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Izvorni znanstveni članak

NUMERICAL MODELLING AS NON-DESTRUCTIVE METHOD FOR THE ANALYSES AND DIAGNOSIS OF STONE STRUCTURES: MODELS AND POSSIBILITIES

Nataša ŠTAMBUK CVITANOVIĆ[1] & Slobodan ŠESTANOVIĆ[2]

[1] *Civil Engineering Institute of Croatia, Business Centre Split, Matice hrvatske 15, HR-21000 Split, Croatia*
[2] *Faculty of Civil Engineering, University of Split, Matice hrvatske 15, HR-21000 Split, Croatia*

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Assuming the necessity of analysis, diagnosis and preservation of existing valuable stone masonry structures and ancient monuments in today European urban cores, numerical modelling become an efficient tool for the structural behaviour investigation. It should be supported by experimentally found input data and taken as a part of general combined approach, particularly non-destructive techniques on the structure/model within it. For the structures or their detail which may require more complex analyses three numerical models based upon finite elements technique are suggested: (1) standard linear model; (2) linear model with contact (interface) elements; and (3) non-linear elasto-plastic and orthotropic model. The applicability of these models depend upon the accuracy of the approach or type of the problem, and will be presented on some characteristic samples.

Ključne riječi: Kamene zidane konstrukcije, Nedestruktivne metode, Numeričko modeliranje

Uz pretpostavku uvažavanja potrebe za analizom, utvrđivanjem zatečenog i projektiranog stanja, te očuvanjem postojećih vrijednih kamenih (i/ili) zidanih konstrukcija i povijesnih građevina u današnjim urbanim jezgrama europskih gradova, numeričko se modeliranje pojavljuje kao učinkovit suvremen »alat« za praćenje stanja konstrukcije. Pri tome bi ulazne podatke trebalo određivati eksperimentalnim putem, a samu numeričku analizu promatrati kao segment u ukupnom kombiniranom i interdisciplinarnom pristupu, posebice kao jednu od nedestruktivnih metoda ispitivanja na konstrukciji/modelu. Za one konstrukcije ili njihove dijelove koji mogu zahtijevati detaljniju analizu predlaže se uporaba triju numeričkih modela po tehnici konačnih elemenata: (1) standardni linearni model; (2) linearni model s kontaktnim (dodirnim) elementima; (3) nelinearni elasto-plastični i ortotropni model. Primjenjivost navedenih modela ovisna je o razini detaljnosti pristupa i tipu problema, te će biti prikazana kroz nekoliko karakterističnih primjera.

Introduction

The role of numerical modelling in structural analysis of historical construction

Assuming that we are interested in safety aspect of existing stone and masonry structures under actual or new (increasing) loading conditions and in preserving architectural and cultural heritage, we need to agree with the tasks imposed by present situation, i.e. to analyse, diagnose and preserve such structures. While some small structures will be reconstructed by simple civil engineering and stone-cutting works, other more significant structures or their details will require complex analyses.

Different from the ancient constructors which built on experience, at present time we use a number of investigation and experimental techniques, together with contemporary calculation and numerical procedures and methods. Here we need to emphasize that for masonry and stone structures the combined use of experimental and numerical techniques is required, as well as the dialogue between different professionals. Combined use of experimental and numerical techniques gives a considerable contribution to the whole study which is consist of *historical analysis*, *direct observation* (geometric survey, crack pattern investigation) support with *continuous monitoring* (installation of a number of static and dynamic sensors on the structure, in order to continuously record settlements, displacements, inclinations, opening of cracks, vibrations, etc.), *experimentation and instrumentation* (determining the static conditions/mechanical parameters using non-destructive or slightly-destructive tests), and finally the *mathematical/numerical model*. This combined approach than enables the assessment of the structural reliability (diagnosis phase) and time evolution of the structural behaviour (prediction phase), which finally lead to defining the possible restoration works (prognosis phase).

Inside the study, and generally, investigation techniques and tests can be described as: *destructive* tests on the sample (laboratory determination of the physical and mechanical characteristics); *slightly-destructive* tests on the structure (coring techniques, borehole video survey, flat jack test); and *non-destructive* testing techniques on the sample or on the structure. The latest mentioned non-destructive investigation on the structure can be recognized as direct (dynamic techniques as sonic measurements, sonic tomography, radar investigation, thermographic analysis, rebound test, magnetometric analysis) or indirect through the model of the structure. At this point – if model is numerical model – numerical modelling i.e. numerical analyses and techniques as FEM approach can be considered as a sort of the indirect non-destructive investigation technique on the structure, in order to give an additional information and insight into the structural behaviour identification.

Numerical analyses becomes an efficient »tool« connected to the knowledge derived from the physical observation, which often cannot be totally included in the constitutive laws of the materials. Therefore, the criticisms about the use of numerical analysis is basically due to the suspicion that the physical reality and the constitutive laws are not appropriately taken into account, but some recent works (Haiman & Žagar, 1997, IABSE, 1993; Loo & Yang, 1991; Lotfi & Shing, 1991; Lourenço, 1996; Naraine & Sinha, 1992) affirm the possibility of successful use of several numerical models with good experimental confirmation of the results. Moreover, due to the complexity of this structures itself and a large number of influence factors, the

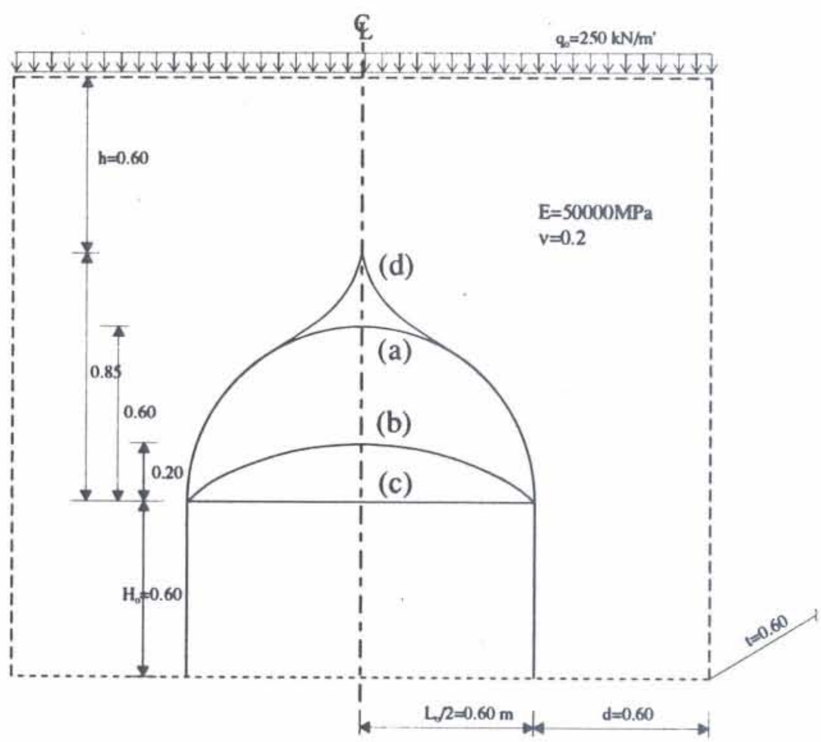

Fig. 1. Samples for wall opening analysis: type (a) semi-circular, (b) segment, (c) flat, (d) gotic

analytical solution is impossible and numerical methods are imposed as a reasonable practical approach.

Macro-modelling and micro-modelling

Reliable numerical models are necessary to assess and strengthen existing stone structures. This paper presents three numerical models based upon finite element techniques which can be used to perform numerical analyses of stone masonry structures (and other structures consist of units and joints). Each of them can be applied to a certain extent considering the accuracy of the approach and type of the problem (Štambuk, 1996). Consequently, stone structures can be generally analyzed according to the macro-modelling or micro-modelling principles.

In *macro-modelling* the stone structures are treated as a homogenous anisotropic continuum with average properties of stone units (blocks) and joints, both for in each element and for the entire finite element mesh which is used to discretize the structure. This approach, with the model of smeared cracks, makes it possible to reproduce successfully the effects of material non-linearity and progressive local failure, so that the effects of local failure on the blocks and/or joints are »smeared« over the finite element either fully or over its respective part. This technique is sufficient for the analysis of stone structures as large panels, globally, and it is applicable up to that level of accuracy when local effects become significant – e.g. in the region adjacent to a concentrated load where stress gradients become high or when details of the stress field and/or displacement should be solved. Summarily, macro-models are applicable when the structure is composed of solid walls with sufficiently large dimensions so that the stresses across or along a macro-length will be essentially uniform.

Higher level of accuracy of the approach to the analysis of stone structures can be obtained by *micro-modelling* where the stone blocks are modelled by the continuum elements, whereas the joints, as a source of weakening and non-linearity, are separately modelled by interface elements. Hence, interface elements are used as materially and geometrically non-linear element at the place of geometrical discontinuities or joints, while the remaining part is treated on the linear (or non-linear) analysis.

Clearly, macro-modelling is more practice oriented due to the reduced time and memory requirements as well as user-friendly mesh generation. This type of modelling is most valuable when a compromise between accuracy and efficiency is needed.

The application of the presented models will be illustrated by the analysis of several characteristic details of stone structures. All the examples were analyzed for the case of plane stress state.

Models for the stone structures analysis

The three mentioned FEM models for numerical analyses of stone structures are presented in this paper in a short form (only regarding the main features of the material behaviour modelling), and include the following: MODEL I – standard linear model; MODEL II – linear model with specific contact (interface) elements for the joints; and MODEL III – non-linear elasto-plastic and orthotropic model.

MODEL I – linear model

The application of the general or standard linear model with finite element method based on the displacement on stone structures is limited, i.e. it can be applied only within the limit of elastic behaviour, until the appearance of the first cracks. Nevertheless it can be applied in predicting the behaviour for the low levels of loading, which is a frequent case for the existing stone structures. It also represents the first step in the non-linear analysis which enable a quick insight into the problem and represents the »mental control« of the solution. Consequently, it is recommended that a linear analysis should be performed first. The reasons for such a suggestion can be additionally expressed by the following:

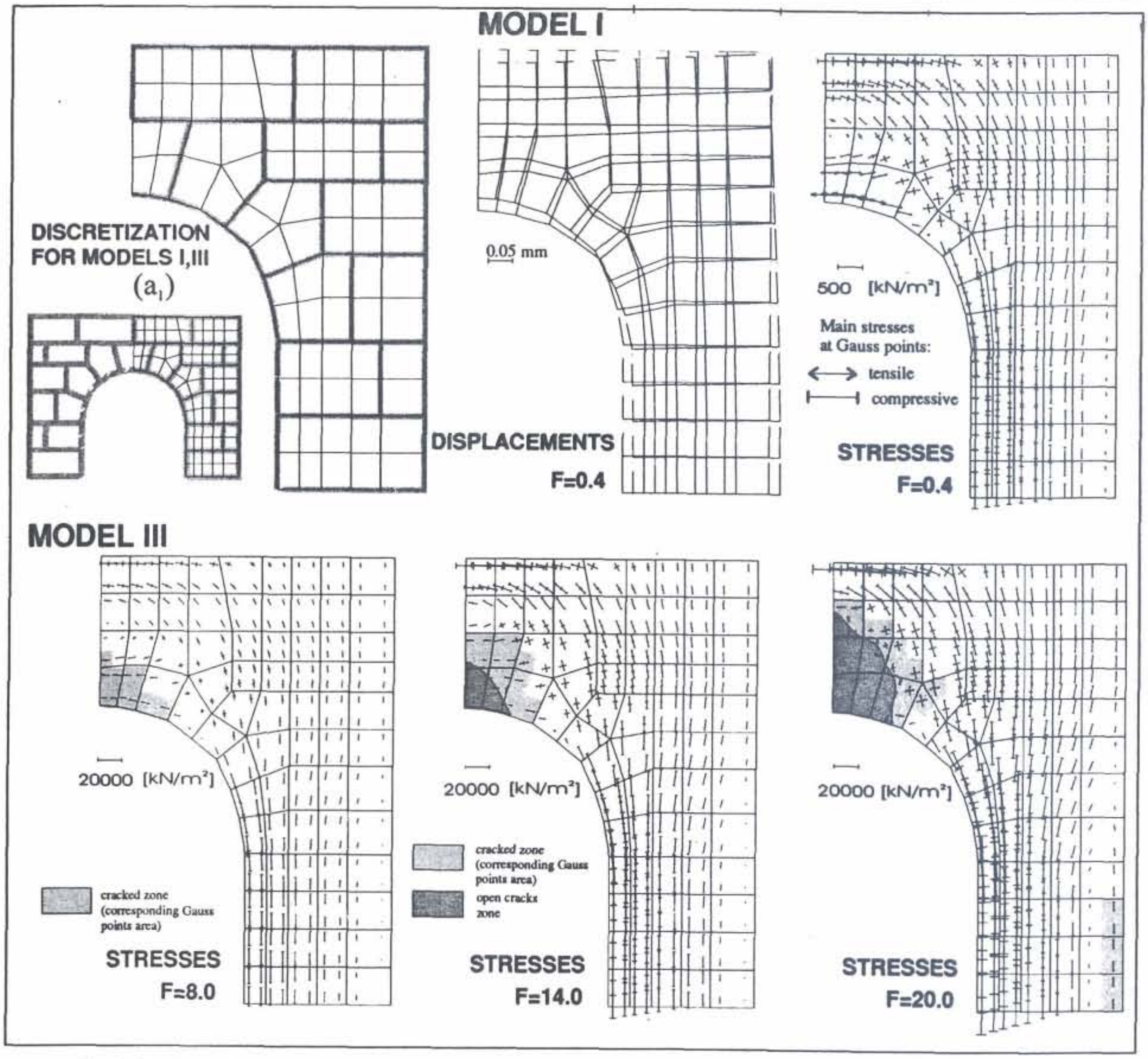

Fig. 2. Semi-circular opening (a_1) analysis

- improved **linear analysis** with the failure criterion or tensile strength built in may provide a reasonable description of the process leading to the crack pattern for which we are concerned,
- **nonlinear analysis** is relatively long and expensive; furthermore, the result should be checked with great care, and numerical instabilities may appear when cracking intervenes.

Subsequently, only the basic features are presented without considering the details of the linear model, which represent the standard of the numerical analysis and has been described in detail in literature (Hinton & Owen, 1977; Jović, 1993). Considering the limits of the model applicability, it should noted that linear behaviour (in 2-D models) for the stone is valid only within the von Mises initial yield surface in biaxial compression-compression stress state, i.e. within the tension cut-off failure surface in biaxial tension-tension and tension-compression. Beyond these limits the elastic linear model cannot be applied and elasto-plastic model becomes relevant (in biaxial compression-compression up to the failure surface), as well as orthotropic non-linear model for cracked stone (in all stress state beyond the failure surface defined by von Mises failure envelope and tensile cut-off).

Elastic behaviour is described by the incremental stress-strain relationship:

$$d\sigma = \mathbf{D} d\varepsilon \qquad (1)$$

where **D** is elastic material matrix; $d\sigma$ and $d\varepsilon$ incremental stress and strain vectors.

Thus, it is possible to define the material model, which together with the standard finite element procedure based on the displacement and frontal algorithm for solving the system of equations, after respective post-processing, form and entire model.

The application of the linear model or MODEL I is illustrated by the analyses of a different shaped opening in the wall, Figures 1, 2, 3, 4, 5, 6 where displacements and stress fields for elastic level (loading scaling factor F=0.4) are shown, together with the finite element mesh.

MODEL II – linear model with interface elements

This model deals with the stone blocks and their joints separately. The previously mentioned linear model can be applied to the continuum elements which form the blocks. The formulation of 1-D non-linear interface ele-

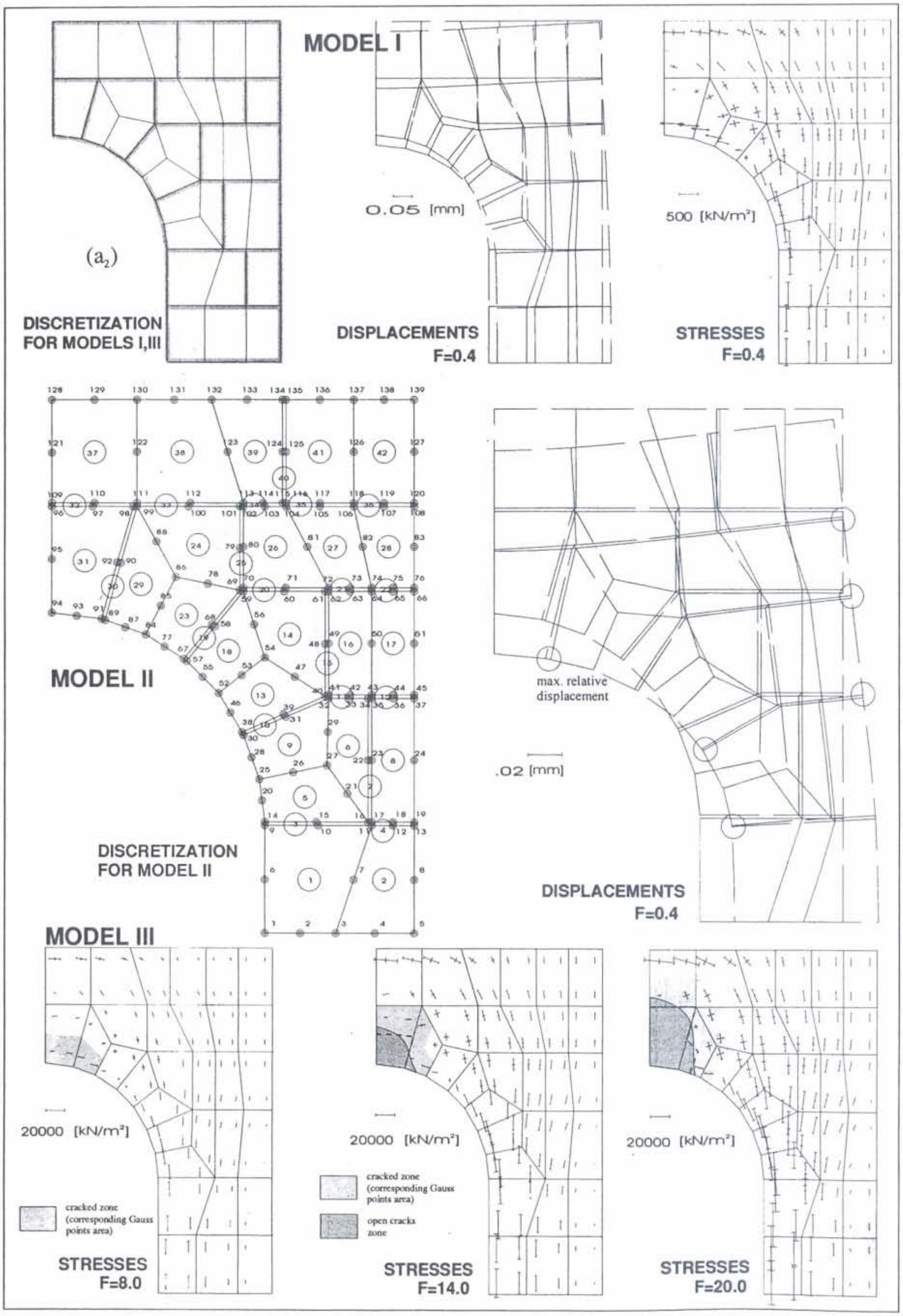

Fig. 3. Semi-circular opening (a₂) analysis

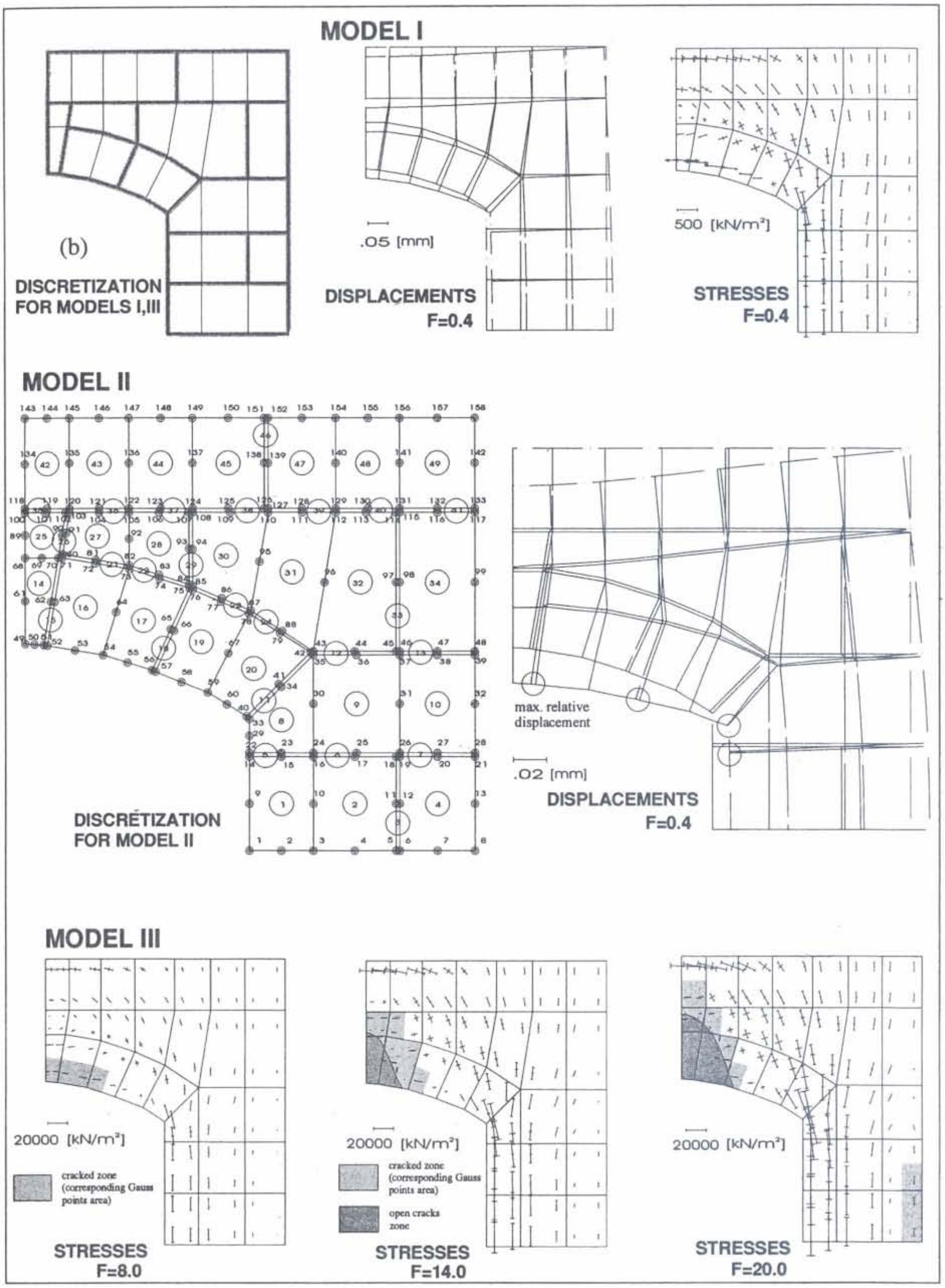

Fig. 4. Segment-shaped opening (b) analysis

ments was described in detail in Gotovac et al., (1992 a. & 1992 b). Different possibilities for interface behaviour are included into the model: contact without sliding, sliding, moving away (opening of crack).

Finite elements meshes on Figures 3,4 5, 6 present the principle for the model development. Each stone block consists of several continuum elements, while interface elements which model the joints as geometrical discontinuities, are placed along the block boundaries (determined

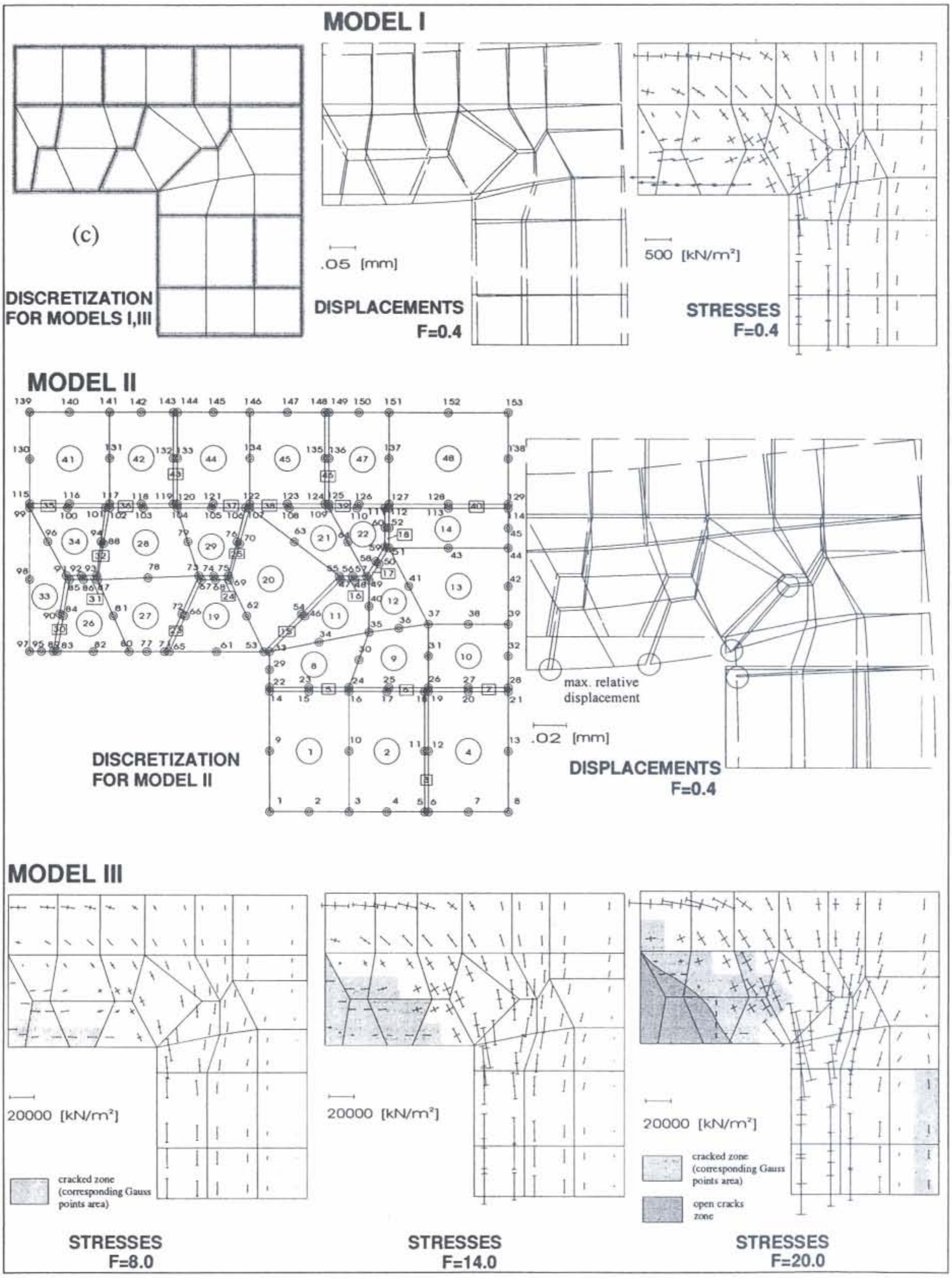

Fig. 5. Flat opening (c) analysis

by bold lines), so that relative displacements between the blocks are made possible. This can be noted from the presentation of displacements, Figures 3, 4, 5, 6. In all examples the characteristic vertical displacement (the top of the opening) is grater than in macro-modelling.

MODEL III – non-linear elasto-plastic and orthotropic model

The appearance of cracks due to tensile stresses in the system is one of the main causes of material non-linearity

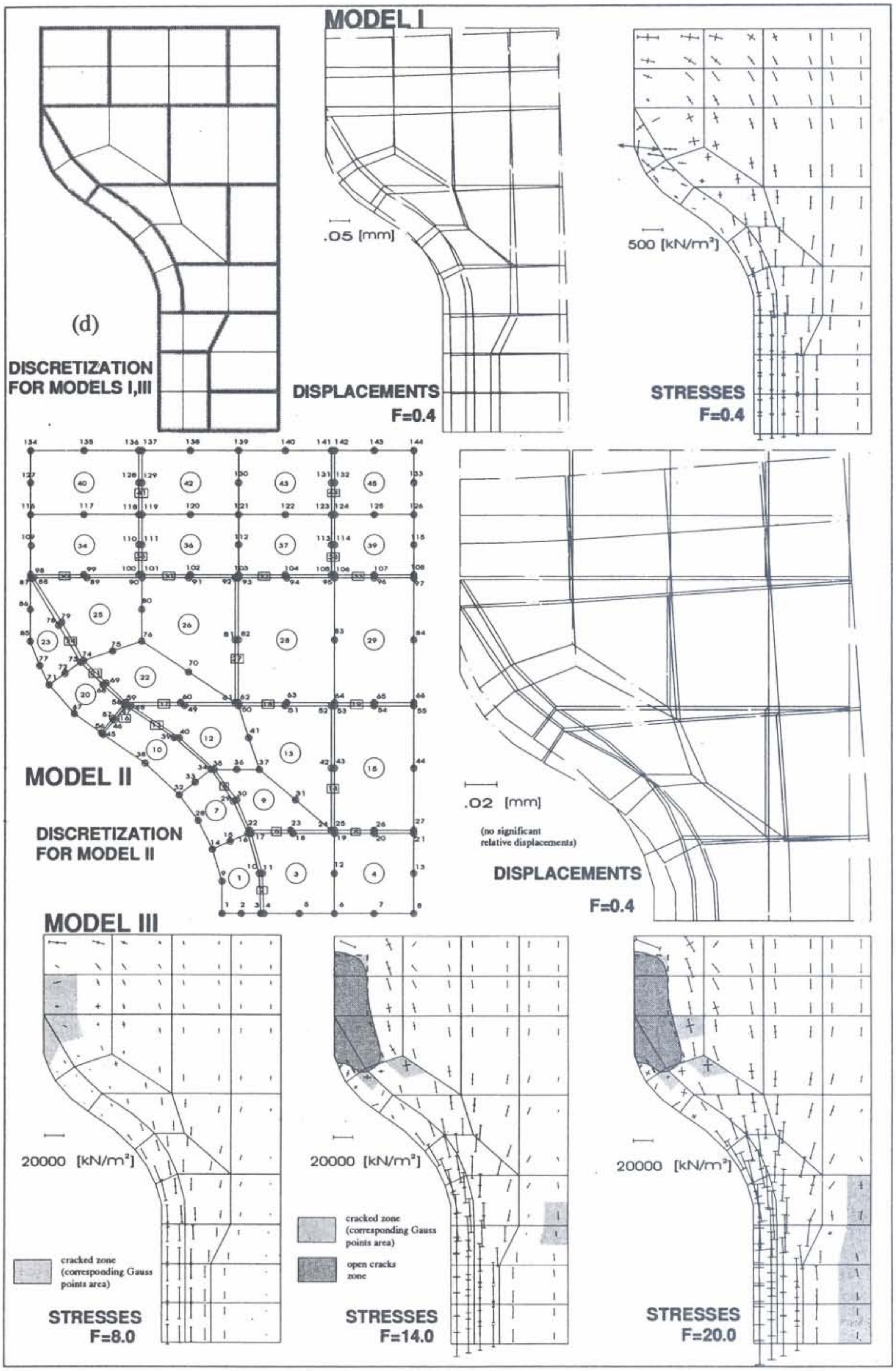

Fig. 6. Gotic opening (d) analysis

in stone structures. Consequently it is necessary to introduce into the model, in addition to the average non-linear behaviour (of the bond) unit-joints, also the appropriate treatment of the non-tension material and cracks behaviour, which enable the analysis of the local failure with the propagation of cracks under incremental loading. Thus, it is possible to respect the principle that accurate mathematical description of the mechanical behaviour of the material is a necessary condition for any calculations for the structures, and, at the same time, it is possible to satisfy the requirement that stone structures are studied at a level higher than the one possible with the linear model. In addition to the expansion of the cracking zones it is possible to solve the problems of the critical and yield loads, actual safety factor, the weakest points and yield zones, the failure mechanism – for specific levels of loads F in incremental approach.

With this objective in mind, the model for the non-linear analysis of stone structures was developed according to the principle of macro-modelling and smeared cracks, in accordance with similar models in the world (Lotfi & Shing, 1991; Lourenço, 1996) according to the basic elasto-plastic model. The model was presented with more details by Štambuk (1996).

Until the failure criterion (defined by von Mises parabola through the point of uniaxial compressive strength f_c' and tensile cut-off through uniaxial tensile strength f_t' in the system of principal stresses) standard elasto-plastic J_2 model is valid. The material matrix **D** from equation (1) is substituted by elasto-plastic $\mathbf{D}_{ep}$ which is of practical importance only in biaxial compression-compression. After reaching the failure criterion it is considered that cracks are formed both in tension-tension and tension-compression, and crushing occurs in compression-compression. The material becomes orthotopic, with orthotropy axes *n-t* normal and tangential with regard to the direction of the crack.

For a single crack, the incremental stress-strain relationship in the local n-t coordinate system can be expressed as:

$$d\sigma^{loc} = \mathbf{D}_c^{loc} d\varepsilon^{loc} \qquad (2)$$

in which $d\sigma^{loc} = \{d\sigma_n, d\sigma_t, d\tau_{nt}\}^T$ and $d\varepsilon^{loc} = \{d\varepsilon_n, d\varepsilon_t, d\gamma_{nt}\}^T$. The local tangent stiffness matrix $\mathbf{D}_c^{loc}$ of the cracked material is generally of the following form:

$$\mathbf{D}_c^{loc} = \begin{bmatrix} E_{nn} & O & O \\ O & E_{tt} & O \\ O & O & G_{nt} \end{bmatrix} \qquad (3)$$

where E_{nn}, E_{tt} and G_{nt} are the tangent moduli according to the correspondent 1-D diagrams, either tensile or compressive, and the respective principle strains, in the *n* and *t* directions. Poisson's effects in the cracking zone is neglected. Actual values depend upon the »zone« of principle stresses, i.e. the type of cracking and local failure (tension-tension, tension-compression, compression-compression).

The transformation into the global system follows after the determination of the local matrix:

$$\mathbf{D}_c = \mathbf{T}^T \mathbf{D}_c^{loc} \mathbf{T} \qquad (4)$$

where **T** is the transformation matrix dependent upon the cracks direction with regard to the global coordinate directions. Finally, the relationship stress-strain in the global system is:

$$d\sigma = \mathbf{D}_c d\varepsilon \qquad (5)$$

If the laws of softening are added, so that the cracked zones extend due to the redistribution of stresses, it is possible to develop a complete material model which should be implemented in the standard displacement-based incremental-iterative finite element formulation. Because of the softening behaviour, the conventional Newton-Raphson and modified Newton-Raphson iteration schemes are not feasible and, consequently, the displacements are evaluated with the initial stiffness method.

Some results for MODEL III, namely displacements and stresses with crack positions, for different load factors F, obtained by a specially adapted post-processor, are presented in Figures 2, 3, 4, 5, 6. The mesh-sensitivity, which is introduced into the model with smeared crack approach, is checked on the semi-circular opening sample through two separated discretizations (a_1) and (a_2), Figures 2, 3. It can be observed from obtained results that the »coarse« mesh (a_2) with a smaller number of finite elements is fine enough. That conclusion was used for further discretizations (b), (c), (d); respective Figures 4, 5, 6.

Conclusion

Since when the rehabilitation of existing stone and historic structures has become a big interest for the building industry, great effort was given to the study of the mechanical behaviour of masonry structures, and consequently, relevant numerical methods.

The application of the previously presented models depends upon the required level of accuracy of the approach and actual structure. There is no general solutions which could be applied to all stone structures. The linear model is sufficient only for a quick insight into the structure state for low levels of loading or as an approximate calculation and a first phase of non-linear analysis. However, more detailed approaches require the use of micro-modelling. General non-linear model will yield more data than a linear models at the macro-modelling level. Macro-models will be satisfactory when normal stresses are dominant, but they will not be sufficient if failure occurs due to **shear and in seismic analyses**.

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