Using an Evolutionary Algorithm for Harmonic Load Modeling by Norton and Thevenin Equivalents

Preliminary Communication

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Abstract *– The Norton equivalent model for harmonic load modeling is used widely. The parameters of the model are usually calculated analytically. These calculations are based on measured or simulated voltage/current waveforms. In this case, harmonic analysis is needed to obtain the harmonic spectra. After this step, model parameter values are calculated for each harmonic separately. Relative phase shifts of all measured harmonic voltages and currents are most important for correct calculation of Norton model parameters. Different estimations in the power network are used to determine the phase shifts. The use of an evolutionary algorithm to determine the voltage and current harmonic phase shifts is researched in the paper. The proper optimization problem is defined and an evolutionary algorithm is used to solve the problem. The verification of the method will be done by comparing simulated waveforms obtained by a computer program.*

Keywords *– heuristic optimization, highdimensional optimization, nature-inspired algorithms, optimization techniques.*

1. INTRODUCTION

The Norton equivalent model is very useful for harmonic load modeling. It is used for steady state analysis of the electrical power distribution system in the presence of harmonic sources [1]. In addition, other methods are used for harmonic simulation in distribution networks [2], [3].

The Norton equivalent method belongs to frequency domain methods for harmonic analysis. The Norton equivalent is based on harmonic analysis of measured voltages at a network node and currents through a network section (line) for two different network states [1], [4]. In the method applied in this paper, the Norton and Thevenin equivalent sources are used. The Norton equivalent is modeled by a current source in parallel with Norton impedance. The Thevenin equivalent is modeled by a voltage source in series with Thevenin impedance. The load side containing the harmonic source is modeled by the Norton equivalent in the method. The supply side of the distribution network is modeled by the Thevenin equivalent in the method. Phase shifts of the measured voltages and currents are very important data for the application of the method [1]. These phase shifts must be determined relative to the phase angle that remains the same in different distribution network states. This is usually the phase angle of the fundamental voltage measured at a network node [1]. The measured voltage and current phase shifts relative to the referent phase angle can be calculated. The data of the complete distribution network must be known in this case. In cases where network data are unknown, they are approximated based on common values in distribution networks [1].

To overcome this problem, the application of an evolutionary algorithm is proposed in the paper. The application of the Evolutionary Strategy (ES) is researched to estimate unknown network data.

Evolutionary algorithms and evolutionary strategies are used to solve similar problems related to harmonic analysis in [5]-[7].

2. MODELING DISTRIBUTION NETWORK BY NORTON AND THEVENIN EQUIVALENTS

The method based on Thevenin and Norton equivalent sources in distribution networks for harmonic analysis is presented in [1]. The method is based on modeling the supply side of the network by the Thevenin equivalent and the load side of the network by the Norton equivalent with respect to a network node.

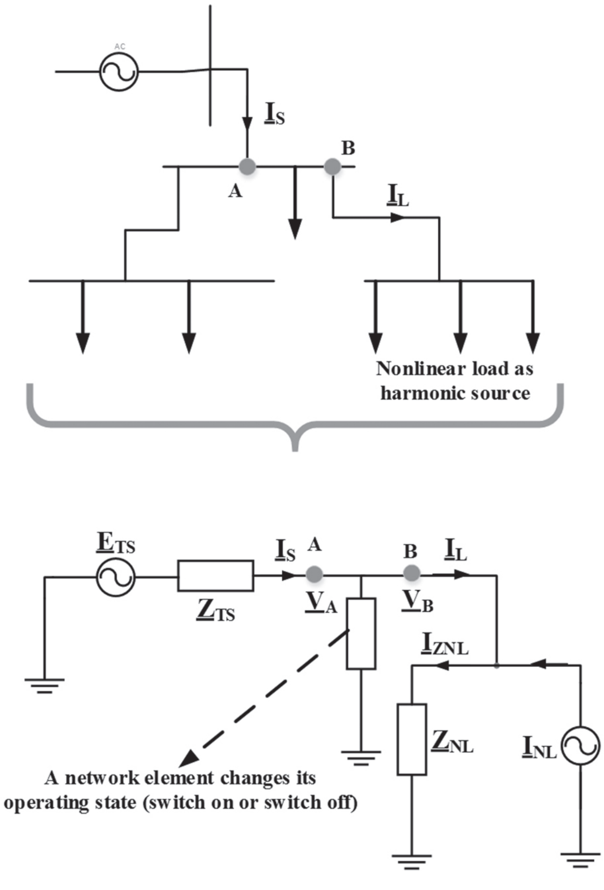

Fig. 1. Modeling supply (source) side by Thevenin equivalent (with respect to node A) and load side by Norton equivalent (with respect to node B)

The measured currents and voltages for two different network states are required to determine the parameters of the Norton and the Thevenin equivalent [1], [4], [8]. The basic idea of the method is shown in Fig. 1.

According to Fig. 1, the supply side of the network is modeled by the Thevenin equivalent with respect to network node A. The load side (contains nonlinear load as a harmonic source) of the network is modeled by the Norton equivalent. Modeling of the load side must be done for each harmonic separately. Two different steady states of the network are required to determine the parameters of the Thevenin and the Norton equivalent [1]. Changes of voltages and currents are caused by switching a network element. This can be a shunt capacitor bank or a parallel transformer. According to [1], switching of a network element should cause significant changes in harmonic currents and voltages.

According to Fig. 1, using Kirchhoff laws (KCL and KVL) for different steady states in the phasor (complex) domain (frequency domain), the expressions can be defined (zero phase angle of Thevenin voltage is taken as a referent angle) as follows.

$$\begin{aligned} E_T &= I_{S,1}\angle\alpha_1 \cdot Z_{TS}\angle\varphi_Z + V_{A,1}\angle\beta_1 \\ E_T &= I_{S,2}\angle\alpha_2 \cdot Z_{TS}\angle\varphi_Z + V_{A,2}\angle\beta_2 \end{aligned} \tag{1}$$

Voltages $V_{A,1}$ and $V_{A,2}$, and currents $I_{S,1}$ and $I_{S,2}$ are measured quantities.

$$\begin{aligned} \underline{I}_{NL} &= \underline{I}_{ZNL,1} - \underline{I}_{L,1} \quad ; \ \underline{I}_{ZNL,1} = \frac{\underline{V}_{B,1}}{\underline{Z}_{NL}} \\ \underline{I}_{NL} &= \underline{I}_{ZNL,2} - \underline{I}_{L,2} \quad ; \ \underline{I}_{ZNL,2} = \frac{\underline{V}_{B,2}}{\underline{Z}_{NL}} \end{aligned} \tag{2}$$

Voltages $V_{B,1}$ and $V_{B,2}$, and currents $I_{L,1}$ and $I_{L,2}$ are measured quantities. As can be seen from (1) and (2), the phasors of measured voltages and currents are required. This means that the phase angles of measured voltages and current must be known to determine parameters of the Thevenin and the Norton equivalent.

According to [1], it is imperative to measure phase angles of all measured voltages and currents with respect to the same reference phase angle. This is very difficult to provide in practice because it requires a synchronized measurement of many quantities in different network nodes. On the other hand, voltages and current in the same network node (for example V_A and I_S or V_B and I_L) can be measured simultaneously. Phasor determination can be done based on the measured voltage and current waveforms. In this case, the phase angle between voltage and current in a network node can be obtained easily. The zero reference phase angle of the Thevenin voltage in (1) is usually taken. In this case, (1) can be rearranged as:

$$\begin{aligned}&I_{S,1}\cdot Z_{TS}\cdot cos\left(\varphi_Z\right)+j\cdot I_{S,1}\cdot Z_{TS}\cdot sin\left(\varphi_Z\right)+\\&V_{A,1}\cdot cos\left(\beta_1-\alpha_1\right)+j\cdot V_{A,1}\cdot sin\left(\beta_1-\alpha_1\right)-\\&E_T\cdot cos\left(-\alpha_1\right)-j\cdot E_T\cdot sin\left(-\alpha_1\right)=0\\&I_{S,2}\cdot Z_{TS}\cdot cos\left(\varphi_Z\right)+j\cdot I_{S,2}\cdot Z_{TS}\cdot sin\left(\varphi_Z\right)+\\&V_{A,2}\cdot cos\left(\beta_2-\alpha_2\right)+j\cdot V_{A,2}\cdot sin\left(\beta_2-\alpha_2\right)-\\&E_T\cdot cos\left(-\alpha_2\right)-j\cdot E_T\cdot sin\left(-\alpha_2\right)=0\end{aligned}\tag{3}$$

The Thevenin impedance $\underline{Z}_{TS}$ is:

$$\begin{aligned}\underline{Z}_{TS}=Z_{TS}\cdot cos\left(\varphi_Z\right)+j\cdot Z_{TS}\cdot sin\left(\varphi_Z\right)=\\R_{TS}+j\cdot X_{TS}\end{aligned}\tag{4}$$

where R_{TS} is the resistance of the Thevenin equivalent (it is the whole network resistance between the supply network point and the network node the Thevenin equivalent refers to) and X_{TS} is the reactance of the Thevenin equivalent (it is the whole network reactance between the supply network point and the network node the Thevenin equivalent refers to).

In (3) and (4), $I_{S,1}$, $I_{S,2}$, $V_{A,1}$, $V_{A,2}$, β_1-α_1 and β_2-α_2 are known and R_{TS}, X_{TS}, E_T, α_1 and α_2 are unknown. Equation (3) is a nonlinear system of equations. The assumption of the X_{TS} over R_{TS} ratio based on the common X over R ratio (X/R) of the distribution network is used to solve (3) in [1]. The optimization approach based on an evolutionary algorithm is proposed to solve (3) in the paper.

Equations in (2) can be rearranged to determine the parameters $\underline{I}_{NL}$ and $\underline{Z}_{NL}$ of the Norton equivalent as:

$$\begin{aligned}\underline{Z}_{NL}&=\frac{\underline{V}_{B,1}-\underline{V}_{B,2}}{\underline{I}_{L,1}-\underline{I}_{L,2}}\\\underline{I}_{NL}&=\frac{\underline{V}_{B,2}}{\underline{Z}_{NL}}-\underline{I}_{L,2}\end{aligned}\tag{5}$$

Here it should be noticed that (5) needs to be determined for each harmonic of the measured load current and voltage waveform. The phase angles of each harmonic of the measured voltage and current in (5) need to be defined with respect to the referent phase angle as in the case of (3). This means that either the measured voltages or currents on the supply and the load side needs to be same ($\underline{V}_A=\underline{V}_B$ or $\underline{I}_S=\underline{I}_L$). This is necessary because the phase angle of the supply side voltage or current is determined with respect to the reference angle of the Thevenin voltage by solving (3). The proposed optimization approach based on an evolutionary algorithm is described below.

3. OPTIMIZATION PROBLEM DEFINITION FOR DETERMINATION OF THE THEVENIN EQUIVALENT PARAMETERS

Problem (3) is defined in optimization form to solve the system of nonlinear equations. The real and the imaginary component of (3) are expressed as parts of the optimization objective for this purpose:

$$\begin{aligned}f1&=I_{S,1}\cdot R_{TS}+V_{A,1}\cdot cos\left(\beta_1-\alpha_1\right)-\\&\quad E_T\cdot cos\left(-\alpha_1\right)\\f2&=I_{S,1}\cdot X_{TS}+V_{A,1}\cdot sin\left(\beta_1-\alpha_1\right)-\\&\quad E_T\cdot sin\left(-\alpha_1\right)\\f3&=I_{S,2}\cdot R_{TS}+V_{A,2}\cdot cos\left(\beta_2-\alpha_2\right)-\\&\quad E_T\cdot cos\left(-\alpha_2\right)\\f4&=I_{S,2}\cdot X_{TS}+V_{A,2}\cdot sin\left(\beta_2-\alpha_2\right)-\\&\quad E_T\cdot sin\left(-\alpha_2\right)\end{aligned}\tag{6}$$

According to (3), the right-hand sides of the expressions are zero. Because of that, the minimization optimization problem is defined as:

$$\begin{aligned}&f(R_{TS},\,X_{TS},\,E_T,\alpha_1,\alpha_2)=\\&f1+f2+f3+f4\rightarrow min\\&subject\quad to:\\&X_{TS}>R_{TS}\\&0\le\alpha_1\le2\pi\\&0\le\alpha_2\le2\pi\\&0.5U_n\le E_T\le2U_n\end{aligned}\tag{7}$$

where U_n is the nominal voltage at the network node where voltage is measured. The first constraint in (7) is very important for optimization by an evolutionary algorithm. This constraint is based on the fact that the X over R ratio in MV distribution networks is greater than 1. The evolutionary algorithm is able to find more different decision variable solutions and constraints are very important in order to direct the search of the solution space in a physically acceptable area of the solution space. Testing of the proposed method shows that the first constraint in (7) is most significant for this. The last three constraints in (7) are defined in order to decrease the solution space needed to be searched by the optimization algorithm.

4. EVOLUTIONARY STRATEGY

The evolutionary strategy (ES) is used here to solve optimization problem (7). A brief description of the most important ES procedure is given. The main steps in the ES are shown in Fig. 2.

Details about crossover and mutation operators in the ES can be seen in [9]. The discrete crossover is used here. The (μ+λ) type of the ES is used and the maximum number of generations is used as a stopping criterion. The parameters of the ES used in simulations below are: the number of generations 200, the number of in-

dividuals (μ) 50, and the number of offsprings (λ) 2,000. The ES is run 20 times and the best results are used.

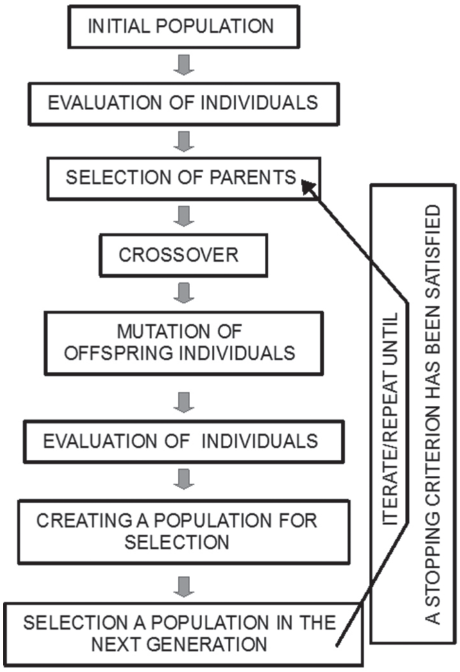

Fig. 2. Steps in ES

The individual in the ES consists of decision variables of optimization problem (7) and it is shown in Fig. 3.

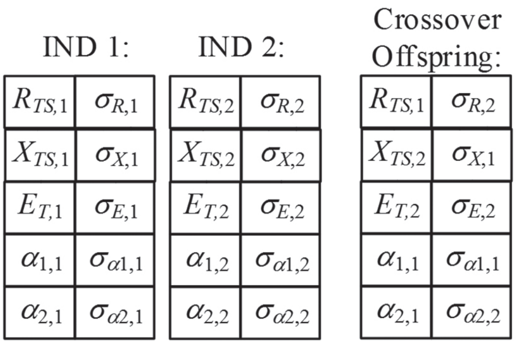

Fig. 3. Individual and discrete crossover in the ES (σ - s are strategy variables in ES [9])

Details about crossover and mutation operators in the ES can be seen in [9]. The discrete crossover is used here. The (μ+λ) type of the ES is used and the maximum number of generations is used as a stopping criterion. The parameters of the ES used in simulations below are: the number of generations 200, the number of individuals (μ) 50, and the number of offsprings (λ) 2,000. The ES is run 20 times and the best results are used.

5. TESTING AND SIMULATION RESULTS

An example of the distribution network is used to demonstrate the proposed method. The simulation program ATP is used to "measure" voltage and current waveforms. Testing the proposed method is performed using the following steps:

Step 1: measured current and voltage waveformsgenerated by ATP for the estimation of the Thevenin and the Norton equivalent parameters,

Step 2: harmonic analysis of measured currents and voltages in ATP,

Step 3: estimation of parameters of the Thevenin equivalent and phase angles of measured currents and voltages by solving optimization problem (7),

Step 4: calculation of parameters of the Norton equivalent for each significant harmonic detected in Step 2 according to (5),

Step 5: constructing an equivalent circuit in ATP of the original network by using the Thevenin and the Norton equivalents,

Step 6: comparison of the load current waveformsobtained by simulations of the original and equivalent networks.

An example of the network used in simulations is taken from [8] and it is shown in Fig. 4. Data of the network are given in Table 1. All transformer resistances and reactances in Table 1 are referred to the secondary side of transformers. The harmonic distortion is obtained synthetically by harmonic current sources (HFS 01 and HFS 02 in Fig. 4) with 5-th and 7-th harmonics. The network element that changes the operating state of the network is a capacitor bank (CapBank) connected in network node X0027. The measured voltage and current used for Thevenin equivalent estimation are at network node X0027 (voltage) and between nodes X0008– X0027 (current).

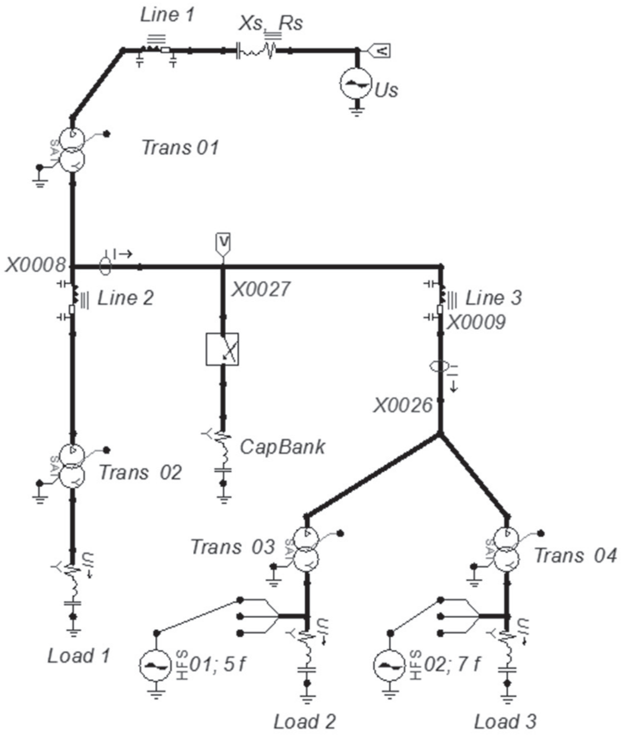

Fig. 4. Test network in ATP

Table 1. Test network data.

Network element	Value	Network element	Value
f [Hz]	50	*N – Trans 02*	10/0.4
*U*s [V]	35 000	*R* [Ω]– *Trans 02*	0,001
*R*s [Ω]	0.1	*X* [Ω] – *Trans 02*	0.00942
*X*s [Ω]	0.94	*N – Trans 03*	10/0.4
R [Ω/km] – Line 1	0.48	*R* [Ω]– *Trans 03*	0.001
X [Ω/km] – Line 1	0.862	*X* [Ω] – *Trans 03*	0.00942
C [μF/km] – *Line 1*	0.008	*N – Trans 04*	10/0.4
l [km] – *Line 1*	10	*R* [Ω]– *Trans 04*	0.001
R [Ω/km] – Line 2	0.83	*X* [Ω] – *Trans 04*	0.00942
X [Ω/km] – Line 2	0.86	*R* [Ω] – Load 1	0.5
C [μF/km] – *Line 2*	0.008	*X* [Ω] – Load 1	0.22
l [km] – *Line 2*	4	*R* [Ω] – Load 2	0.5
R [Ω/km] – Line 3	0.83	*X* [Ω] – Load 2	0.2512
X [Ω/km] – Line 3	0.86	*R* [Ω] – Load 3	0.5
C [μF/km] – *Line 3*	0.008	*X* [Ω] – Load 3	0.2569
l [km] – Line 3	5	C [μF] - CapBank	50
N – Trans 01	35/10	*I* [A] – HFS 01	100
R [Ω]– *Trans 01*	0.04	*f* [Hz] – HFS 01	250
X [Ω] – *Trans 01*	0.471	*I* [A] – HFS 02	50
		f [Hz] – HFS 02	350

The measured voltage and current used for Norton equivalent estimation are at network node X0027 (voltage) and between nodes X0009 – X0026 (current). The operating condition of the network is changed at time *t*=0.05 s by switching off the capacitor bank. The voltage and current waveforms for these two network states are shown in Fig. 5 and Fig. 6, respectively. The harmonic voltage and current spectra for these two network states for significant harmonics are given in Table 2. The waveshape of current flows between nodes X0009 and X0026 is shown in Fig. 7 for both network states. Current harmonic spectra for this current are given in Table 3.

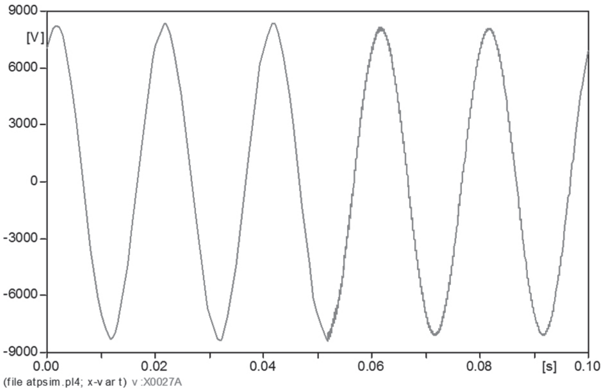

Fig. 5. Voltage at node X0027 before and after the capacitor bank is switched off (at *t*=0.05 s)

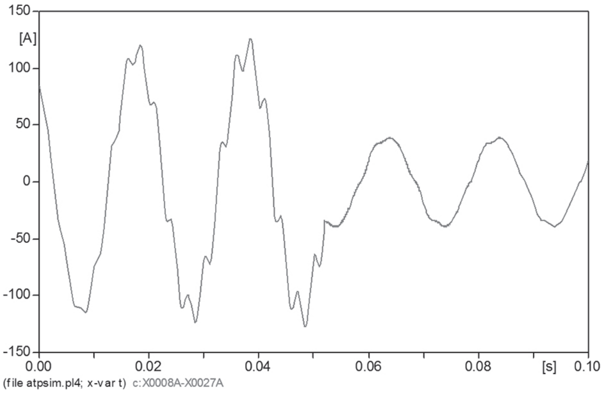

Fig. 6. Current flows from node X0008 to node X0027 before and after the capacitor bank is switched off (at *t*=0.05 s)

Table 2. Harmonic values for the estimation of the Thevenin equivalent

Network state	Harmonic	Voltage [V] at X0027	*β*[°] voltage angle	Current [A] X0008 -X0027	*α*[°] currentangle
Capacitor bank connected (index 1 in text)	1	5 806	59.27	80.32	131.9
	5	11.30	-106.5	2.371	-8.52
	7	54.25	166	5.849	-96.74
Capacitor bank disconnected (index 2 in text)	1	5 685	59.67	27.49	28.56
	5	9.72	-93.95	1.502	-0.733
	7	6.81	-92.57	0.751	-0.451

Parameters of Norton equivalents calculated from (5) are shown in Table 4. Estimated parameters of the Thevenin equivalent obtained by solving optimization problem (7) are shown in Table 5. Estimated parameters of the Norton equivalent calculated according to (5) taking into account referent angles from Table 5 (zero phase angle of the Thevenin equivalent) are given in Table 4. Current and voltage data in Tables 4 and 5 are effective values. The equivalent circuits of the original network from Fig. 4 based on Thevenin and Norton equivalents are shown in Fig. 8 for each harmonic. In Fig. 9, details of the original and estimated current waveforms are shown. Both operational states of the network, with a connected and a disconnected capacitor bank, are included.

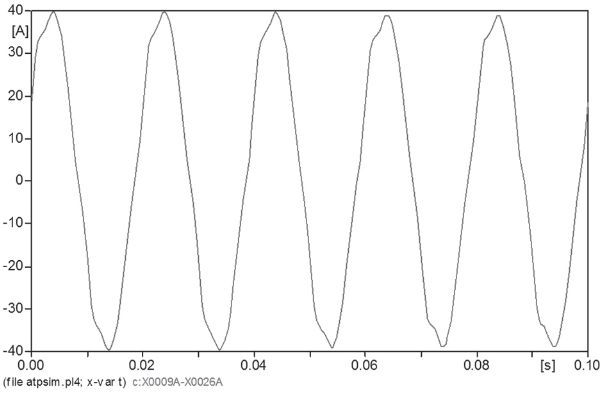

Fig. 7. Current flows from node X0009 to node X0026 before and after the capacitor bank is switched off (at *t*=0.05 s)

Table 3. Harmonic values for the estimation of the Norton equivalent

Network state	Harmonic	Voltage [V] at X0027	*β*[°] voltage angle	Current [A] X0008 -X0027	*α*[°] currentangle
Capacitor bank connected	1	5 806	59.27	28.11	28.03
	5	11.30	-106.5	1.492	-0.736
	7	54.25	166	0.7581	4.871
Capacitor bank disconnected	1	5 685	59.67	27.53	28.44
	5	9.72	-93.95	1.5	-0.723
	7	6.81	-92.57	0.7492	-0.436

Table 4. Estimated parameter values of Norton equivalents for all harmonics

Harmonic	R_N [Ω]	X_N [Ω]	$\underline{Z}_N$ [Ω]	$\underline{I}_N$[A]
1	177.15	109.73	208,38 <31.77°	0,36 <-165.47°
5	115.81	240.64	267.206 <64.30°	1,534 <119.84°
7	209.46	729.58	759.06 <73.98°	0.758 <119.77°

Table 5. Estimated parameter values of the Thevenin equivalent and phase angles of measured currents

E_T[V]	R_T [Ω]	X_T [Ω]	α_1[°]	α_2[°]
5707	0.143	1.33	72.20	-31.40

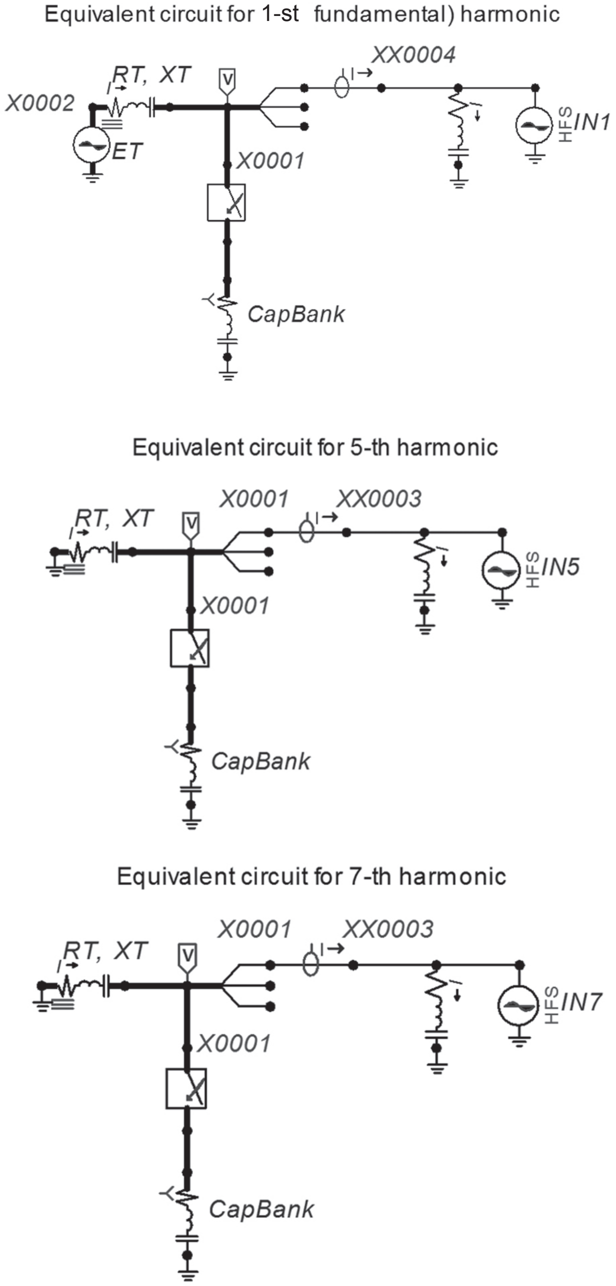

Fig. 8. Equivalent circuits based on Thevenin and Norton equivalents

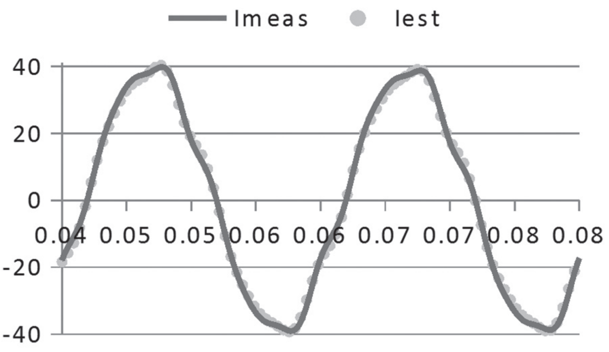

Fig. 9. Comparison of measured (*I*meas) and estimated (*I*est) current

As can be seen from Fig. 9, there are slight differences between the original current (current X0009-X0026 in Fig. 4) and the estimated corresponding current (current X0001-XX0004 in Fig. 8). The total distortion factors (THD) for these two currents are given in Table 6.

Table 6. Harmonic spectra and THD of the original (measured) and the estimated current

Harmonic	*I*meas [A] capacitor connected	*I*est [A] capacitor connected	*I*meas [A] capacitor disconnected	*I*est [A] capacitor disconnected
1	28.11	27.62	27.54	27.04
5	1.483	1.462	1.49	1.499
7	0.763	0.7466	0.7473	0.7489
THD [%]	5.935	5.944	6.056	6.197

6. CONCLUSION

The application of the Thevenin and the Norton equivalent for load harmonic modeling is presented in the paper. The most important procedure in the method application is to determine phase angles of measured voltages and currents with respect to the reference angle. In this paper, a new approach based on the application of the ES as a procedure for phase angle estimation is presented. Simulation results indicate that the proposed procedure is applicable to the Norton load harmonic approach. The use of evolutionary strategies or other population-based near global optimum optimization methods enables us to estimate parameters of the Thevenin and Norton equivalents without knowing impedance of the supply network.

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