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NEKONZERVATIVNI PROBLEMI STABILNOSTI ŠTAPNIH KONSTRUKCIJA

NON-CONSERVATIVE PROBLEMS OF THE STABILITY OF BAR STRUCTURES

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Stručni članak

Sažetak: *U članku je predstavljen algoritam za korištenje numeričko-analitičke verzije metode graničnih elemenata (MGE) za rješavanje nekonzervativnih problema stabilnosti štapnih konstrukcija kao što su kontinuirane grede i ravninski okviri. Cilj ovog rada je proučiti ponašanje kompleksnih mehaničkih sustava pod djelovanjem nekonzervativnih tlačnih sila. Slični problemi se javljaju u visoko opterećenim konstrukcijama te je karakter njihovog ponašanja od velikog znanstvenog značaja. Pri rješavanju nekonzervativnih problema korištena je dinamička metoda i algoritam MGE. Pokazano je da je ponašanje složenih štapnih konstrukcija pod djelovanjem tlačnih nekonzervativnih sila kvalitativno se razlikuje od ponašanja pojedinačnih stupova i greda. Rezultati rada dopunjuju i proširuju znanje o ponašanju različitih konstrukcija što se može iskoristiti u razradi projektnih rješenja u visokotlačnim cjevovodima, izgradnji strojeva, avio i brodogradnji, gradnji raketa, u izgradnji bušaćih strojeva i sl. Proračuni kritičnih sila napravljeni su u programu MATLAB.*

Ključne riječi: *MATLAB, metoda graničnih elemenata, nekonzervativni problem stabilnosti, štapnih sustavi*

Professional paper

Abstract: *An algorithm for the use of numerical-analytic version of boundary element method (MGE) for solving the problems of stability of non-conservative bar systems such as continuous beams and plane frames is presented. The aim of this work is to learn the behavior of complex mechanical systems, loaded with non-conservative compressive forces. Similar problems occur in heavily loaded structures and their behavior is of great scientific and practical importance. Dynamic method and algorithm of MGE are applied in solving the non-conservative stability problems. It is shown that the behavior of complex systems under the action of the bar non-conservative compressive forces are qualitatively different from the behavior of individual bars and beams. The results of the work complement and extend the knowledge about the behavior of different designs that can be used in such design solutions as high-pressure pipelines, machine building, aerospace, shipbuilding, rocket, on drilling rigs, etc. Calculations of the critical forces are made in the MATLAB environment.*

Keywords: *boundary element method, MATLAB, non-conservative problems of stability, rod system*

1. INTRODUCTION

Mechanical systems with non-conservative forces are widely used in the life of modern society. Systems with internal sources of energy can be attributed to such systems: these are rockets, planes, space stations, oil derricks and platforms, turbines, internal combustion engines, metal-cutting machines, various cranes, high-pressure pipelines and etc.

The conservative problems of the stability can be solved only with a static method while the non-conservative tasks can be served with the help of dynamic method only [1]. The main element of BEM for the dynamic method is solution of Cauchy problem for the bar lateral oscillations considering the longitudinal force. The given solution is convenient to be introduced in the matrix form [1].

$$\begin{bmatrix} EI\,\upsilon(x) \\ EI\,\varphi(x) \\ M(x) \\ Q(x) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & -A_{13} & -A_{14} \\ A_{21} & A_{11} & -A_{23} & -A_{13} \\ -A_{31} & -A_{21} & A_{33} & A_{23} \\ -A_{41} & -A_{31} & A_{43} & A_{33} \end{bmatrix} \begin{bmatrix} EI\,\upsilon(0) \\ EI\,\varphi(0) \\ M(0) \\ Q(0) \end{bmatrix} + \int_0^x \begin{bmatrix} A_{14}(x-\xi) \\ A_{13}(x-\xi) \\ -A_{23}(x-\xi) \\ -A_{33}(x-\xi) \end{bmatrix} q(\xi)d\xi, \quad (1)$$

where fundamental orthonormalized functions have the following view:

$$A_{11}=\frac{\beta^2 ch\alpha x+\alpha^2\sin\beta x}{\alpha^2+\beta^2};$$
$$A_{12}=\frac{\beta^3 sh\alpha x+\alpha^3\sin\beta x}{\alpha^3\beta+\alpha\beta^3};\; A_{13}=\frac{ch\alpha x-\cos\beta x}{\alpha^2+\beta^2};$$
$$A_{14}=\frac{\beta^2 sh\alpha x-\alpha\sin\beta x}{\alpha^3\beta+\alpha\beta^3};\; A_{21}=\alpha^2\beta^2 A_{14};$$
$$A_{23}=\frac{\alpha sh\alpha x+\beta\sin\beta x}{\alpha^2+\beta^2};\; A_{31}=\alpha^2\beta^2 A_{13}; \qquad (2)$$
$$A_{33}=\frac{\alpha^2 ch\alpha x+\beta^2\cos\beta x}{\alpha^2+\beta^2};\; A_{41}=\alpha^2\beta^2 A_{23};$$
$$A_{43}=\frac{\alpha^3 sh\alpha x-\beta^3\sin\beta x}{\alpha^2+\beta^2};$$
$$\alpha=\sqrt{-\frac{F}{2EI}+\sqrt{\left(\frac{F}{2EI}\right)^2+\frac{\omega^2 m}{EI}}};$$
$$\beta=\sqrt{\frac{F}{2EI}+\sqrt{\left(\frac{F}{2EI}\right)^2+\frac{\omega^2 m}{EI}}}.$$

Expressions (2) proceed to the famous A.N. Krylov functions at $F = 0$.

In contrast to the static method, critical force in the dynamic method is defined in the point where two neighboring frequencies of free oscillations become equal (merge). Therefore, the initial data of compressing force is input in the program and frequencies (at least two) of free oscillations from the BEM frequency equation are defined [2],

$$\left|A_*(F,\omega)\right|=0 \qquad (3)$$

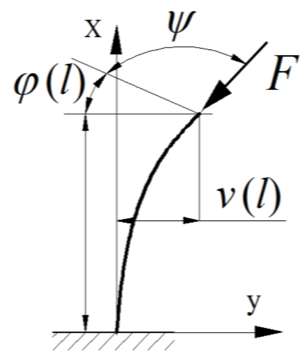
where $A_*(F,\omega)$ – is the frequency matrix of the bar structures built according to the BEM algorithm. Then the value of the compressing force is increased and the frequency change is backtracked. The process continues until two neighboring frequencies become equal with the certain precision. At the same time, the value of the compressing force will be critical and the whole process of its identification is comfortable to be presented in the form of diagrams.

2. ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

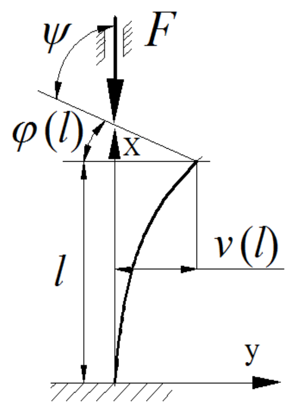
Literature analysis on solution of non-conservative problems of steadiness shows [1], that behavior of complex bar systems such as continuous beams and frames. Therefore, let us show BEM algorithm ideally suits for identical problem-solving models with any structure of elastic system. A random set of bars can be a model of the object. Each of those bars can have an infinite number of degrees of freedom. Moreover, displacement, rotation inertia, inside and inner friction, optional laws of mass and rigidity change, longitudinal forces and other factors. Non-conservativeness of compressing forces in BEM is considered with a proper formulation of boundary conditions for an elastic system. Boundary conditions are non-linear for a separate bar and are linearized considering a smallness of proper dislocation, i.e. the equalities are true.

$$\sin\psi=tg\,\psi=\psi;\cos\psi=1, \qquad (4)$$

where ψ – is an angle between the force of F and a normal to the bar axis (Fig. 1).



a)



b)

Figure 1. Alternatives of behavior for the compressing non-conservative forces: a) following up the rotation angle of the beam section; в) the force of F has a fixed line of action

Boundary conditions for the force of F in Fig.1, a are quite simple:

$$\psi=\frac{\pi}{2}; EI\upsilon(l)\neq 0; EI\varphi(l)\neq 0; \qquad (5)$$
$$M(l)=0;\, Q(l)=0$$

In boundary section of the beam for the force of F in Fig.1, b there arise the bending moment and shear.

$$\psi = \frac{\pi}{2} - \varphi(l); EI\upsilon(l) \neq 0; EI\varphi(l) \neq 0;$$
$$M(l) = F\upsilon(l); Q(l) = F\sin\varphi(l) \qquad (6)$$

If non-conservative pressure forces are applied to the assemblies of continuous beams and frames, then their behavior should be considered with boundary conditions of bars that are a part of this assembly. Let us review the tasks of bar structures stability on the force application in Fig.1.

3. THE PURPOSE OF WORK

The purpose of work is to solve new non-conservative tasks of continuous beams and plate frames and learn the behavior of these sys-tems at the application of increasing non-conservative compressing forces.

4. MAIN PURPOSE OF THE ARTICLE

Continuous beam (Figure 2). According to the BEM [1] algorithm, it is divided into separate bars, assemblies are numerated and the start and end of each bar are marked with arrows. Then the matrices of initial and finite beam parameters are formed. These matrices consider the end conditions, boundary conditions and a connection between the boundary elements in assemblies.

Load non-conservativeness in Fig. 2, a is shown in the Matrix Y in the rows from 13 to 16. If we transfer all parameters from Y to the matrix X_*, we will receive the matrix $A_*(F,\omega)$ of equation (3) for the given beam.

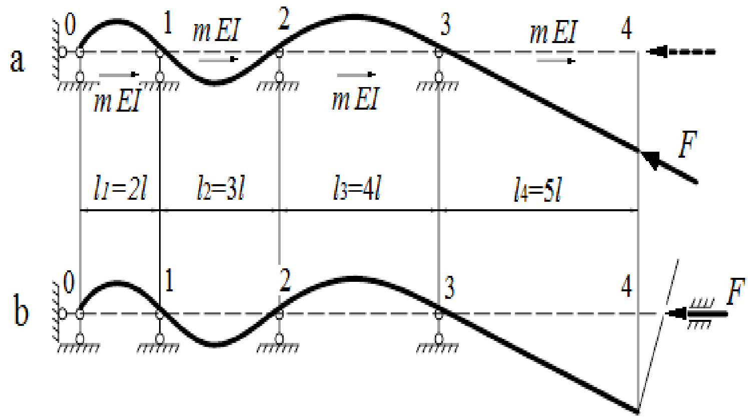


Figure 2. Continuous beam loaded with non-conservative forces

$X_* =$; $Y =$ (7)

	X_*		Y
1	$EI\upsilon_{(0)}^{0-1}=0;\ Q^{0-1}(l_1)$	1	$EI\upsilon_{(l_1)}^{0-1}=0$
2	$EI\varphi_{(0)}^{0-1}$	2	$EI\varphi_{(l_1)}^{0-1}=EI\varphi_{(0)}^{1-2}$
3	$M_{(0)}^{0-1}=0;\ Q^{1-2}(l_2)$	3	$M_{(l_1)}^{0-1}=M_{(0)}^{1-2}$
4	$Q_{(0)}^{0-1}$	4	$Q_{(l_1)}^{0-1}$
5	$EI\upsilon_{(0)}^{1-2}=0;\ Q^{2-3}(l_3)$	5	$EI\upsilon_{(l_2)}^{1-2}=0$
6	$EI\varphi_{(0)}^{1-2}$	6	$EI\varphi_{(l_2)}^{1-2}=EI\varphi_{(0)}^{2-3}$
7	$M_{(0)}^{1-2}$	7	$M_{(l_2)}^{1-2}=M_{(0)}^{2-3}$
8	$Q_{(0)}^{1-2}$	8	$Q_{(l_2)}^{1-2}$
9	$EI\upsilon_{(0)}^{2-3}=0;\ EI\upsilon_{(l_4)}^{3-4}$	9	$EI\upsilon_{(l_3)}^{2-3}=0$
10	$EI\varphi_{(0)}^{2-3}$	10	$EI\varphi_{(l_3)}^{2-3}=EI\varphi_{(0)}^{3-4}$
11	$M_{(0)}^{2-3}$	11	$M_{(l_3)}^{2-3}=M_{(0)}^{3-4}$
12	$Q_{(0)}^{2-3}$	12	$Q_{(l_3)}^{2-3}$
13	$EI\upsilon_{(0)}^{3-4}=0;\ EI\varphi_{(l_4)}^{3-4}$	13	$EI\upsilon_{(l_4)}^{3-4}$
14	$EI\varphi_{(0)}^{3-4}$	14	$EI\varphi_{(l_4)}^{3-4}$
15	$M_{(0)}^{3-4}$	15	$M_{(l_4)}^{3-4}=0$
16	$Q_{(0)}^{3-4}$	16	$Q_{(l_4)}^{3-4}=0$

$$A_* = \qquad (8)$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1		A_{12}		$-A_{14}$												
2		A_{11}		$-A_{13}$		-1										
3		$-A_{21}$		A_{23}			-1									
4	-1	$-A_{31}$		A_{33}												
5						A_{12}	$-A_{13}$	$-A_{14}$								
6						A_{11}	$-A_{23}$	$-A_{13}$		-1						
7						$-A_{21}$	A_{33}	A_{23}			-1					
8			-1			$-A_{31}$	A_{43}	A_{33}								
9										A_{12}	$-A_{13}$	$-A_{14}$				
10										A_{11}	$-A_{23}$	$-A_{13}$		-1		
11										$-A_{21}$	A_{33}	A_{23}			-1	
12					-1					$-A_{31}$	A_{43}	A_{33}				
13									-1					A_{12}	$-A_{13}$	$-A_{14}$
14													-1	A_{11}	$-A_{23}$	$-A_{13}$
15														$-A_{21}$	A_{33}	A_{23}
16														$-A_{31}$	A_{43}	A_{33}

For the case of applying force of F according to Fig. 2, b, there will be added next elements to the matrix

$$A_*(15,9) = -\frac{F}{EI};\; A_*(16,13) = -\frac{F}{EI}. \qquad (9)$$

Let us review a plane frame, which studdings 4 times longer than cross-bars with the same masses of *m* and rigidity of *EI* (Fig.3). Simularly, when building matrices with initial and finite data and considering boundary data and frame topology, we will come to a dynamic stability matrix (10) for the frame in Fig. 4, *a*. Calculation of inertia force of linear movable bars 0-1 and 1-2 leads to the increase of the bars 1-3 and 4-2 distributed mass, i.e. m_*^{1-3} and m_*^{4-2} [3].

$$\boldsymbol{A}_* = \qquad (10)$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1		B_{12}		$-B_{14}$																	
2		B_{11}		$-B_{13}$													-1				
3		$-\lambda^4 B_{14}$		B_{12}				1										-1			0-1
4		$-\lambda^4 B_{13}$		B_{11}					-1											-1	
5										-1									1		
6							B_{12}	$-B_{13}$	$-B_{14}$												
7							B_{11}	$-B_{12}$	$-B_{13}$												
8	-1						$-\lambda^4 B_{14}$	B_{11}	B_{12}												2-1
9			-1				$-\lambda^4 B_{13}$	$\lambda^4 B_{14}$	B_{11}												
10					-1					1											
11													$-B_{13}$	$-B_{14}$		1					
12							-1						$-B_{12}$	$-B_{13}$							
13								-1					B_{11}	B_{12}							4-2
14										1			$\lambda^4 B_{14}$	B_{11}							
15									-1						1						
16																A_{11}	A_{12}	$-A_{13}$	$-A_{14}$		
17																A_{21}	A_{11}	$-A_{23}$	$-A_{13}$		
18						-1										$-A_{31}$	$-A_{21}$	A_{33}	A_{23}		1-3
19											-1					$-A_{41}$	$-A_{31}$	A_{43}	A_{33}		
20												-1								1	

Here we should use A.N. Krylov functions from the bars that are free from compressing forces:

$$\lambda = \sqrt[4]{\frac{\omega^2 m}{EI}};\quad B_{11} = \frac{\mathrm{ch}\lambda x + \cos\lambda x}{2};$$

$$B_{12} = \frac{\mathrm{sh}\lambda x + \sin\lambda x}{2\lambda}; \qquad (11)$$

$$B_{13} = \frac{\mathrm{ch}\lambda x - \cos\lambda x}{2\lambda^2};\quad B_{14} = \frac{\mathrm{sh}\lambda x - \sin\lambda x}{2\lambda^3};$$

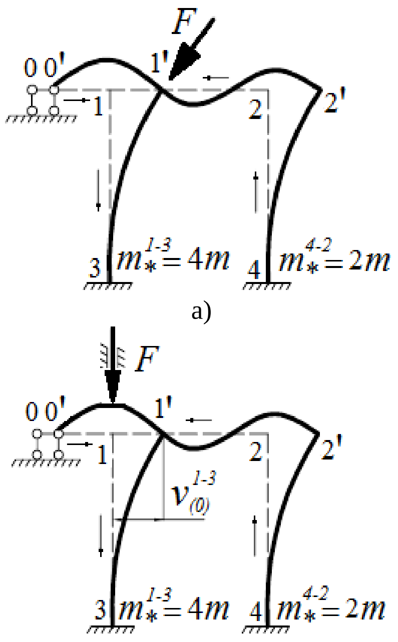


Figure 3. Plane frame: a) force of F follows up the assembly rotation angle 1; b) force of F has a fixed action line.

As in the case with continuous beam, force of F consideration in accordance with Fig. 3, в leads to extension of elements:

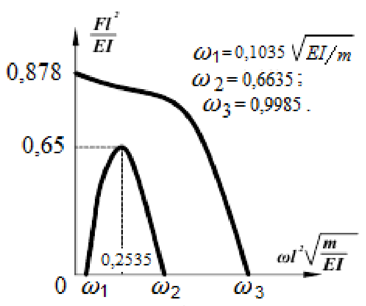
$$A_*(3{,}16) = -\frac{F}{EI};\ A_*(5{,}17) = -\frac{F}{EI}.$$

4. RESULTS

Diagrams representing the changes in frequencies of beam free oscillations during force of F increase are provided in Fig. 4.

The calculations are made in the MATLAB environment at $m = EI = l = 1$ [4]. From the diagrams in Fig.4 it follows that two types of compressing forces lead to the same behaviour of continuous beam. First ensues buckling mode due to non-conservative forces (flatter or divergence) and then, if the beam is not destroyed, comes Euler's buckling mode. Critical non-conservative forces are 0,65/0,06468=10 times higher than the first critical force of beam compared to Gallilei's "dead" force.

The relationships between compressing forces and frequencies of free oscillations are provided in Fig.5 (the calculations are made in the MATLAB environment at $m = EI = l = 1$[4]).



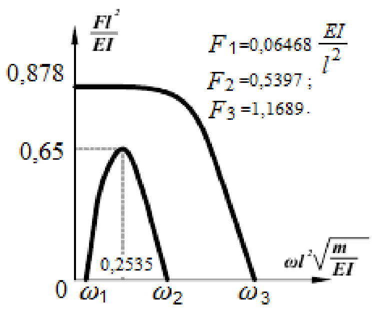
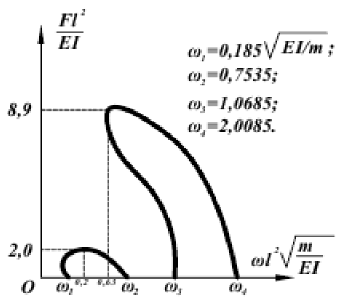


Figure 4. Dependencies of the beam oscillation frequency from compressing forces:
a) - force follows the slope of the elastic curve;
b) force has a fixed line of action.



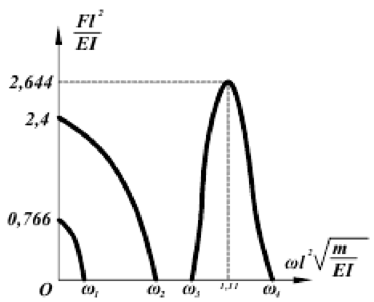


Figure 5. Dependencies of the frame oscillation frequency from compressing forces: a) force follows up the assembly rotation angle 1;
b) force has a fixed line of action.

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