TWO-FLUID 2.5D MHD-CODE FOR SIMULATIONS IN THE LOWER SOLAR ATMOSPHERE

I. PIANTSCHITSCH[1], U. AMERSTORFER[1], J. THALMANN[1],
D. UTZ[1,2], A. HANSLMEIER[1], M. BÁRTA[3],
S. THONHOFER[1,2] and B. LEMMERER[1]

[1]*IGAM/Institute of Physics, University of Graz,*
Universitätsplatz 5, A–8010 Graz, Austria
[2]*Instituto de Astrofísica de Andalucía, CSIC,*
Granada, Spain
[3]*Astronomical Institute of the Academy of Sciences,*
Fricova 298, 251 65, Ondřejov, Czech Republic

Abstract. We investigate magnetic reconnection due to the evolution of magnetic flux tubes in the solar chromosphere. We developed a new numerical two-fluid magnetohydrodynamic (MHD) code which will perform a 2.5D simulation of the dynamics from the upper convection zone up to the transition region. Our code is based on the Total Variation Diminishing Lax-Friedrichs scheme and makes use of an alternating-direction implicit method, in order to accommodate the two spatial dimensions. Since we apply a two-fluid model for our simulations, the effects of ion-neutral collisions, ionization/recombination, thermal/resistive diffusivity and collisional/resistive heating are included in the code. As initial conditions for the code we use analytically constructed vertically open magnetic flux tubes within a realistic stratified atmosphere. Initial MHD tests have already shown good agreement with known results of numerical MHD test problems like e.g. the Orszag-Tang vortex test.

Key words: MHD simulation - chromosphere - reconnection

1.   Introduction

One possible source for magnetic reconnection is the dynamics of small scale magnetic fields in the photosphere. In order to describe and explain the motion of these magnetic fields and their effects in the chromosphere, we implemented a two-fluid 2.5D MHD code, in which the two fluids are coupled predominantly through ion-neutral collisions but also through ionization and recombination effects. There already exist several MHD codes which are able to simulate different phenomena within the chromosphere (Gudiksen *et al.* 2011; Freytag *et al.* 2010) but are usually based on single-fluid models.

What is innovative about our newly developed 2.5D simulation code is the inclusion of the two-fluid model of Smith & Sakai (2008) in combination with the use of analytically constructed vertically open magnetic flux tubes (Gent *et al.* 2013) in order to observe magnetic reconnection within our simulation of the chromospheric dynamics. The motivation for the use of the two-fluid approach lies on the one hand in the results of Smith & Sakai (2008), who already pointed out that comparisons of MHD and two-fluid simulations show significant differences in the measured rates of magnetic reconnection. On the other hand Zaqarashvili *et al.* (2011) could show that for time-scales less than ion-neutral collision time the single-fluid approach fails and the two-fluid description is the better approximation.

2. Simulation Model

The following set of equations describes the two-fluid model we use, the subscripts p and n refer to the ion (proton) and neutral fluid. The mass density, velocity, pressure and magnetic field are given by ρ, v, p and B. First, we consider the set of equations for the neutral fluid:

$$\frac{\partial \rho_n}{\partial t} + \nabla \cdot (\rho_n v_n) = -S_1 \tag{1}$$

$$\frac{\partial (\rho_n v_n)}{\partial t} + \nabla \cdot (\rho_n v_n v_n) + \nabla \cdot p_n = -S_2 \tag{2}$$

$$\frac{\partial e_n}{\partial t} + \nabla \cdot \left[(e_n + p_n) \, v_n \right] - q_n = -S_3 \tag{3}$$

where the neutral heat flux is given by $q_n = \nabla^2 (\lambda p_n / \rho_n)$, the neutral energy density can be described as $e_n = \frac{p_n}{\gamma - 1} + \frac{\rho_n |v_n|^2}{2}$ and the adiabatic constant is $\gamma = 5/3$. In equations (1)-(3) the source terms S_1, S_2 and S_3 characterize the effects of ionization/recombination, ion-neutral drag and collision heating:

$S_1 = -\rho_p (\alpha_r \rho_p - \alpha_i \rho_n)$,
$S_2 = \alpha_c \rho_p \rho_n (v_n - v_p) - \rho_p (\alpha_r \rho_p v_p - \alpha_i \rho_n v_n)$,
$S_3 = \alpha_c \rho_p \rho_n (v_n - v_p) v_p$,

where the coefficients α_i, α_r and α_c denote the effects of ionization, recombination and ion-neutral collisions.

Secondly, the equations for the ion fluid can be described as

$$\frac{\partial \rho_p}{\partial t} + \nabla \cdot (\rho_p v_p) \quad = \quad S_1 \tag{4}$$

$$\frac{\partial (\rho_p v_p)}{\partial t} + \nabla \cdot (\rho_p v_p v_p) - J \times B + \nabla p_p \quad = \quad S_2 \tag{5}$$

$$\frac{\partial B}{\partial t} - \nabla \times (v_p \times B) \quad = \quad \eta \nabla^2 B \tag{6}$$

$$\frac{\partial e_p}{\partial t} + \nabla \cdot [(e_p + p_p) \, v_p] - q_p \quad = \quad S_3 \tag{7}$$

where the ion heat flux is given by $q_p = \nabla^2(\lambda p_p/\rho_p) + \mu_0\eta \mid J \mid^2$ and includes thermal conduction as well as Joule heating. The current density is given by $J = (\nabla \times B)/\mu_0$ and η is the magnetic diffusivity. The heat transfer constant is denoted by λ and μ_0 is the vacuum permeability. The ion energy density can be written as $e_p = \frac{p_p}{\gamma-1} + \frac{\rho_p|v_p|^2}{2} + \frac{|B|^2}{2\mu_0}$.

3.   Initial Conditions

In order to implement the initial conditions we use the model of an analytically constructed vertically open magnetic fluxtube (Gent et al. 2013) which represents one footpoint of a coronal loop. In Cartesian coordinates the components of the magnetic field can be written as

$$B_x \quad = \quad -x\left(\frac{\partial B_{bz}}{\partial z} + B_{0z}G\frac{\partial B_{0z}}{\partial z}\right), \tag{8}$$

$$B_y \quad = \quad -y\left(\frac{\partial B_{bz}}{\partial z} + B_{0z}G\frac{\partial B_{0z}}{\partial z}\right), \tag{9}$$

$$B_z \quad = \quad B_{0z}^2 G + 2B_{bz}, \tag{10}$$

where

$$B_{bz} \quad = \quad b_{00}\exp\left(-\frac{z}{z_b}\right), \tag{11}$$

$$B_{0z} \quad = \quad b_{01}\exp\left(-\frac{z}{z_1}\right) + b_{02}\exp\left(-\frac{z}{z_2}\right), \tag{12}$$

$$G \quad = \quad \frac{2l}{\sqrt{\pi}f_0}\exp\left[-\left(\frac{f}{f_0}\right)^2\right]. \tag{13}$$

Here B_{0z}, G and $f = rB_{0z}$ represent the self-similar expanding axially symmetric magnetic flux tube and r prescribes the radial distance to the centre of the flux tube. B_{bz} denotes a vertically diminishing background term and b_{01}, b_{02}, b_{00} are constants which control the strength of the vertical component of the magnetic field. Here l is the scaling length whereas f_0 is used to scale the horizontal length. Moreover, z_1 and z_2 are used for scaling the magnetic field strength and z_b is included to scale the external magnetic field.

This magnetic flux tube configuration in magnetohydrostatic equilibrium is embedded within a realistic solar atmosphere which is based on the combination of the results of Vernazza, Avrett & Loeser (1975, Table 12, VALI-IIC) and McWhirter, Thonemann & Wilson (1975, Table 3). Furthermore, the equation of pressure is integrated analytically to find the pressure and density correction required to preserve the magnetohydrostatic equilibrium.

4.   Numerical Scheme

The general formulation of the MHD equations (1)-(7) can be written as

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} + \frac{\partial G(U)}{\partial y} = S(U) \tag{14}$$

where U denotes the vector of conserved variables, which are built of the primitive variables mass density, pressure, velocity and magnetic field given by ρ_i, p_i, v_i ($i = n$, p) and B. Moreover, $F(U)$ and $G(U)$ represent the fluxes in x- and y-direction, whereas $S(U)$ denotes the source term which includes all the source terms of (1)-(7).

It can be shown (Toro 2009) that (14) can be solved by using the so called alternating-direction implicit method (ADI), which means that one applies one-dimensional methods in each coordinate direction. For solving the homogeneous one-dimensional equations we make use of the so called Total Variation Diminishing Lax-Friedrichs (TVDLF; Tóth and Odstrčil 1996) scheme which is based on the idea that the total variation of the analytical solution of a linear system of hyperbolic equations does not increase in time. It is the simplest version of a TVD scheme and is built on the first order Lax-Friedrichs scheme. Furthermore, our algorithm includes the well known Hancock-predictor step which is based on the following idea: Within our computational grid, first cell averages are used for predicting

the values of the conserved quantities at cell edges at an auxiliary time step. Subsequently, these predicted values are used to update the solution. By applying this Hancock-scheme we achieve temporally second order accuracy. In order to get second order spatial accuracy, linear approximations of U and the corresponding fluxes at the boundary interfaces are incorporated in the algorithm. Due to numerical issues, the order of the one-dimensional TVDLF-operators is changed in every time step.

There are several advantages of applying the TVDLF-method:

- no spurious oscillations are generated

- the method behaves well near discontinuities

- no Riemann-solvers are needed

- fully explicit method, which leads to benefits regarding the parallelization process

To include the source term in our algorithm, we make use of a simple four-stage Runge-Kutta method to solve the inhomogeneous ordinary differential equation. For numerical reasons, the operator of the source term is splitted up by calculating one half of a time step before and one half time step after applying the two different TVDLF-operators .

In order to maintain the $\nabla \cdot B = 0$ constraint, which has to be fulfilled in every single time step of the algorithm, we apply a field-interpolated central differencing scheme (Field-CD) for the induction equation (Tóth 2000). The complete numerical scheme can therefore be described as follows:

$$
\begin{array}{ll}
1.\ \frac{\partial U^n}{\partial t} + \frac{\partial F(U^n)}{\partial x} = 0 \Longrightarrow \overline{U}^{n+1} & \text{(TVDLF)} \\[2ex]
2.\ \frac{\partial \overline{U}^{n+1}}{\partial t} + \frac{\partial G(\overline{U}^{n+1})}{\partial y} = 0 \Longrightarrow \hat{U}^{n+1} & \text{(TVDLF)} \\[2ex]
3.\ \frac{d}{dt}\hat{U}^{n+1} = S(\hat{U}^{n+1}) \Longrightarrow U^{n+1} & \text{(Runge-Kutta)} \\[2ex]
4.\ \nabla \cdot B = 0 & \text{(Field-CD)}
\end{array}
$$

Two full time-steps can be formulated as

$$U^{n+2} = DM_{\triangle t/2}L_xL_yM_{\triangle t/2}DM_{\triangle t/2}L_yL_xM_{\triangle t/2}U^n \qquad (15)$$
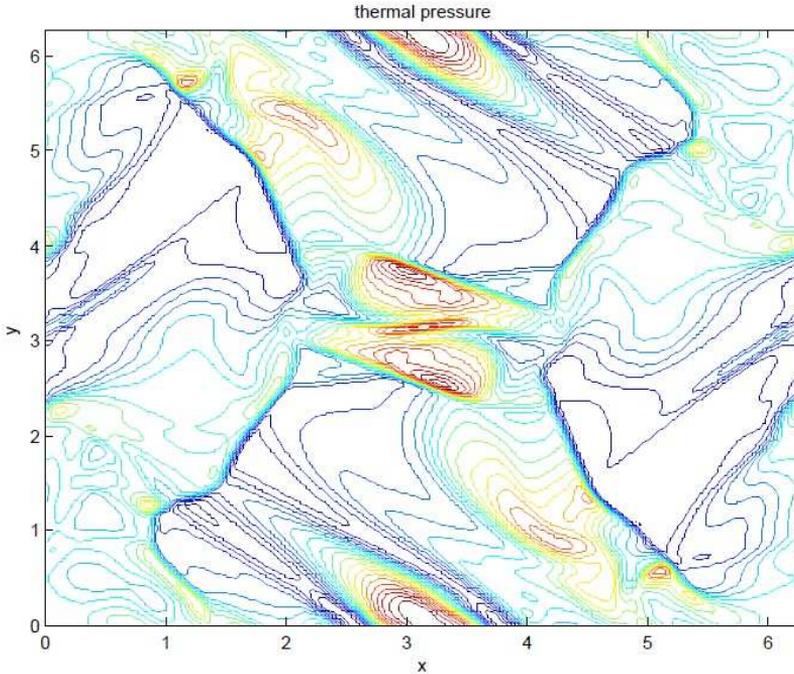
Figure 1: Thermal pressure of the MHD Orszag-Tang vortex system. The initial conditions for the 192x192 grid with $0 < x, y < 2\pi$ lead to a system of supersonic MHD turbulence. The algorithm stops at the time of $t = 3.1$ and the thermal pressure is plotted with 60 contour levels. The boundary conditions are periodic everywhere.

with

L_x ... 1D TVDLF-operator in x-direction,

L_y ... 1D TVDLF-operator in y-direction,

M ... Runge-Kutta-operator,

D ... Field-CD-operator.

5. Code Tests

In order to test our code, we implemented e.g. the well known Orszag-Tang vortex test which was performed by Tóth and Odstrčil (1996). It is a common test of numerical MHD codes in two spatial dimensions. The initial conditions lead to a system of supersonic MHD turbulence (see Fig.

1), which makes this problem an appropriate test of the algorithm's ability to handle the formation of such turbulence and MHD shocks.

6. Outlook

As a next step of research we will continue performing comprehensive code tests especially concerning the code's ability to handle magnetic reconnection. Subsequently we will apply our newly developed two-fluid 2.5D code to the above described initial conditions and observe the time evolution of the magnetic flux tubes as well as potential reconnection events within our computational grid. Also the single-fluid approach will be applied to the initial conditions and the subsequent output will be compared to the results of the two-fluid approach. The code will be extended to three spatial dimensions and we plan to augment the already existing parallelized versions of the code to a hybrid-version, i.e. a combination of the MPI (Message Passing Interface)-based and the OpenMP (OpenMultiProcessing)-based version of the code, in order to reduce computing time.

Acknowledgements

References

Freytag, B., *et al.*: 2010, *Astrophysics Source Code Library.*
Gudiksen, B. V., *et al.*: 2011, *Astron. Astrophys.* **531**, 154.
Gent, F. A., Fedun, V., Mumford, S. J., and Erdélyi, R.: 2013, *Mon. Not. R. Astron. Soc.* **435**, 689.

McWhirter, R. W. P., *et al.*: 1975, *Astron. Astrophys.* **40**, 63.

Smith, P. D. and Sakai, J. I.: 2008, *Astron. Astrophys.* **486**, 569.

Toro, E. F.: 2009, *Riemann Solvers and Numerical Methods for Fluid Dynamics*, Springer, Third Edition.

Tóth, G.: 2000, *J. Comput. Phys.* **161**, 605.

Tóth, G. and Odstrčil, D.: 1996, *J. Comput. Phys.* **128**, 82.

Vernazza J. E., Avrett E. H., and Loeser R.: 1981, *Astrophys. J., Suppl. Ser.* **45**, 635.

Zaqarashvili T., *et al.*: 2011, *Astron. Astrophys.* **529**, 85.