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AN AGENT-BASED MODEL FOR OPTIMISATION OF ROAD WIDTH AND PUBLIC TRANSPORT FREQUENCY

ABSTRACT

An urban passenger transportation problem is studied. Municipal authorities and passengers are regarded as participants in the passenger transportation system. The municipal authorities have to optimise road width and public transport frequency. The road consists of a dedicated bus lane and lanes for passenger cars. The car travel time depends on the number of road lanes and passengers' choice of travel mode. The passengers' goal is to minimize total travel costs, including time value. The passengers try to find the optimal ratio between public transport and cars. The conflict between municipal authorities and the passengers is described as a game theoretic model. The existence of Nash equilibrium in the model is proved. The numerical example shows the influence of the value of time and intensity of passenger flow on the equilibrium road width and public transport frequency.

KEY WORDS

game theory; bus lane; travel mode choice; traffic congestion;

1. INTRODUCTION

The developing nations are facing a number of problems, of which the transport problem is especially acute. The traffic is getting heavier, but the infrastructure is not developing so fast. Not so long ago the greater part of the population had few chances to own and use cars because of low income.

Gradually the situation has changed. The increasing income of citizens enabled more people to own cars. A steady increase in car fleet has already resulted in the decline of public transport passenger flow [1]. Thus, public transportation is under pressure in this situation; round trip time increase, expenditure growth, passenger flow decline and outdated vehicles maintenance costs question the very existence of public transport.

The municipal authorities have to solve two costly problems, i.e. how to develop public transport and transport infrastructure. These problems have no simple solutions. Passengers' travel mode choice and route choice may lead to paradoxical results. The widely known Braess's paradox [2] states that adding extra capacity to a network when the moving entities selfishly choose their route, can in some cases reduce overall performance. The Downs-Thomson paradox [3] shows that increasing road capacity can make traffic congestion worse. Wide roads shift additional passengers into cars; therefore, public transport frequency will decline and traffic congestion will be worse than before.

Sometimes authorities make wrong decisions which only aggravate the situation. However, many cities try their hardest to improve the public transport service, e.g., by dedicating lanes and ways for buses [1].

The given paper is about mathematical modelling concerning how municipal authorities make their decisions in conditions of passengers' travel mode choice. It is the most complicated problem solved on the game theory basis; therefore, it singles out two participants.

The transport scientists' interest for game theory is increasing greatly [4]. Many papers considered competition among transport operators [4-7] or between the transport operator and the government [4]. Paper [5] introduced such participants as "potential passengers", "operators" and "public authority", but the statement of the problem was presented as a non-game model. The urban transportation system [6] was represented as a coalition-free game between a set of private companies (which change frequency) and a set of passenger flows (that make travel mode choice). The competition between operators with optimisation of bus capacity, fares and frequency was researched in [7].

The peculiarity of this paper is the construction of the conflict model between the authorities and passengers with the traffic congestion taken into account. In this paper the traffic congestion is modelled by the well-known Greenshields' equation [8, 9].

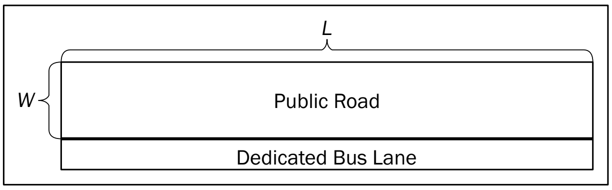

Figure 1 – Road width

The traffic model depends on the road structure. In the paper the road consists of two parts (*Figure 1*): dedicated bus lane (only for public transport) and for passenger cars. In this case public transport has priority and its travel time decreases. In [10] simulation models were developed in order to examine several scenarios of dedicated bus lanes and bus priority schemes so that the buses can provide the desired level of service with the minimal impact on the rest of the traffic.

Such road structure is spreading worldwide. For example, modern bus rapid transit system is constructed on busways [1]. However, the optimisation of public transport frequency in bus way was also considered in many papers i.e. [10-12]. In paper [11] a microeconomic model for the operation of a bus corridor was constructed that minimises total cost (users and operator) and has five decision variables: frequency, capacity of vehicles, station spacing, fare payment system and running speed. In [12] an extension of Jansson's model for a single period is developed analytically, including the effect of vehicle size on operating costs and the influence of crowding on the value of time.

In the present paper we are trying to optimise both the public transport frequency and the width of road for passenger cars.

2. STATEMENT OF THE PROBLEM FOR PEAK PERIOD

Car congestion depends on traffic which increases during peak period. Thus, the road capacity must be optimised for this period.

The scheme in *Figure 2* describes the transportation management system for peak period. Municipal authorities optimise public road width and public transport frequency (or interval) which depends on passengers' travel mode choice. The authorities' solution determines the travel time by car and by public transport. This information is required for passengers' decision-making. The information about passengers' travel mode choice is passed to the authorities.

The model of the transportation system consists of the following parameters:

- t_t – average travel time on public transport (not including waiting time);
- t_w – average waiting time for public transport;
- t_c – average travel time by car (reaching the parking from the place where the need for transportation occurred; leaving the parking; parking; reaching the destination). Time for transportation by car is not included as it depends on car velocity;
- v – average car velocity;
- c_f – fare on public transport;
- c_c – car travel costs ($c_c > c_f$);
- γ – average passengers' value of time;
- L – road length;
- p – probability that the travellers choose a car;
- λ – passenger flow intensity.

2.1 Passenger flow

Travel mode choice (TMC) theory describes passengers' decision-making which depends on a lot of parameters. For example, logit and probit distributions are usually used for TMC models [13]. The logit models describe decisions made by the passengers in accordance with survey data, but these models do not describe the objective functions. To construct the objective function in this research we developed further the model from [6].

The model described passengers' decision depending on their time value, which has the exponential distribution [6]. Passengers with low income (less than γ') prefer public transport, but those with high income (more than γ') use passenger cars. Such approach was used in [7, 14] where the uniform distribution of passengers' value of time is used. But the

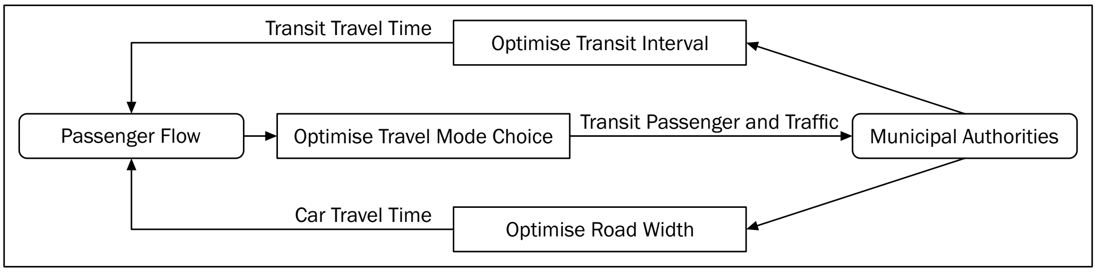

Figure 2 – Transportation management system during peak period

presented paper is based on exponential distribution with average γ.

The average value of time of the passengers who use public transport (mathematical expectation on condition that value of time is less than γ').

$$\frac{\int_0^{\gamma'} \frac{x}{\gamma}\exp\left\{-\frac{x}{\gamma}\right\}dx}{\int_0^{\gamma'} \frac{1}{\gamma}\exp\left\{-\frac{x}{\gamma}\right\}dx} = \frac{\left[\gamma - \gamma'\exp\left\{-\frac{\gamma'}{\gamma}\right\} - \gamma\exp\left\{-\frac{\gamma'}{\gamma}\right\}\right]}{1-\exp\left\{-\frac{\gamma'}{\gamma}\right\}}$$

The probability that travellers choose car p is related to γ' as follows: $\gamma' = -\gamma\ln(p)$, therefore, the average value of time for passengers who use public transport is

$$\frac{\left[\gamma + \gamma p\ln(p) - \gamma p\right]}{1-p}$$

There is a simple formula for total travel time by public transport $t_w + t_t$. Then time costs per travel by public transport are

$$\frac{\left[\gamma + \gamma p\ln(p) - \gamma p\right]}{1-p}(t_w + t_t) \tag{1}$$

Using similar manipulation the time costs per travel by car are obtained.

$$\left[\gamma - \gamma\ln(p)\right]\left(t_c + \frac{L}{v}\right) \tag{2}$$

The passenger's average total spending per travel is

$$G_0(t_w,p,v) = \left[\gamma + \gamma p\ln(p) - \gamma p\right](t_w + t_t) + c_f(1-p) + \\ + \left[\gamma p - \gamma p\ln(p)\right]\left(t_c + \frac{L}{v}\right) + c_c p \to \min. \tag{3}$$

Formula (1) includes full travel costs on public transport (time costs (1) and fare c_f), which are taken with probability $1-p$, and full travel costs by car (time costs (2) and car travel costs) which are taken with probability p.

The main property of function (3) is convex downwards on variable p. The proof of the property is based on the second derivative in (3). Also we must take into account that the car mode requires less time than public transport.

$$G_0''(t_w,p,v) = \frac{\gamma}{p}\left[t_w + t_t - t_c - \frac{L}{v}\right] \geq 0. \tag{4}$$

This conclusion ensures the existence and uniqueness of the solution to problem (3). The decision made by passengers depends on public transport interval and car velocity. The solution of (3) determines car traffic intensity λp and the number of passengers in public transport $\lambda(1-p)$.

2.2 Municipal authorities

Optimisation of road width and public transport frequency depend on the car traffic and the number of passengers in public transport, which, in turn, are determined by passengers' decision (3).

First of all, the road width (which is synonymous with capacity) must be optimised, as the road capacity influences velocity. The basic Greenshields' model [9] determines velocity as

$$v = v_0\left(1 - \frac{\rho}{\rho_j}\right), \tag{5}$$

where v_0 is free speed, ρ_j - jam density, i.e. the density when traffic is so heavy that it is at a complete standstill, ρ - average traffic density.

Traffic congestion may occur only in public roads (*Figure 1*). The average traffic density depends on traffic intensity and velocity as follows

$$\rho = \frac{\lambda p}{v}.$$

The jam density (number of cars per distance unit) depends on the road width w (*Figure 1*), lane width a and vehicle length b.

$$\rho_j = \frac{w}{ab}.$$

Thus Greenshields' formula (5) is rewritten as follows

$$v = v_0\left(1 - \frac{\lambda pab}{vw}\right). \tag{6}$$

Let us express road width w in terms of the function of variable v (i.e. velocity determines the road width)

$$w = \frac{\lambda pabv_0}{v(v_0 - v)}. \tag{7}$$

It is simple to prove the convex downwards of (5) on variable v. The second derivative is

$$2\lambda pab\left(\frac{1}{v^3} + \frac{1}{(v_0 - v)^3}\right) > 0.$$

Let us introduce additional parameters:

- c_r – spending on road expansion (width increase);
- d_t – city environmental damage and spending on public transportation (per trip);
- d_c – city environmental damage and spending on car (per trip).

The number of public transport trips depends on the interval. The dedicated lane ensures that public transport traffic is a deterministic flow. Thus, the interval is equal to $2t_w$ and the frequency is

$$\frac{1}{2t_w}.$$

The objective function of the municipal authorities is to reduce time losses for passengers (1) and curb spending on the infrastructure (7) and transport maintenance:

$$F_0(t_w,p,v) = \left[\gamma + \gamma p\ln(p) - \gamma p\right](t_w + t_t) + \\ + \left[\gamma p - \gamma p\ln(p)\right]\left(t_c + \frac{L}{v}\right) + \frac{c_r\lambda pabv_0}{v(v_0 - v)} + \\ + \frac{d_t}{2t_w} + d_c\lambda p \to \min. \tag{8}$$

The (6) includes the variables t_w and v in the different addends. Therefore, it is easy to prove convexity downwards on (8).

2.3 Statement of the problem

The decision of municipal authorities depends on passengers' decisions. Also passengers use the authorities' decision (public transport interval and car velocity) for their own decision-making. The interference between decision-making of passengers and that of the authorities leads to game theoretical formulation of the problem.

$$\begin{cases} G_0(t_w,p,v) \to \min\limits_{p} \\ F_0(t_w,p,v) \to \min\limits_{t_w,v} \end{cases}. \tag{9}$$

The Nash theorem [15] is used to prove the existence of equilibrium. Let's consider the basic conditions of the theorem.

First of all, the set of strategies is compact and convex: $p \in [0,1]$, $v \in [0,v_0]$ and $t_w \in [0,\overline{t_w}]$, where $2\overline{t_w}$ is maximal interval for public transport during peak period.

Secondly, the participants' payoff functions are convex upwards functions on their own strategies and continuous functions on each participant's strategies. These conditions were proved in sections 2.1 and 2.2 (but we used loss functions (payoff functions with the minus) which are convex downwards).

3. STATEMENT OF THE PROBLEM FOR OFF-PEAK PERIOD

Passenger flow varies considerably. During on-peak period it reaches its maximum, thus, road capacity must be increased for such periods. As for off-peak period, the municipal authorities have to manage only public transportation frequency during this time.

The transportation management system is shown in *Figure 3*. Passengers predetermine car traffic, but any changes in traffic have an impact on car travel time, which influences passengers' decision-making.

3.1 Passenger flow

It is better to use the car driving time as a variable:

$$t_d = \frac{L}{v}. \tag{10}$$

The objective function of passenger flow (1) is changed by substituting (11)

$$G(t_w,p,t_d) = [\gamma + \gamma p \ln(p) - \gamma p](t_w + t_t) + \\ + c_f(1-p) + [\gamma p - \gamma p \ln(p)](t_c + t_c) + c_c p \to \min. \tag{11}$$

Passengers try to cope with two tasks: choosing travel mode and reducing car travel time. But these parameters are linked by Greenshields formula (6). The solution of quadratic equation (6) determines the velocity

$$v = \frac{v_0 + \sqrt{v_0^2 - \frac{4\lambda pabv_0}{w}}}{2}. \tag{12}$$

The driving time is not less than

$$t_d \geq \frac{L}{v} = \frac{2L}{v_0 + \sqrt{v_0^2 - \frac{4\lambda pabv_0}{w}}}. \tag{13}$$

The equality (12) is attained by solution of (11). Also, the driving time (10, 12) is not greater than

$$t_d \leq \frac{2L}{v_0}. \tag{14}$$

The conditions (13, 14) describe the convex set, because the second derivative of (13) is

$$\frac{16\lambda^2 a^2 b^2 v_0^2 L}{w^2\left(v_0 + \sqrt{v_0^2 - \frac{4\lambda pabv_0}{w}}\right)^3 \left(v_0^2 - \frac{4\lambda pabv_0}{w}\right)} \cdot \\ \cdot\left(1 - \frac{v_0 + \sqrt{v_0^2 - \frac{4\lambda pabv_0}{w}}}{2\sqrt{v_0^2 - \frac{4\lambda pabv_0}{w}}}\right) > 0. \tag{15}$$

3.2 Municipal authorities

Municipal authorities have only one variable during off-peak period – public transport interval. Thus, (8) may be simplified by removing unessential summand

$$F(t_w,p) = [\gamma + \gamma p \ln(p) - \gamma p](t_w + t_t) + \frac{d_t}{2t_w} \to \min. \tag{16}$$

It is evident that (15) is convex downwards function on t_w.

3.3 Statement of the problem

The statement of the problem for off-peak period is based on the conflict between the passengers and the

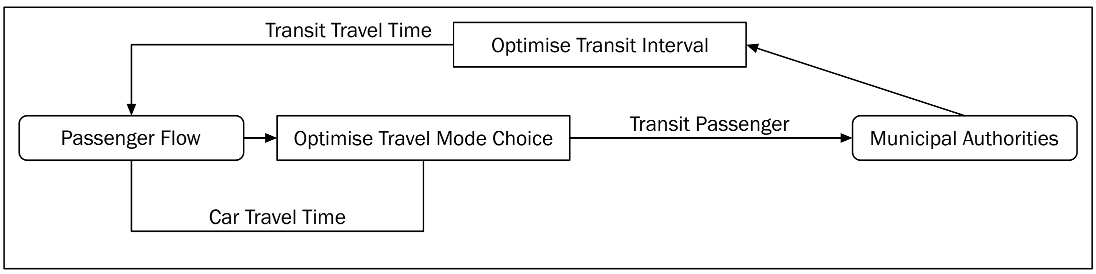

Figure 3 – Transportation management system during off-peak period

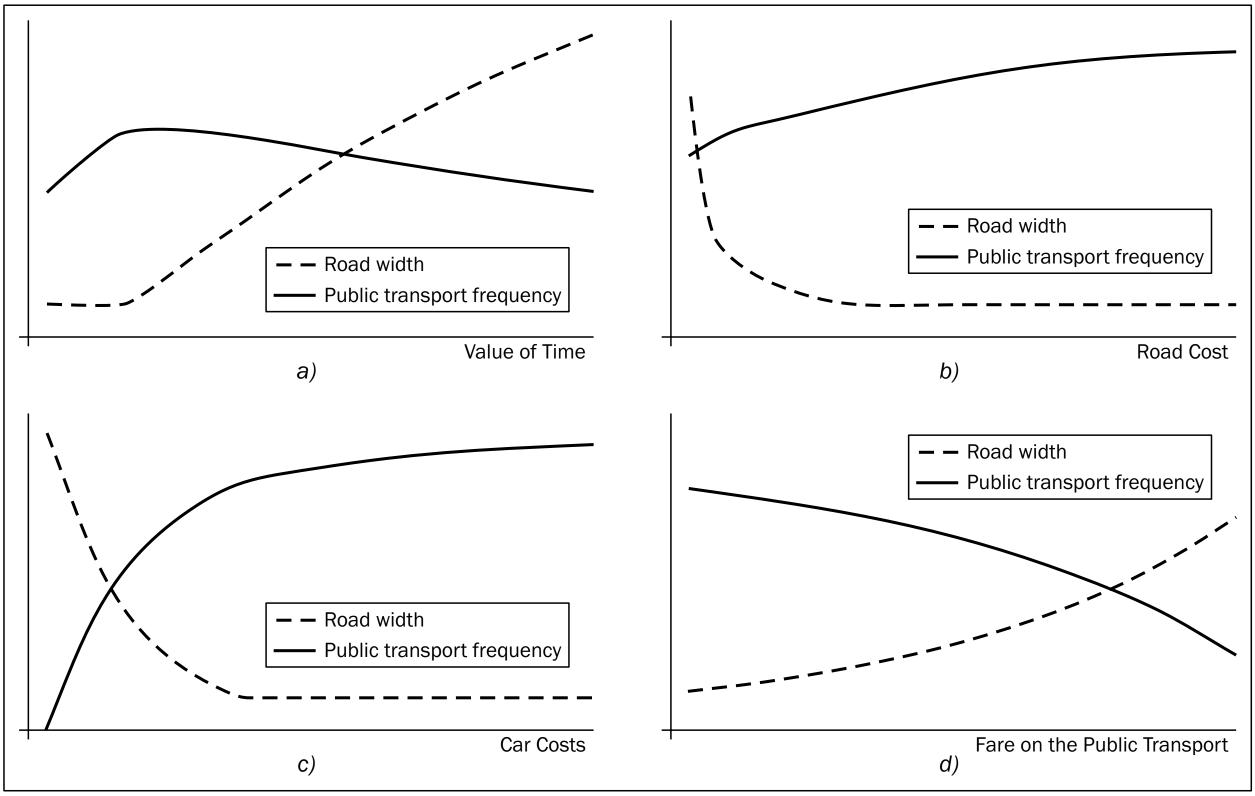

Figure 4 – Influence of the parameters of the models on the equilibrium frequency of public transport and road width concerning: a) value of time; b) road construction costs; c) car ownership costs; d) fare on public transport

authorities. The passengers have two variables while the municipal authorities have one.

$$\begin{cases} G(t_w, p, t_d) \to \min\limits_{p, t_d} \\ F(t_w, p) \to \min\limits_{t_w} \end{cases}. \tag{17}$$

The solution of (17) is Nash equilibrium. The conditions of Nash theorem are met. The proving is based on the results of the last sections (3.1 and 3.2).

4. NUMERICAL EVIDENCE

The specific data values are different for each nation. So the numerical evidence shows only the influence of the parameters on the solution. The solution for peak is considered in *Figure 4*. The variables of the municipal authorities are the road width for pasenger cars and frequency of public transportation (inverse value of waiting time).

*Figure 4*a shows that the increase in value of time leads to the increase in the road width. But the frequency is up for low income (when the possibility of car using emerges) and down for high income (passengers prefer cars). *Figure 4*b shows that the increase in road construction costs leads to the increase in frequency (the authorities stimulate passengers to use public transport). *Figure 4*c shows that the increase in

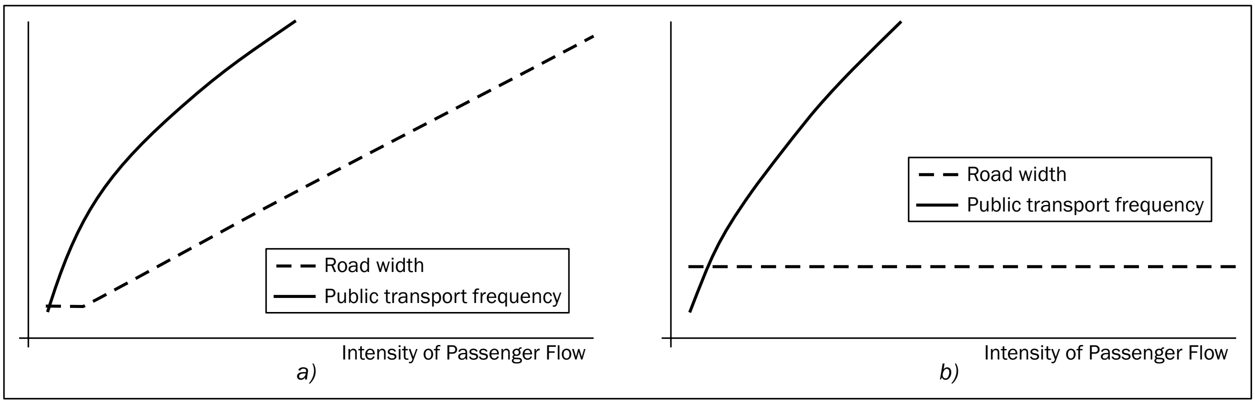

Figure 5 – Influence of the passenger flow intensity on the equilibrium frequency of public transport and road width for a) peak period b) off-peak period

car ownership costs leads to the increase in frequency and decrease in infrastructure expenditure. Obviously, increase in fares leads to degradation of public transport.

The off-peak and peak periods have the same parameters with the exception of the passenger flow intensity. Thus, *Figure 5* shows that the frequency increases during off-peak period more rapidly than during peak period, because the authorities may increase not only the frequency but the number of lanes, too.

5. CONCLUSION

Vigorous social and economic changes in developing countries lead to transportation problems resulting from the possibility of citizens to choose the travel mode. At present the level of motorization is growing; therefore, the choice is getting very sensitive to time value and travel time by each mode. Municipal authorities have to decrease travel time for the sake of the city sustainable development, therefore dedicated bus lanes should be used. This problem solution requires two parameters, road width and public transportation frequency.

The mutual influence of road width, public transportation frequency, travel time, congestion and travel mode choice leads to constricting complicated mathematical models. It must be noted that the general goal function for authorities (which takes into account passengers' decision-making) is not convex. Therefore, such an approach cannot be a true way to the solution of the problem for a real-sized city.

The present paper suggests a decomposition of the models. Two participants are used; they are the passengers and the municipal authorities. In this case the problem solution is the Nash equilibrium. The fundamental result of the paper is the proof of the existence of the Nash equilibrium for the presented models.

The models will be developed further for the solution of traffic problems in real-sized cities. The participants of such a model will be road segments, public transport lines and passengers flows. The existence of the Nash equilibrium allows constructing fast algorithms for solution of the problem of cities in the developing nations.

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АННОТАЦИЯ

АГЕНТНЫЙ ПОДХОД ПРИ ОПТИМИЗАЦИИ ШИРИНЫ ПРОЕЗЖЕЙ ЧАСТИ ДОРОГИ И ЧАСТОТЫ ДВИЖЕНИЯ ОБЩЕСТВЕННОГО ТРАНСПОРТА

Исследуется проблема управления городским пассажирским транспортом. Участниками транспортной системы являются муниципальные органы власти и пассажиры. Муниципальные органы власти должны оптимизировать ширину проезжей части дороги и частоту движения общественного транспорта. Проезжая часть состоит из двух частей: выделенная полоса для общественного транспорта и полосы для движения личного автотранспорта. Время передвижения на личном автотранспорте зависит от количества полос и выбора пассажирами способа передвижения. Цель пассажиров минимизировать стоимость передвижения включающую стоимость времени пассажира. Пассажиры находят оптимальное соотношение между общественным и личным транспортом. Конфликт между муниципальными органами власти и пассажирами описывается с помощью теоретико-игровой модели. Доказано существование равновесие Нэша для данной модели. Численный пример показывает влияние стоимости времени пассажира и интенсивности пассажиропотока на равновесные ширину проезжей части дороги и частоту движения общественного транспорта.

КЛЮЧЕВЫЕ СЛОВА

теория игр; выделенная полоса; выбор способа передвижения; автомобильные заторы;

REFERENCES

[1] Wright L, Hook W. Bus Rapid Transit Planning Guide. New York: Institute for Transportation and Development Policy; 2007. Available from: https://go.itdp.org/display/live/Bus+Rapid+Transit+Planning+Guide+in+English

[2] Braess D. Über ein Paradoxon aus der Verkehrsplanung. Unternehmensforschung. 1969;12:258-268. Available from: http://homepage.ruhr-uni-bochum.de/Dietrich.Braess/paradox.pdf

[3] Downs A. The Law of Peak-Hour Expressway Congestion. Traffic Quarterly. 1962;16:393-409.

[4] Hollander Y, Prashker JN. The Applicability of Non-Cooperative Game Theory in Transport Analysis. Transportation. 2006;33(5):481-496. Available from: http://link.springer.com/article/10.1007%2Fs11116-006-0009-1

[5] Evans AW. A Theoretical Comparison of Competition with Other Economic Regimes for Bus Services. Journal of Transport Economics and Policy. 1987;21:7-36.

[6] Koryagin ME. Competition of public transport flows. Automation and Remote Control. 2008:69(8):1380-1389. Available from: http://link.springer.com/article/10.1134%2FS0005117908080109

[7] Dodgson JS, Katsoulacos Y. Quality competition in bus services. Journal of Transport Economics and Policy. 1988;22:263-281.

[8] Kerner BS. Introduction to Modern Traffic Flow Theory and Control: The Long Road to Three-Phase Traffic Theory. Berlin: Springer; 2009. Available from: http://www.springer.com/engineering/mechanical+engineering/book/978-3-642-02604-1

[9] Greenshields BD. A Study of Traffic Capacity. Highway Research Board Proceedings. 1935;14:448-477.

[10] Papageorgiou G, Ioannou PA, Pitsillides A, Aphamis T, Maimaris A. Development and Evaluation of Bus Priority Scenarios Via Microscopic Simulation Models. Proceedings of the 12th IFAC Symposium on Control in Transportation; 2009 Sep 2-4; Redondo Beach, CA, USA. p. 434-441. Available from: http://www.ifac-papersonline.net/Detailed/40461.html

[11] Tirachini A, Hensher DA. Bus congestion, optimal infrastructure investment and the choice of a fare collection system in dedicated bus corridors. Transportation Research Part B: Methodological. 2011;45(5):828-844. Available from: http://dx.doi.org/10.1016/j.trb.2011.02.006

[12] Jara-Díaz S, Gschwender A. Towards a general microeconomic model for the operation of public transport. Transport Reviews. 2003;23(4):453-469. Available from: http://dx.doi.org/10.1080/0144164032000048922

[13] Horowitz JL, Koppelman FS, Lerman SR. A Self –Instructing Course in Disaggregate Mode Choice Modeling. Technology Sharing Program. Washington: U. S. Department of Transportation; 1986. Available from: http://www.transportation.northwestern.edu/docs/koppelman/publications/Self-InstructingCourseDisaggregateModeChoice.pdf

[14] Koryagin ME. Game theory approach to optimising of public transport traffic under conditions of travel mode choice by passengers. Transport Problems. 2014;9(3):117-124. Available from: http://transportproblems.polsl.pl/pl/Archiwum/2014/zeszyt3/2014t9z3_13.pdf

[15] Glicksberg IL. A Further Generalization of the Kakutani Fixed Point Theorem with Application to Nash Equilibrium. Proceedings of the American Mathematical Society. 1952;3(1):170-174.