Error Analysis of Inertial Navigation Systems Using Test Algorithms

Analiza greške inercijskih navigacijskih sustava pomoću test algoritama

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Summary

Content of this contribution is an issue of inertial sensors errors, specification of inertial measurement units and generating of test signals for Inertial Navigation System (INS). Given the different levels of navigation tasks, part of this contribution is comparison of the actual types of Inertial Measurement Units. Considering this comparison, there is proposed the way of solving inertial sensors errors and their modelling for low – cost inertial navigation applications. The last part is focused on mathematical testing and simulations of the inertial navigation. Given issue is a partial result, which is part of dissertation thesis Integration architectures of navigation systems.

KEY WORDS
sensor error
inertial navigation system
simulation

Sažetak

Tema ovog rada je problem grešaka inercijskih senzora, specifikacija inercijskih mjernih jedinica i generiranje test signala za intercijski navigacijski sustav (INS). S obzirom na različite razine navigacijskih zadataka, jedan dio ovog rada bavi se usporedbom postojećih tipova inercijskih mjernih jedinica. Imajući u vidu ovu usporedbu predložen je način rješavanja grešaka inercijskih senzora i njihovog modeliranja kako bi se omogućila primjena jeftinije inercijske navigacije.

KLJUČNE RIJEČI
greška senzora
intercijski navigacijski sustav
simulacija

INTRODUCTION

The issue of the objects movement and the accuracy of the positioning is still current scientific and technical field. Independently on type of the moving object, there are many systems that allow to determine the position and other navigational parameters. The principle are basic physical and mathematical procedures, which create a technical system operating in real time and perform a specific navigation task. The content of this contribution is the issue of inertial navigation, which is based on the first and second Newton´s laws. Autonomy of the inertial navigation system is one of the basic advantages of application, as well as great possibility of integration with other navigation systems. Inertial navigation is constantly being developed, particulary in the aviation, marine and military applications. Structure of this contribution closely describes inertial measurement unit IMU, which is source of information for subsequent complex navigation calculations.

ERRORS OF INERTIAL SENSORS

The INS is characterized by suitable dynamic of navigation information and relatively small errors in determination of navigation parameters during short time period. This characteristic is directly dependent on accuracy and characteristics of the inertial unit. Sensor error is generally its inherent feature. This property is result of production inaccuracies, effects of the environment and other factors. For accurate function of the inertial system, which is based on integration of sensor output, accurate sensors should be applied. However, the accuracy is dependent on the price of the sensor measurement unit, which may be undesirable for some applications. Sensor applications with lower accuracy are determined by an accurate and long-term analysis of the sensor. More about sensor accuracy and errors in [1][2].

This contribution in terms of analysis describes error approximation using stochastic precesses, or completly describes sensor by means of mathematical relationships. In terms of stochastic processes, the main task of description and error modeling is to include funfamental aspects of error in particular model. Description of stochastic processes dealing with authors in publications [1] [2]. In this case, when the main problem is to determine error characteristics of low – cost sensors, the main error sources are bias and noise [2]. Long-term measurements reveal the behavior of mentioned phenomena and their possible long-term temporal variations. From this perspective is suitable to apply generated white noise and stochastic processes to model of sensor output, thereby achieving approximation of the slow phenomena. Slow phenomena can model also small changes in the constant value of sensor bias. Gauss – Markov process of first order or random walk are stochastic processes, which are used for slow phenomena modeling. The resulting error model consists of constant bias, noise part and part of long – term temporal variations. Mathematical descriptions of the noise model elements are as follows [2]:

$$\dot{x} = w \tag{1}$$

$$\dot{x} = -\frac{1}{T}x + w \tag{2}$$

In relations (1) and (2) performs variable *x* and its time derivation. In the first case (1), *x* defines the change description of random walk based on the current value of white noise *w* [2]. Second case (2) defines Gauss- Markov process of first order. For modelling of Gauss – Markov process is necessary to apply white noise *w* and to know correlation time *T*. More accurate description of the accelerometer error (3) or gyroscope error (4) is based on the following equations [3].

$$\delta f^b = N_f f^b + S_f f^b + b + \mu_f + w_f \tag{3}$$

$$\delta \omega_{ib}^b = N_\omega \omega_{ib}^b + S_\omega \omega_{ib}^b + d + \mu_\omega + w_\omega \tag{4}$$

In relations (3) and (4) are used following variables:
N – nonorthogonality matrix
b, d – sensor biases
w – sensor noise
S – Scale factor matrix
μ – other errors

These relationships are important for a comprehensive description of the sensor. The models include deterministic and stochastic components of sensor errors.

INERTIAL MEASUREMENT UNITS

For navigation applications is essential analysis of the unit based on long-term measurements. Long-term measurements allow us to create better models of stochastic components of sensor faults and thus have a positive impact on the accuracy of the task being performed. The most common tool for the analysis of stochastic components is Allan variance and power spectral density (PSD). Another important aspect in terms of choice and application of specific sensor unit is estimated rate of movement of the subject and integration with support systems (GNSS etc.) [9]. Following part is an example of IMU separation, which is based on features of sensor unit.

According to Table 1., IMU nAX5 is described by following parameters.

Table 2 IMU nAX5

Properties	Gyroscope	Accelerometer
In Run Bias	0,007 °/s	0,2 mg
Range	± 350 °/s	± 18 g
Random Walk	2 °/sqrt(h)	0,2 m/s/sqrt(h)

The information in Table 2 with respect to the information set out in Table 1 categorize described IMU unit in class MEMS, as confirmed by the used source [5]. The analysis of the sensor is often based on application of the Allan variance or power spectral density. For better illustration is, based on the information in [5], provided an overview of the results of Allan variance. The results [5] are based on 10 hour measurement of static data using sampling frequency of 10 Hz. Standard, which is the Allan variance is based on the IEEE and allows us to analyze seven types of noise. Careful analysis based on Allan variance is based on a large number of data obtained at specified and immutable conditions. Given the extent of the contribution is shown the result for the gyroscope of unit nAX5.

Table 1 IMU characteristics based on [4]

Sensor	Accelerometer			Gyroscope		
Grade of IMU	Navigation	Tactical	MEMS	Navigation	Tactical	MEMS
In Run Bias	0,025 mg	1 mg	2,5 mg	0,0022 °/h	1 °/h	1040 °/h
Turn On Bias	-	-	30	-	-	5400 °/h
Scale Factor	100 PPM	300 PPM	10000 PPM	5 PPM	150 PPM	10000 PPM
Approximate Cost	>$90000	>$20000	<$2000	>$90000	>$20000	<$2000

Table 3 Allan variance for nAX5 gyroscope (values express slope-lines)

Noise Term	X - axis	Y - axis	Z - axis
Quantization Noise (rad)	0,00003035	0,00003436	0,00002024
Angular Random Walk (rad/sqrt(s))	0,002087	0,002073	0,002039
Bias Instability (rad/s)	0,000226	0,000191	0,000183
Angular Rate Random Walk (rad/s/sqrt(s))	0,000007059	0,000002178	0,000009021
Rate Ramp (rad/s2)	0,00000008735	0,00000001825	0,0000001182

TEST SIGNALS FOR INERTIAL NAVIGATION SYSTEM

The sensor outputs corrupted with noise, as well as mathematical models of sensors, are the beginning of INS navigation process. Inertial navigation model for the moving object is necessary to test considering more accurate reference inertial measurement unit. Mathematical simulation can be tested not only on the basis of real reference data but also using the generated input signals. If the equation for calculating the rate of velocity change in the navigation reference frame has following form [6] [8]:

$$f^n = C_b^n f^b - (\Omega_{en}^n + 2\Omega_{ie}^n)v^n - g^n \quad (5)$$

Then, for the generation of accelerations at the given input conditions is applied modified navigation equation in following form:

$$f^b = C_n^b(f^n + (\Omega_{en}^n + 2\Omega_{ie}^n)v^n + g^n) \quad (6)$$

Those relationships (5), (6) are based on knowledge and modeling of the following parameters:

C_b^n – tranformation matrix from body (b) to navigation (n) reference frame

C_n^b – inverse transform matrix of C_b^n

f^b – vector of specific forces expressed in body frame

f^n – vector of specific forces expressed in navigation referenceframe

Ω_{en}^n – skew – symmetric matrix of angular rates between navigation (n) and earth (e) reference frame expressed innavigation reference frame

Ω_{ie}^n – skew – symmetric matrix of angular rates between earth (e) and inertial (i) reference frame expressed in navigation reference frame

v^n – velocity expressed in navigation reference frame

g^n – gravity vector expressed in navigation reference frame

The basis for the modeling of complex signals is based not only on acceleration (specific forces), but also on model of the gyro output signal. It is according to [6] based on the following formula:

$$\Omega_{ib}^b = C_n^b(\Omega_{ie}^n + \Omega_{en}^n + \Omega_{nb}^n) \quad (7)$$

Ω_{ib}^b – skew – symmetric matrix of angular rates between body (b) and inertial (i) reference frame expressed in body reference frame

Ω_{nb}^n – skew – symmetric matrix of angular rates between body (b) and navigation (n) reference frame expressed in navigation reference frame

These equations are the basic idea for solution of inertial signals generator. It is important to note that this generated signal is noise - free. System errors in the case of application have only numerical origin. Benefit of this process is directed to the possibility of testing any noise model simply by adding noise to the modeled output [7] [10]. Following figures represent generated outputs of accelerometers and gyroscopes to simulate the test movement.

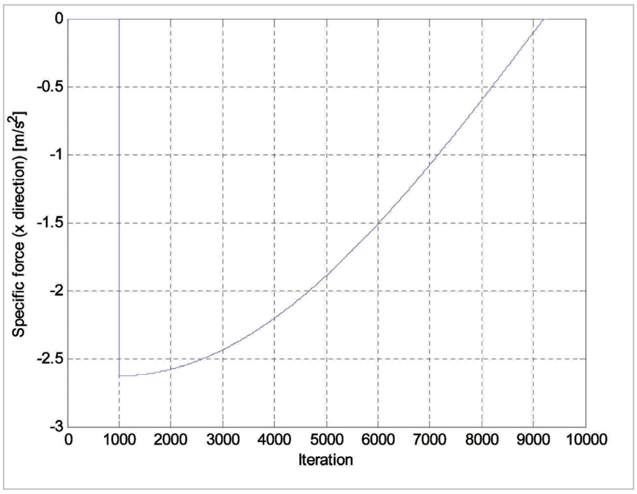

Figure 1 Generated specific force (x – direction)

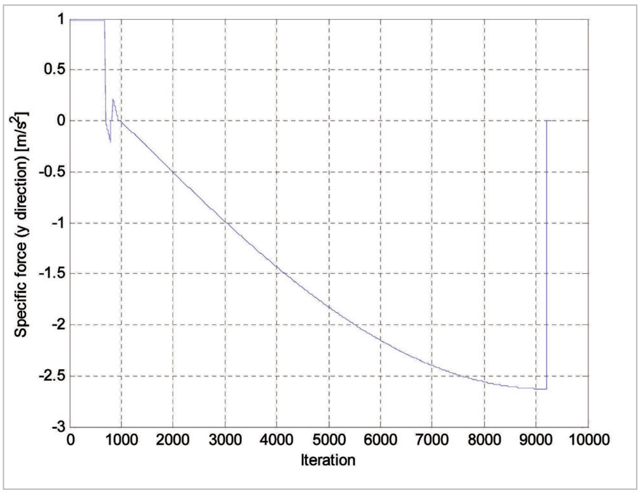

Figure 2 Generated specific force (y – direction)

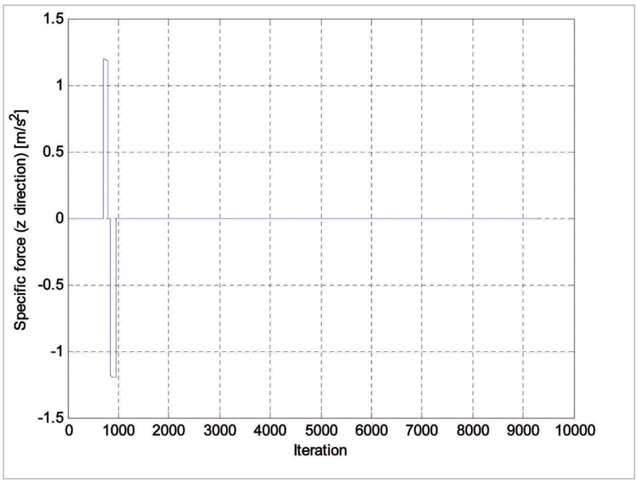

Figure 3 Generated specific force (z – direction)

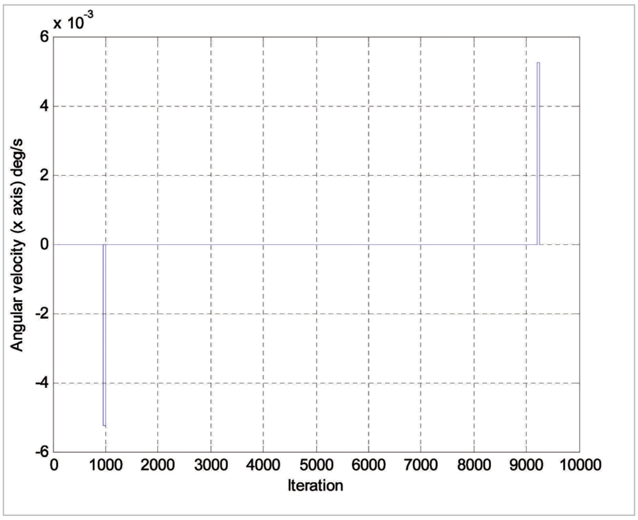

Figure 4 Generated angular rate (x – rotation)

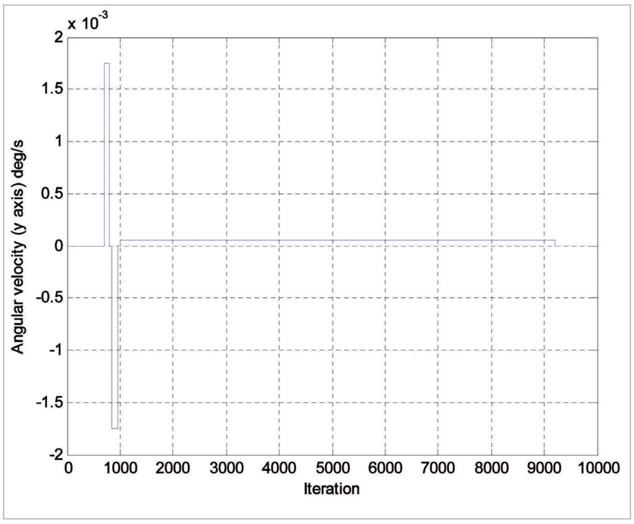

Figure 5 Generated angular rate (y – rotation)

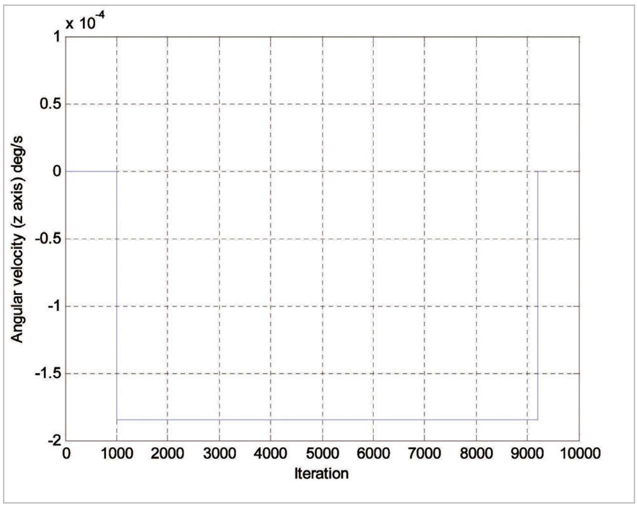

Figure 6 Generated angular rate (z – rotation)

Outputs and their changes are specified by the user, as well as the beginning of the navigation task and changes of attitude angles. The outputs of the gyroscopes are decisive for determining the position angles and compile the transformation matrix between body and navigation reference frame. Simulation using the ideal signal was performed on the basis of sensor outputs generated with the orientation axes ENU (East - North - Up). This generated movement can replace comparison with reference IMU data. For simulation requirements of the navigation algorithm is also generated time vector. This is derived in terms of the number of cycles and is itself essential to the discrete time operation in the tested model.

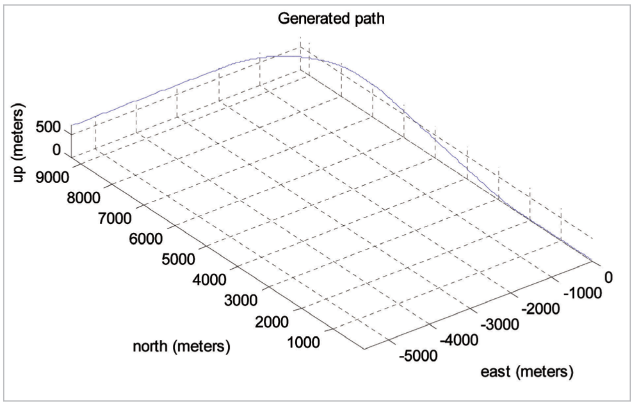

Figure 7 Generated path based on generated signals of accelerometers and gyroscopes

CONCLUSION

Contribution titled Error Analysis of Inertial Navigation Systems Using Test Algorithms represents a closer look at the error and system tests of INS. Errors are simply represented by two basic ways for sensors, which accuracy is significantly different. Modeling errors and error analysis include a complex issue, essential for the proper function of simulation and real implementation of navigation systems. INS system testing in this case is contained in the program draft to generate the ideal output of accelerometers and gyroscopes. Corresponding outputs are shown in this article. Generation of trajectory based on modeled output of the sensors represents a reference against which the output is compared when INS model also include error model of sensors. Separation of inertial measurement units better approximates the potential ranges of modeled errors, which should be included in error model of given INS simulation. Further development of this issue would lead by modeling of errors and their application to the input of the navigation system. Comparison of generated navigation outputs with the reference trajectory provides a graphical overview of the impact of sensor errors. This type of testing, which is based on modeled sensor output and sensor error models, is suitable for all types of moving objects.

REFERENCES

[1] E. H. Shin, "Accuracy Improvement of Low Cost INS/GPS for Land Applications", University of Calgary 2001.

[2] E. H. Shin, "Estimation Techniques for Low – Cost Inertial Navigation", University of Calgary 2005.

[3] M. Sotak, M. Sopata, R. Breda, J. Rohac, L. Vaci, "Navigation system integration", *Monograph*, ISBN 80 – 969619 – 9 – 3, Kosice 2006.

[4] A. Angrisano, "GNSS/INS Integration Methods", Dipartimento di Scienze Applicate, Universita Degli Studi Di Napoli, 2010.

[5] L. Yiang, Y. Li, Y. Wu, C.Rizos, "An enhanced MEMS – INS/GNSS integrated system with Fault Detection and Exclusion (FDE) capability for land vehicle navigation in urban areas", University of New South Wales, Sydney, Australia.

[6] R. Christensen, N. Fogh, "Inertial Navigation System", Department of Control Engineering, Aalborg University 2008.

[7] D. H. Titterton, J. L. Weston, "Strapdown Inertial Navigation Technology – 2nd Edition", 2004 The Institution of Electrical Engineers, ISBN 0 86341 358 7.

[8] P. D. Groves, "Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems", Artech House 2008, ISBN – 13: 978 – 1 – 58053 – 255 – 6.

[9] J. Rohac, M. Sipos, J. Simanek, O. Teren, "Inertial Reference Unit in a Directional Gyro Mode of Operation" In IEEE SENSORS 2012 – Proceedings. Piscataway: IEEE Service Center, 2012, p. 1356 – 1359. ISBN 978 – 1 – 4577 – 1765 – 9.

[10] T. Hirner, P. Farkas, S. Kirle, "One Unequal Error Control Method For Telemetric Data Transmission", Journal of Electrical Engineering (JEEEC), Bratislava, Vol. 62, No 3, pp. 166 – 170., 2011. (ISSN 1335 – 3632).