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INTEGRAL EQUATION APPROACH FOR PROXIMITY EFFECT IN A TWO-WIRE LINE WITH ROUND CONDUCTORS

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Preliminary notes

This paper presents an analysis of the proximity effect in a line consisting of two identical solid conductors of circular cross section. Integral equation for the current distribution in one of the conductors is solved approximately by assuming the current density in the form of a finite double polynomial series and by applying the point matching procedure. The current distribution obtained by the presented method has been checked by implementing the FEMM 4.0 software package. Also, the a.c. to d.c. resistance ratio is calculated and the results are compared to the available ones, and a good agreement is exhibited.

Keywords: *a.c. to d.c. resistance ratio; circular cross section conductors; current distribution; proximity effect*

Metoda integralnih jednadžbi za analizu efekta blizine u dvožičnom vodu s kružnim vodičima

Prethodno priopćenje

U ovom radu predstavljena je analiza efekta blizine u vodu sastavljenom od dva identična vodiča kružnog poprečnog presjeka. Integralna jednadžba za raspodjelu gustoće struje u jednom od vodiča riješena je približno uzimajući da je gustoća struje u obliku konačnog dvojnog stupanjskog reda s nepoznatim koeficijentima koji su određeni metodom podešavanja u točkama. Raspodjela struje dobivena prikazanom metodom provjerena je uporabom softverskog paketa FEMM 4.0. Također, izračunat je omjer otpornosti pri izmjeničnoj i istosmjernoj struji, a rezultat je pokazao dobro poklapanje s raspoloživim rezultatima.

Ključne riječi: *omjer otpornosti pri izmjeničnoj i istosmjernoj struji; provodnici kružnog poprečnog presjeka; raspodjela struje; efekt blizine*

1 Introduction

When two parallel conductors carrying time-varying currents are close to each other, eddy currents are induced in one conductor by the transverse magnetic field of the other. These currents affect the current distribution in the cross section of the conductors. This phenomenon is referred to as the proximity effect. Since this effect causes an increase of the conductor resistance it is of concern in many applications.

Combined skin and proximity effect analysis is very involved. This is due to the fact that the determination of the current distribution in a system of several parallel conductors with time-varying currents requires solution of a system of differential or integral equations. There are very few cases where a closed-form solution for proximity effect can be obtained. Some of them are considered in [1 ÷ 4]. In all of these cases one of the conductors is thin.

Current distribution in a system of several conductors with sinusoidal currents can be determined by the method of integral equations. Here, the current densities are described by a system of integral equations. Manneback [4] was the first to derive such a system for two cylindrical conductors. Dwight [1] followed Manneback's approach and considered some systems of two or three conductors of different cross sections, and gave exact and approximate solutions for current distribution. Both Dwight and Manneback used successive approximations for solving integral equations. In [5], the authors of the present paper introduced a direct generalization of Manneback's system of integral equations for the system of *n* parallel solid conductors of arbitrary cross section, in somewhat modified, but equivalent form. The current density in a cross section of each conductor is approximated by a linear combination of conveniently chosen base functions. For determination of unknown coefficients, a point matching method is used. This procedure leads to solving systems of low-order linear algebraic equations, from which the unknown coefficients were determined. This method was used to analyse proximity effect in different systems of parallel conductors, including thin tubular conductors, strip conductors and conductors of rectangular cross section [6 ÷ 9].

In this paper we investigate proximity effect in a line consisting of two identical solid conductors of circular cross section, carrying equal sinusoidal currents in opposite directions. Due to the mirror symmetry of the system geometry it suffices to determine current distribution in one of the conductors. In this paper the current density is assumed in the form of a finite polynomial series and then the point matching method is applied for determining the unknown coefficients. Once the approximate current distribution is known one readily finds the a.c. to d.c. resistance ratio.

Some other methods for solving the problem of current distribution are possible, such as boundary integral equation formulation [10, 11], modal functions [12], differential equation approach in terms of the magnetic vector potential [13 ÷ 16], etc.

2 Integral equation for current distribution

Geometry of the system under consideration is shown in Fig. 1.

The radii of the conductors are a and the distance between the conductor axes is D. Sinusoidal currents of angular frequency ω and r.m.s. value I flow through the conductors in opposite directions. Due to the mirror symmetry $J_2(x', y') = J_1(D - x', y')$ so we need to consider only the integral equation for the current density in conductor 1 [5]:

$$J_1(x,y)=\frac{k^2}{4\pi}\left[\int_{S_1} J_1(x',y')\ln\left(\frac{\overline{P_1P_1'}}{a}\right)^2 \mathrm{d}x'\mathrm{d}y' - \right.$$
$$\left. -\int_{S_2} J_1(D-x',y')\ln\left(\frac{\overline{P_1P_2'}}{a}\right)^2 \mathrm{d}x'\mathrm{d}y'\right]+K, \tag{1}$$

where $k^2=j\omega\mu_0\sigma$, K is an unknown constant and distances $\overline{P_1P_1'}$ and $\overline{P_1P_2'}$ are:

$$\left(\overline{P_1P_1'}\right)^2=(x-x')^2+(y-y')^2;\; x,\,y,\,x',y'\in S_1 \tag{2}$$

$$\left(\overline{P_1P_2'}\right)^2=(x-x')^2+(y-y')^2;\; x,\,y\in S_1;\; x',y'\in S_2 \tag{3}$$

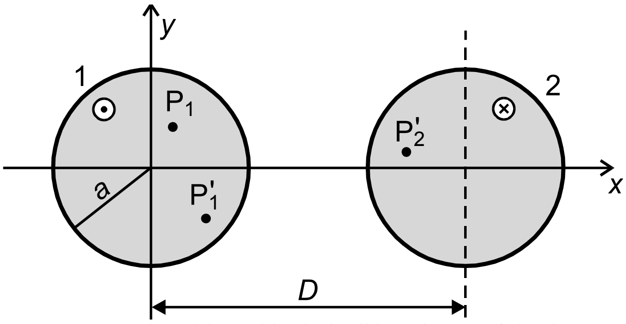

Figure 1 Line with two identical solid conductors of circular cross section

By using normalised coordinates

$$x_n=\frac{x}{a}, \tag{4}$$

$$y_n=\frac{y}{a}, \tag{5}$$

and substituting (2) ÷ (5) into (1) we obtain

$$J_1(x,y)=\frac{k^2a^2}{4\pi}\left[\int_{S_1} J_1(x',y')\ln\left[(x-x')^2+(y-y')^2\right]\mathrm{d}x'\mathrm{d}y' - \right.$$
$$\left. -\int_{S_2} J_1\left(\frac{D}{a}-x',y'\right)\ln\left[(x-x')^2+(y-y')^2\right]\mathrm{d}x'\mathrm{d}y'\right]+K. \tag{6}$$

Eq. (6) has a dimensionless form, which means that x, y, x' and y' stand for the normalized coordinates i.e. the actual coordinates divided by the value of a (subscript n was omitted for brevity). Also, $J_1(x_n\cdot a,\, y_n\cdot a)$ in real coordinates is identified by $J_1(x_n,y_n)\equiv J_1(x,y)$ in normalized coordinates.

Note: In case of currents flowing in the same direction, the *minus* sign in Eq. (6) should be replaced by a *plus* sign.

3 Approximate solution of the integral equation

We seek for an approximate solution of the integral Eq. (6) in the form of a finite polynomial series

$$J_1(x,y)=\sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn}x^m y^{2n},$$
$$-1\le x\le 1,\, -\sqrt{1-x^2}\le y\le +\sqrt{1-x^2}, \tag{7}$$

where a_{mn} ($m=0,\ 1,\ 2,\ ...,\ M$; $n=0,\ 1,\ 2,\ ...,\ N$) are unknown coefficients. Note that only even powers of y are taken due to the symmetry with respect to the x-axis. By substituting (7) into (6) we obtain

$$\sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn}x^m y^{2n}=$$
$$=\frac{k^2a^2}{4\pi}\left[\sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn}\int_{S_1} x'^m y'^{2n}\ln\left[(x-x')^2+(y-y')^2\right]\mathrm{d}x'\mathrm{d}y' - \right. \tag{8}$$
$$\left. -\sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn}\int_{S_2}\left(\frac{D}{a}-x'\right)^m y'^{2n}\ln\left[(x-x')^2+(y-y')^2\right]\mathrm{d}x'\mathrm{d}y'\right]+K,$$

which can be written in the form

$$\sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn}F_{mn}(x,y)=K \tag{9}$$

where

$$F_{mn}(x,y)=x^m y^{2n}-\frac{k^2a^2}{4\pi}\left[G_{mn}^{(1)}(x,y)-G_{mn}^{(2)}(x,y)\right], \tag{10}$$

and $G_{mn}^{(1,2)}(x,y)$ are double integrals defined as

$$G_{mn}^{(1)}(x,y)=\int_{S_1} x'^m y'^{2n}\ln\left[(x-x')^2+(y-y')^2\right]\mathrm{d}x'\mathrm{d}y' \tag{11}$$

where $-1\le x,x'\le 1$, $-\sqrt{1-x^2}\le y,y'\le +\sqrt{1-x^2}$, and

$$G_{mn}^{(2)}(x,y)=\int_{S_2}\left(\frac{D}{a}-x'\right)^m y'^{2n}\ln\left[(x-x')^2+(y-y')^2\right]\mathrm{d}x'\mathrm{d}y' \tag{12}$$

where $-1\le x\le 1$, $-\sqrt{1-x^2}\le y\le +\sqrt{1-x^2}$, $\frac{D}{a}-1\le x'\le\frac{D}{a}+1$, $-\sqrt{1-\left(x-\frac{D}{a}\right)^2}\le y'\le +\sqrt{1-\left(x-\frac{D}{a}\right)^2}$.

To determine the unknown coefficients a_{mn} in (7) we use two conditions.

Condition 1 – Currents flowing through the conductors are known:

$$
\begin{aligned}
I &= \int_{S_1} J_1(x,y)\mathrm{d}x\mathrm{d}y = \sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn} \int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{+\sqrt{a^2-x^2}} \left(\frac{x}{a}\right)^m \left(\frac{y}{a}\right)^{2n} \mathrm{d}x\mathrm{d}y = \\
&= a^2\pi \sum_{m=0}^{M}\sum_{n=0}^{N} a_{mn} \frac{m!\,[2(n+1)]!}{2^{m+2n+1}(2n+1)\left(\frac{m}{2}\right)!\,(n+1)!\left(\frac{m}{2}+n+1\right)!}
\end{aligned}
\tag{13}
$$

where we use only **even** values of m.

Condition 2 (Point matching)

We stipulate that Eq. (9) be satisfied at $(M+1)(N+1)$ different points (x_k,y_k), ($x_k \in (-1,1)$, $y_k \in \left(-\sqrt{1-x^2}, +\sqrt{1-x^2}\right)$) of the cross section S_1. The Eq. (9) turns into the system of $(M+1)(N+1)$ linear algebraic equations from which, together with (13), the unknown coefficients a_{mn} and constant K can be found. Once coefficients a_{mn} are known, the current distribution in conductor 1 is approximately determined by (7).

4 Resistance ratio R_{ac}/R_{dc}

The a.c. to d.c. resistance ratio is the same for both conductors. In this case it can be calculated as [5]

$$
\frac{R'_{ac}}{R'_{dc}} = a^2\pi \frac{\int_S |J(x,y)|^2 \mathrm{d}x\mathrm{d}y}{\left|\int_S J(x,y)\mathrm{d}x\mathrm{d}y\right|^2}
\tag{14}
$$

where $J(x,y) \equiv J_1(x,y)$.

5 Numerical results

Fig. 2 shows the current distribution, i.e. lines of equal magnitudes of the current densities. It was taken a = 1 cm, D = 3 cm, f = 50 Hz and $\sigma = 57\times10^6$ S/m. The same distribution obtained by implementing the FEMM 4.0 software package is depicted in Fig. 3.

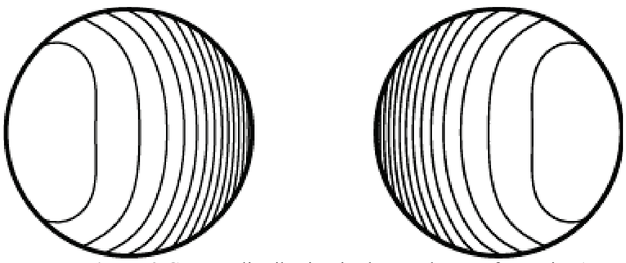

Figure 2 Current distribution in the conductors from Fig. 1 (obtained by the presented method)

Fig. 4 shows the current distribution for the case, when currents flow in the same direction, while the results obtained by implementing the FEMM 4.0 software package are depicted in Fig. 5.

Fig. 6 shows the a.c. to d.c. resistance ratio for the system from Fig. 1 versus frequency f, for different distances of the conductors. It was taken a = 1cm and $\sigma = 57\times10^6$ S/m.

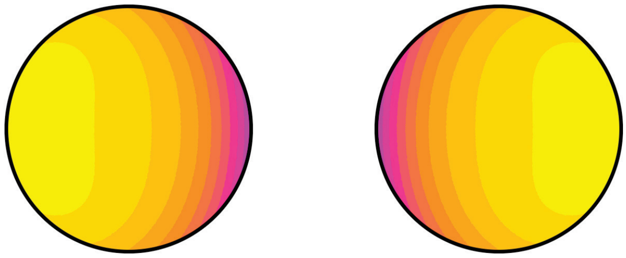

Figure 3 Current distribution in the conductors from Figure 1 (obtained by implementing the FEMM 4.0 software package)

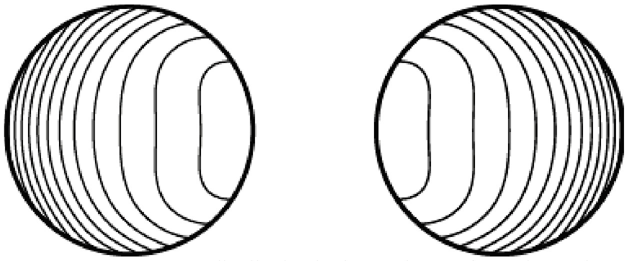

Figure 4 Current distribution in the conductors from Fig. 1 when the currents flow in the same direction (obtained by the presented method)

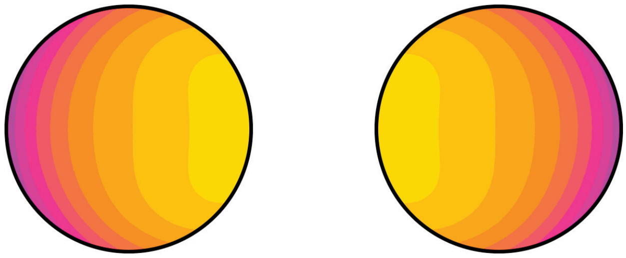

Figure 5 Current distribution in the conductors from Fig. 1 when the currents flow in the same direction (obtained by implementing the FEMM 4.0 software package)

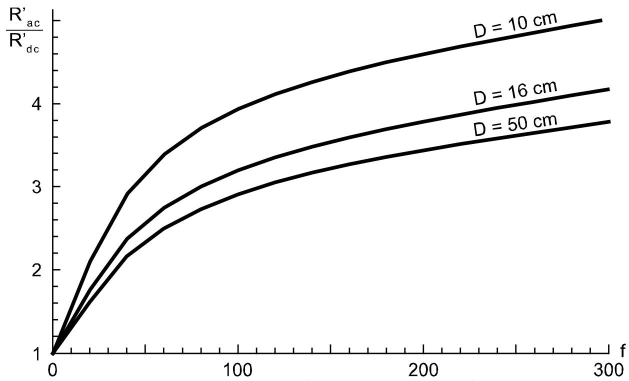

Figure 6 Resistance ratio R_{ac}/R_{dc} for the system from Fig. 1 versus frequency f for different distances of the conductors

We were able to compare our results for resistance ratio R_{ac}/R_{dc} with the results in [1]. In case when the currents flow in opposite directions, for $D/a = 4$ and $b = 10$ (b is a parameter defined as $b = a\sqrt{\omega\mu_0\sigma}$), Dwight [1] obtained $R_{ac}/R_{dc} = 4{,}29$. Our result for $D/a = 4$, $a = 4$ cm, $\sigma = 57\times10^6$ S/m and $f = 139$ Hz (which corresponds to the value $b = 10$) is $R_{ac}/R_{dc} = 4{,}34$. For the same parameters, but for the currents flowing in the same direction, Dwight's result [1] was $R_{ac}/R_{dc} = 4{,}1838$, and the result obtained by the method described in this paper is $R_{ac}/R_{dc} = 4{,}23$.

6 Conclusion

This paper presents an analysis of the proximity effect in a two-wire line consisting of two identical solid conductors of circular cross section. Integral equation for the current density is approximately solved by assuming the current density in the form of a finite double power series with unknown coefficients. These coefficients are determined by the point matching method. Also, a.c. to d.c. resistance ratio is calculated, and the results are compared to the available ones, exhibiting good agreement.

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