_a.Does it follows that the projections pA are confluent relative to the point x?

Let us note that for the usual inverse system the answer to first question is yes [1].

8 Approximate limits of continua without n-ods and w-sets

Let us recall that a proper nondegenerate subcontinuum $K \subseteq Y$ is a W-set if,for every continuum X and map f of X onto Y,some subcontinuum of X is mapped by f onto K.

THEOREM 8.1 *Let A be a proper subcontinuum of a continuum X.Then A is not a W-set if and only if there exists some $\varepsilon > 0$ and a neighborhood G of A such that*

1. for each $x \in G$ there exists a continuum B from x to Bd(G) such that $A \not\subset S(B, \varepsilon)$,and
2. for each decomposition of $Bd(G)=R \cup S$ into disjoint closed sets R and S,there exists a continuum K from R to S with $A \not\subset S(K, \in)$.

Proof. See [3,2.1. Theorem.].■

THEOREM 8.2 *A continuum X is in Class (W) if and only if for every subcontinuum A of X, for each $\varepsilon > 0$ and each neighborhood U of A we have either*
1. there exists $x \in U$ such that for every continuum B from x to Bd(U) in Cl(U) we have that $A \subset S(B, \in)$,or
2. there is a decomposition of $Bd(U)= R \cup S$ into disjoint non- empty closed sets R and S such that for each subcontinuum K of X from R to S we have $A \subset S(K, \varepsilon)$.

Proof. See [3, 2.2. Corollary.].■

THEOREM 8.3 *[19, Theorem 1.]. The continuum Y contains no W-sets and,for some integer $n \geq 3$,has no n-ods if and only if Y is a graph in which each point is contained in a simple closed curve.*

THEOREM 8.4 *[19, Corollary 1.]. If Y is an atriodic continuum that contains no W-sets,then Y is a simple closed curve and conversely.*

A proper nendegenerate subcontinuum K of Y is said to be W´-set if for every map f of Y onto Y,some subcontinuum of Y is mapped onto K.

THEOREM 8.5 *[19, Theorem 3.].The continuum Y contains no W´-sets and,for some integer ≥ 3,has no n-ods if and only if Y is a graf in which each point is contained in a simple closed curve.*

THEOREM 8.6 *Let $\underline{X}=\{X_n, \in_n, p_{mn}, N\}$ be an approximate inverse sequence of the simple closed curves.A space $X=\lim\underline{X}$ has no W-sets if and only if X is a simple closed curve.*

Proof. The space X is atriodic (see Theorem 6.2.).If X has no W-sets,then X is a simple closed curve (see 7.4.).Conversely,if X is a simple closed curve,then (by 8.4.) X has no W-sets.■

REMARK 8.7 *Let us note that from Corollary 5.5. it follows that if $\underline{X}=\{X_n, \in_n, p_{mn}, N\}$ is an approximate inverse sequence of the simple closed curves X_n with limit X,then each subcontinuum of X is the union of 2 or fewer W-sets.*

Similarly,from 6.2. and 8.3. it follows

THEOREM 8.8 *Let $\underline{X}=\{X_n, \epsilon_n, p_{mn}, N\}$ be an approximate inverse sequence of the graphs which for some integer $n \geq 3$ have no n-ods and each point of X_n is contained in a simple closed curve.A space $X=\lim\underline{\mathbf{X}}$ has no W-sets if and only if X is graph in which each point is contained in a simple closed curve.*

QUESTION. *Let $\underline{X}=\{X_a, \epsilon_a, p_{ab}, A\}$ be an approximate inverse system of continua containing no W-sets.Does it follow that $\lim\underline{\mathbf{X}}$ contains no W-sets?*

W - SKUPOVI I APROKSIMATIVNI LIMES

SADRŽAJ

Preslikavanje f:X→Y kontinuuma X na kontinuum Y je *slabo konfluentno* ako za svaki potkontinuum K⊆Y postoji komponenta Q skupa $f^{-1}(K)$ sa svojstvom f(Q)=K. Potkontinuum K kontinuuma X je *W-skup* ako za svako preslikavanje f:Y→X na X postoji kontinuum L⊆Y sa svojstvom f(L) = K. Prostor X je iz klase W *(X∈Class W)* ako je svako preslikavanje f:Y→X kontinuuma Y na kontinuum X konfluentno.

U drugom odjeljku dokazujemo da je aproksimativni limes kontinuuma kontinuum te da je limes lančast ako su svi prostori sistema lančasti.

Treći odjeljak posvećen je točno (n,1)-preslikavanjima. U četvrtom odjeljku dokazujemo da je aproksimativni limes u klasi W ako su takvi prostori aproksimativnog sistema. Taj je rezultat nov i za obične inverzne sisteme.

U petom odjeljku je taj rezultat dokazan u općenitijoj situaciji.

Aproksimativni limesi konačnih grafova proučavani su u šestom odjeljku.

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