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Izvorni znanstveni članak

PARAMETERS OF TRANSPORT OF NON-NEWTONIAN FLUIDS THROUGH THE PIPES

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The Institute for Mining,Geotechnology and Environment has developed a tube viscometer which is enabling estblishment of the rheological properties. This paper is describing how it is possible, by the help of the experimental results, to calculate the pressure drop of the concentrated suspension by transport through the pipes, being the basis for dimensioning the pipes.

Ključne riječi: Viskozitet suspenzija, Cijevni viskozimeter, Relogija, Transport cjevovodom

U Institutu za rudarstvo, geotehnologiju i okolinu (IRGO) razvili smo vlastiti cijevni viskozimetar pomoću kojega je moguće odrediti reološka svojstva suspenzija. Prikazano je na koji je način moguće pomoću eksperimentalnih rezultata izračunati pad tlaka pri transportu gustih suspenzija kroz cjevovod što služi za osnovu dimenzioniranja cjevovodova.

Introduction

In the mechanical process engineering as well as in the mineral processing the flow of suspension through equipment, tubes etc. is playing an essential role. In mining the transport of tailings has become of special importance in connection with mine backfill and lately also with disposal of problematic waste (slag, ash, flotation tailings, gypsum from process of scrubbing sulfur-laden flue gasses etc.) wich has proved of great importance in environmental protection.

The theoretical principles, wich are serving us as basis for our calculation of transport through the tubes, are gained in our laboratory through rheological examination (viscometry) since the feasibility of transporting a mixture of coarse and fine particles in water seems almost solely dependent on the properties of the suspension of fine particles only (Bothe & Meier, 1987).

At IRGO (Institute for Mining, Geotechnology and Environment) we have developed our own tube- or extrusion rheometer which is enabling us to establish the rheological properties of suspensions on the basis of which we can dimension the tubes for transportation of concentrated suspensions in the indrustrial mining. (Eberl & Eberl, 1995).

Non - Newtonian fluids

The properties of non-Newtonian fluids cannot be described with the Newton's law of viscosity as the viscosity of these fluids proves to be dependent on the rate of shear. Moreover, the viscosity of these fluids can increase or decrease due to the changes of the rate of shear which, again, is subject to the nature of the fluid.

The non-Newtonian fluids are according to their properties, established in the steady flow, divided into the fluids the properties of which are time-dependent and into the fluids the properties of which are time-independent.

In our case we take interest in the time-independent fluids only in which the shear stress proves to be constant at any shear rate all the time. The properties of such fluids depend only on the magnitude of the imposed shear stress and not on the duration of the stress.

The group of non-Newtonian time-independent fluids includes Fig. 1.

Pseudoplastic (shear thinning) fluids encompassing most non-Newtonian fluids. In these fluids the shear stress is increasing proportionally and the viscosity decreasing by growth of the shear rate (gradient of speed). This group practically includes all dispersion systems (e.g. suspensions of particles in water).

In dilàtant (shear thickening) fluids, however, the shear stress is increasing superproportionally and so does the viscosity with growth of the shear rate.

Plastic fluids start to yield only at a certain characteristic minimum yield stress τ_o. If the shear stress $\tau > \tau_o$ linearly increases in dependence on the shear rate we are speaking of the Bingham fluid.

While the dilute and more concentrated suspensions of mineral particles in water behave like pseudoplastic (shear thinning) non-Newtonian fluids, the highly concentrated suspensions (paste) exhibit nearly such properties as the Bingham fluids do.

The theoretical principles of rheological examinations are serving us a basis for calculations of transports of concentrated and highly concentrated suspension (paste) within the laminar flow. For this purpose we need a model conveying the ratio between the shear stress and shear rate. Pastes, consisting of mineral particles and water, are from rheological point of view rated among the pseudoplastic (shear thinning) fluids for which the Bingham's law is applied enabling us to define properties of the concentrated suspensions with a good approximation, viz. (Barnes et al., 1989):

$$\tau = \tau_o + \eta_p \ \mathrm{dv/dr} = \tau_o + \eta_p \dot{\gamma} \qquad (1)$$

where:

τ = shear stress in N/m^2,
τ_o = yield limit or minimum yield stress in N/m^2,
η_p = plastic viscosity in Pa.s,
dv/dr = $\dot{\gamma}$ - shear rate (gradient of speed) in s^{-1}

In the Bingham model the plastic viscosity is represented by the ratio between the shear stress above minimum yield stress and shear rate. The apparent viscosity, however, is the ratio between the shear stress and shear rate if it depends on the rate of shear; it is also called shear viscosity. Since the highly concentrated suspensions, when flowing, do not behave exactly according to the Bingham's law, we would define the viscosity of these fluids as an apparent one.

Buckinham-Reiner equation of volume flow rate of concentrated suspensions through the tubes is as follows (B o t h e & M e i e r, 1987):

$$Q = \frac{\pi D^4 \Delta p}{128 L \eta_p} - \frac{\tau_o \pi D^3}{24 \eta_p} \qquad (2)$$

where:
Q = volume flow rate in m^3/s
D = tube diameter in m,
Δp = required pressure in Pa,
L = tube length in m.

Both authors have for the required pressure formed the following equation:

$$\Delta p = \frac{32 L \bar{v} \eta_p}{D^2} - \frac{16 \tau_o L}{3D} \qquad (3)$$

where:
$\bar{v}$ = average flow rate im m/s.

In the equations (2) and (3) the first term stands for the Newtonian fluids.

Since the flow behaviour of the concentrated suspensions is not or is only partially conforming to the Bingham model we will use another method to calculate parameters for transport of these suspensions through the tubes in laminar conditions.

The maximum shear stress τ_s in the tube is (S u l t a n, 1988; G e r t h, 1981):

$$\tau_s = \frac{D \Delta p}{4L} \quad (Pa) \qquad (4)$$

This relation is valid for all fluids, the Newtonian as well as the non-Newtonian ones.

The shear rate for the Newtonian fluids at the wall is equal to:

$$\dot{\gamma}_a = \left(-\frac{dv}{dr}\right)_s = \frac{8\bar{v}}{D} \quad (s^{-1}) \qquad (5)$$

and for the non-Newtonian ones according to Rabinovich

$$\dot{\gamma}_s = \frac{3n'+1}{4n'} \cdot \dot{\gamma}_a = \frac{3n'+1}{4n'} \cdot \frac{8\bar{v}}{D} \qquad (6)$$

where:

n' = non-Newtonian rheological constant in the flow or shear diagram, which has for ordinate the value
$\tau_s = D\Delta p/4L$ and for abscisse $\dot{\gamma}_s = 8\bar{v}/D$.

$$n' = \frac{d \cdot \ln \tau_s}{d \cdot \ln \dot{\gamma}_s} \qquad (7)$$

The data needed to plot a flow diagram are obtained by means of the tube viscometer (E b e r l & E b e r l, 1995).

If the vallues τ_s, displayed in the diagram, are divided by $\dot{\gamma}$ we get the Poiseuille's equation:

$$\eta = \frac{D\Delta p}{4L} : \frac{8\bar{v}}{D} \qquad (8)$$

η = is apparent viscosity (in the non-Newtonian fluids) in Pa.s.

By means of the flow diagram we can ascertain the tube diameter at a determined pressure drop or vice versa.

The pressure drop is calculated from:

$$\Delta p = \frac{32 \eta L \bar{v}}{D^2} \qquad (9)$$

Rheological properties of suspensions and calculation of transport of concentrated suspensions

Knowing the rheological properties of concentrated suspension we can dimension the pumping facility in terms of the required pressure, diameter and length of tubes as well as transporting rate or we can adapt the rheologic properties of materials to some requested transport concept.

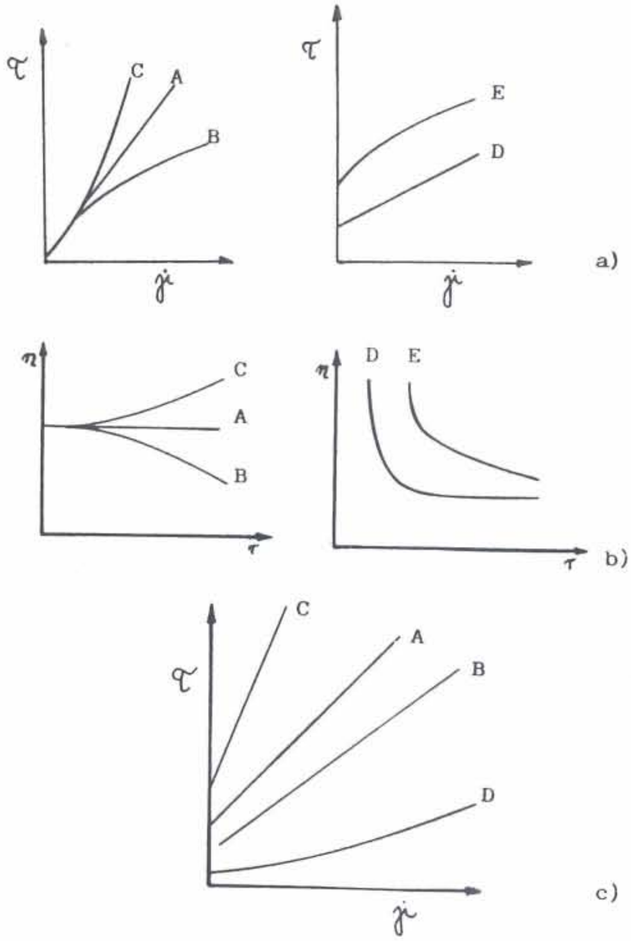
It is obvious that besides the laboratory examinations of rheometry it is also necessary to determine the size distribution of material, its density, mineral properties, sedimentation in suspension as well as other features of material like hardening and binding strength, elutriation, and pH value.

From the industrial point of view it is necessary to check the flow properties of suspension as the data, obtained through lab-examinations, are valid for suspensions with fine granules (<250 μm), while we also have to determine the influence, exerted by the coarse particles in suspension.

In this paper we are focusing solely on dimensioning the tubes on the basis of hydrological researches of concentrated suspension of fine granules.

A fluid can be by means of shear - or flow diagram rated with respect to its rheological properties. The data on the slope of the straight-line in the flow diagram with arithmetic (Fig. 1a) or log-log plot of coordinate axes (Fig. 1c) and march of the viscosity curve resp. (Fig. 1b) are characterizing the flow type.

In the fluid diagram with arithmetic plot all curves, with the exception of the curve representing the Bingham fluid, have got their origin in the coordinate



system. While the Newtonian fluid is representing a straight line and the Bingham fluid almost a straight line the others are developing curves. In the diagram with log-log plot all fluids are represented by straight lines except for the Bingham fluid, which is displayed by a curve.

The shear rate of the Newtonian fluid in laminar flow at the tube wall is $8\bar{v}/D$, while in the non-Newtonian fluid the shear rate in the laminar flow is a function of the value $8\bar{v}/D$, so that we may write down:

$$\frac{D\Delta p}{4L} = \phi\cdot\frac{8\bar{v}}{D} \tag{10}$$

The plotted values, obtained by means of the viscometer are,- in the diagram $D\Delta p/4L$ to $8\bar{v}/D$, for the time-independent non-Newtonian fluids, -entered into the same curve regardless of the diameter or length of the viscometer tube and flow rate. In order to be able to plot this curve we have to know density of the fluid, pressure, bearing on it, and its mass.

Fig. 1. Flow diagrams of various fluids:
A-Newtonian, B-pseudoplastic, C dilatant, D-Bingham, E-plastic;
a), b) arithmetic plot of coordinate axes;
c) log-log plot of coordinate axes,
($\dot{\gamma}$ = shear rate, τ = shear stress, η = apparent viscosity)

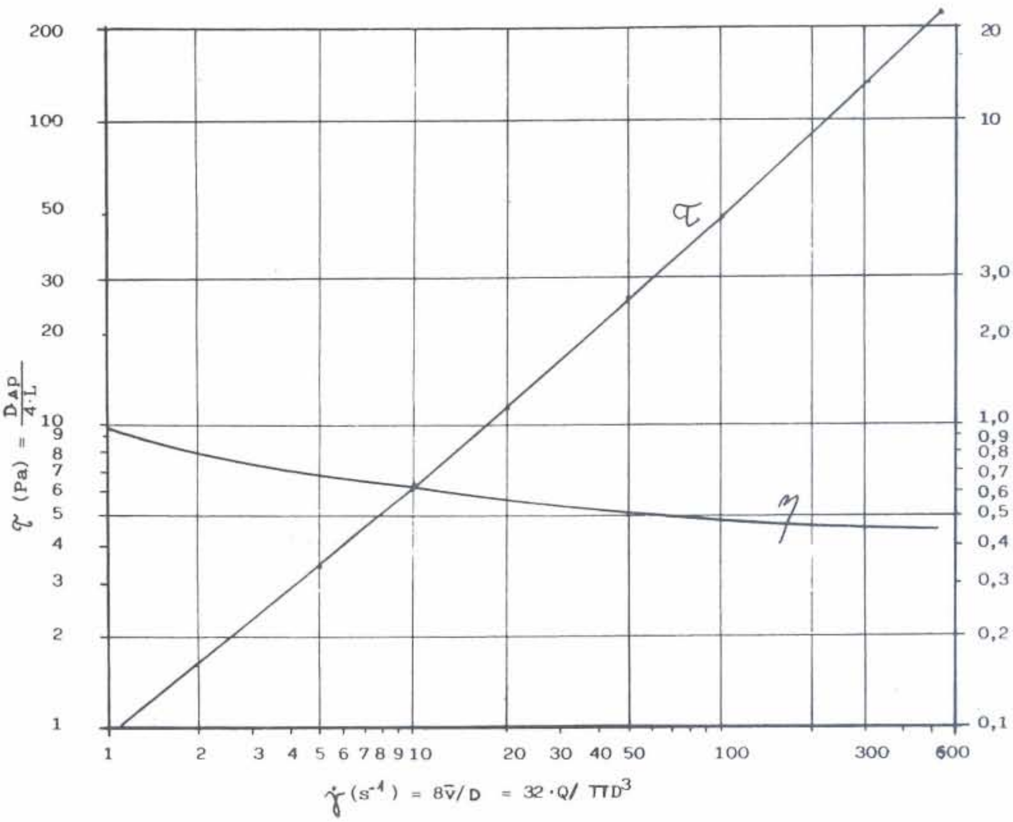


Fig. 2. Flow diagram:
$\tau = D\Delta p/4L$ (Pa) - shear stress, $\dot{\gamma} = dv/dr = 8v/D = 32Q/\pi D^3$ (s^{-1}) - shear rate, η_a - apparent viscosity (Pa.s)

Practical example

We would like to calculate the pressure drop for transport of concentrated suspension (57.5 vol%) of filter ashes quantity $Q=30\ m^3/h\ (8.3x10^{-3}\ m^3/s)$ through a tube with inside diamter $D_i=140$ mm.

We performed the measurements in our lab with tube viscometer, having tube diameters D= 9.0 and 6.0 mm and - lengths L = 900 and 600 mm at various values of pressure over the free suspension surface. On the basis of the obtained results we first calculated the values DΔp/4L and 8v/D and then plotted the diagram (Fig. 2).

It can be seen from the diagram (Fig. 2), showing log-log plots of values γ and τ on the abscisse and ordinate, that the represented value τ forms almost a straight-line which brings us to a conclusion that this suspension does happen to be totally pseudoplastic nor Bingham, yet it comes closer to the pseudoplastic than to the Bingham fluid also τ_0 scarcely occurs in the latter.

One group of data, obtained by experiment with tube viscometer, tube, dia D = 9.0 mm, and lenght L=0.9 m, was $\Delta p=$ 0.9 bar, $\rho=$ 1575 kg/m^3, fluid discharge time t = 23.3 s and catched pulp mass m = 1.373 kg.

$$\frac{D\Delta p}{4L}=\frac{0{,}009\cdot 9\cdot 10^4}{4\cdot 0{,}9}=225{,}0 \quad [\mathrm{Pa}]$$

$$Q=\frac{m}{\rho t}=\frac{1{,}373}{1575\cdot 23{,}3}=3{,}7\cdot 10^{-5}\ \mathrm{m^3/s}$$

$$\frac{8\bar{v}}{D}=\frac{32\ Q}{\pi D^3}=\frac{32\cdot 3{,}7}{10^5\cdot\pi\cdot 0.009^3}=517.0\ \mathrm{s^{-1}}$$

In the same way we obtained values DΔp/4L and $8\bar{v}$/D also at other, lower pressures with tube 9.0 and tube 6.0 mm. These values are displayed in the flow diagram (Fig. 2).

All calculated values, obtained with tubes 9.0 and 6.0 mm, are in compliance with (correspond to) one and the same curve which means that the examined non-Newtonian fluid is time-independent.

The apparent viscosity is according to the equation (8):

$$\eta=\frac{D\Delta p/4L}{8\bar{v}/D}=\frac{225}{517}=0.435\ \mathrm{Pa\cdot s}$$

The apparent viscosity in the same way also caluculated at other shear stresses and displayed in the diagram (Fig. 2).

We now carry out the calculation of the function 8v/D for the tube dia. D_i = 140 mm at flow Q = 8.3 x $10^{-3}m^3/s$. At this value of the function we take a reading from the diagram (Fig. 2) of the apparent viscosity at the flow Q through a given tube.

$$v=\frac{Q}{A}=\frac{Q\ 4}{\pi D_i^2}=\frac{8.3\cdot 4}{10^3\pi\cdot 0.14^2}=0.539\ \mathrm{m/s}$$

$$\frac{8\bar{v}}{D_i}=\frac{8\cdot 0.539}{0.14}=30.81\ \mathrm{s}\ \text{and}\ \eta=0.57$$

The calculation of the pressure drop (equation 9):

$$\Delta p=\frac{32\eta L\bar{v}}{D^2}=\frac{32\cdot 0.57\cdot 1\cdot 0.539}{0.14^2}$$

$$=501.6\ \mathrm{Pa/m}=0.502\ \mathrm{bar/100m}$$

From the Fanning equation we can deduce for pressure drop:

$$\Delta p=\frac{\lambda L\bar{v}^2\rho}{2D} \tag{11}$$

where λ represents friction factor (non-dimensional) for which there is valid for the laminar flow:

$$\lambda=\frac{64}{Re} \tag{12}$$

Re-Reynolds' number, the practical limiting value of which is at the transition from the laminar into the turbulent flow for all Newtonian and non-Newtonian fluids approx. 2100.

$$Re=\frac{\rho D\bar{v}}{\eta} \tag{13}$$

We are able to deduce from this equation for our example:

$$Re=\frac{1575\cdot 0.14\cdot 0.539}{0.57}=208$$

$$\lambda=\frac{64}{208}=0.308$$

$$\Delta p=\frac{0.308\cdot 1.0\cdot 0.539^2\cdot 1575}{2\cdot 0.14}=503.3\ \mathrm{Pa/m}$$

being virtually an equal result to the previously obtained one.

Metzner and Reed carried out an operation for calculation of the above relations in independence of rheological properties of the time-independent fluids (S u l t a n, 1988). This operation is based on a modified friction factor and Reynolds' number for the Newtonian fluids.

The modified Reynolds' number is:

$$Re_m=\frac{\rho\ D^{n'}\ \bar{v}^{(2-n')}}{K'\ 8^{(n'-1)}} \tag{14}$$

where:

K' = fluid consistency index and

n' = non-Newtonian rheological constant

which we can determine from the flow diagram DΔp/4L - $8\bar{v}$/D (Fig. 2) the abscisse and ordinate of which have got a log-log plot, n' is representing the slope of the line and tangent resp. in the curve of this diagram and K' the value DΔp/4L on the ordinate at 8v/D=1.

We can deduce from the equations (11), (12) and (14) the relation:

$$\Delta p = \frac{32K' \cdot 8^{(n'-1)} \cdot L\bar{v}^{n'}}{D^{(n'+1)}} \qquad (15)$$

Since the representation of the ratio $\tau/\dot{\gamma}$ in the flow diagram (Fig. 2) does not constitutes a straight line we have obtained the slope n' with a tangent to this curve (n'=0.863), with K at $\dot{\gamma}=1,0$ (K=0.914) resulting into:

$$Re_m = \frac{0.14^{0.863} \cdot 0.539^{(2-0.863)} \cdot 1575}{0.914 \cdot 8^{(0.863-1)}} = 207$$

and

$$\Delta p = \frac{32 \cdot 0.914 \;\; 8^{(0.863-1)} \cdot 0.539^{0.863}}{0.914 \cdot 8^{(0.863+1)}} = 502$$

Conclusion

In the mechanical process engineering as well as mineral processing the flow of suspensions through equipment and tubes is playing an essential role. The theoretical principles, serving as basis for calculation of transport, are obtained in laboratory by means of rheological examinations (viscometry). At the Institute of Mining, Geotechnology and Environment (IRGO) we have developed our own tube viscometer by the help of which we are able to determine the rheological properties of suspensions. After the rheological parameters of concentrated suspensions have been established we are able to dimension the pumping facility in the terms of required pressure, diameter and length of tubes or, the other way round, we cān adapt the rheological properties of a fluid to some requested concept of transport.

A fluid may be, with respect to its rheological properties, by the help of the shear-and flow diagram resp. referred to a certain rheological classification.

In the laboratory we carried out experiments with tube viscometer of two different tube diameters by which we determined the rheological properties of concentrated suspension of filter ashes in water.

The results are displayed in the diagram DΔp/4L to 8v/D. Since the values have, regardles of the diameter of the viscometer tube, fallen into the same curve, we find that the suspension is a time-independent non-Newtonian suspension. However, as it can be concluded from the form of the curve, the suspension is not totally pseudoplastic nor Bingham yet it is coming closer to the pseudoplastic fluid.

We have presented a practical example of the calculation of pressure drop in the tubes (pipes) for transport of concentrated suspension on the basis of the results of experiments made with the concentrated suspension, serving as basis for dimensioning of the tubes.

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