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AN ENHANCED EQUATION FOR VIBRATION PREDICTION OF NEW TYPES OF SHIPS

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Summary

A simplified approach developed to evaluate the vibration levels of complex structures such as passenger and similar ships with large shell and deck openings and extended superstructures is here presented. The final objective is to give an useful tool to ship designers, to establish since the first design stage the dynamic response of the ship with sufficient precision.

This approach is based on the assumption that the ship hull can be represented as a non uniform section beam. The propeller excitations in terms of pressure pulses and shaft line moments and forces are introduced. To take into account this exciting source in the early design stage a statistical formula for dynamic excitation of propeller was developed. Furthermore the superimposition of local effects has been performed with the use of an analytical formula. The local effect due to the different space topologies such as cabins, public spaces, technical and machinery areas has been taken into account. The transversal beams, longitudinal girders, stiffeners and pillars as supported structural elements are considered in the vibration local response.

The reliability of the results obtained using the formula has been improved with more precise results obtained by FEM analysis. The calculated vibration response has been verified and compared to vibration measurements performed on board of ships.

Key words: *ship vibrations; non-uniform section beam; correction factor; local magnification factor; vibration measurements; error index*

1. Introduction

At the early beginning Kumai developed simple analytical formulas for the calculation of natural frequencies of merchant ships [1]. Modern Pax and Ro-Pax vessels have large shell and deck openings and extended superstructures unlike older merchant ships. The FE method was developed for the design of ships and at the same time classification societies developed FE analysis procedures for static and dynamic investigation [2-9]. Thus these formulas nowadays are not useful for passenger and Ro-Ro passenger ships. In addition the owners and the Classification societies reduced vibration limits defined into the Comfort Class requirements [10-15]. At the early design stage there is not enough time and technical data to develop a FEM

Model. On the other side eventual errors in dynamic design introduce high costs on the total construction [16]. Due to this it has been decided to develop a new formula for ship global and local vibration response prediction.

The objective of this investigation is to present and prepare the data for the concept design phase where the most significant decisions are made regarding the ship dynamic design. It is important to take the most important decisions in early design stages because the earlier a problem is addressed, the easier it is to implement changes, and less costly it is to resolve. On the contrary, if most of the changes happen in late design stages, the cost of making changes will dramatically increase since design freedom is highly limited in these stages [16].

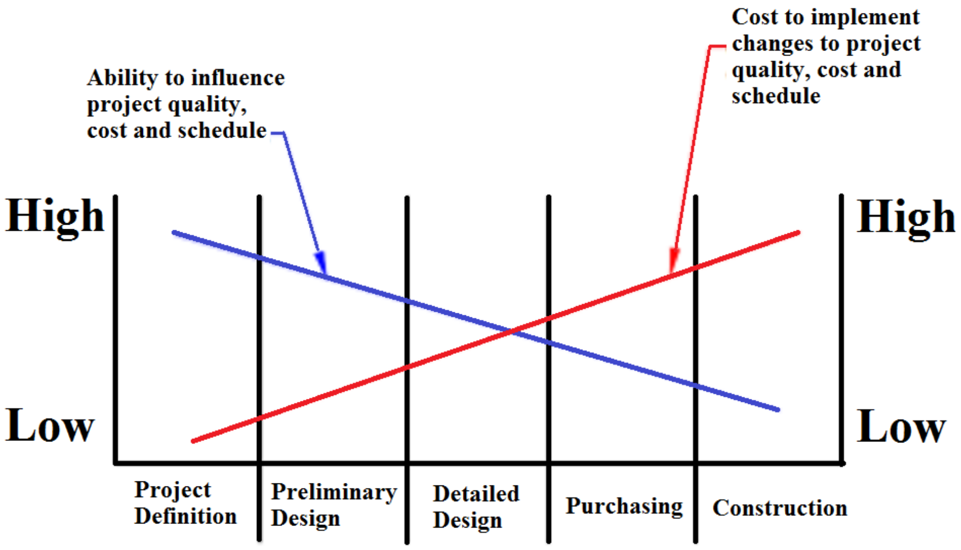

Figure 1-1 Cost Influence [16]

Furthermore this problem is complex and time consuming, this is in contrast with low available time for the basic–concept structural design. This means that huge amount of data required in the 3D FEM models cannot be generated and are not available. Concept design is the phase in structural design when geometry typology and shapes are subject to modification and structural options are investigated in accordance to layout and design [17,18].

The vibration problem is significant for multi-deck ships, passenger cruise ships, RO-RO pax ships, mega yachts with the extended, multi-deck superstructures. Definition of the adequate scantling is also very important for the dynamic design that is correlated to structural weight, achievable clearances regarding heights of the webs; transversal beams longitudinal girders distribution and pillars layout.

A simplified approach to dynamic prediction design is the basic objective of the method here presented, to give acceptable results during the basic design phase. The simplified approach has been developed for evaluating the vibration levels of complex structures such as passenger and similar ships with large shell and deck openings, and large primary structure. The final objective is to give an useful tool to ship designers, to establish in the first design stage the dynamic ship response with sufficient precision.

2. Generalized formulation for a non-uniform beam

It is known that the ship section vary with length. Different authors solved the problem for cargo ships with direct partial differential equation solution [19-21]. The idea to use in any way the uniform section beam theory and expand it for a non-uniform hull ship section has been here developed. The ship was divided in ten sections along her length in accordance with recommended practice from Naval Literature [1]. A regression of the dimensionless area, inertia data, mass and added mass of various ships has been carried out. The cross-section area and inertia data used for the regression refer to transversal sections of ships whose sectional properties had been calculated. Mass distributions are taken from the author's database.

The solution of a two stepped beam can be viewed as expansion of the uniform beam as separate beams with continuity conditions applied at the joints. The basic idea developed by Koplov [22,23] for two-stepped beams has been expanded to ten-stepped beams, that has been used for a global ship dynamic response. The solution is found in terms of dynamic force and coupled excitation. The calculated results have been compared to author's experimental data obtained during sea trials.

In general for the i^{th} section of a non-uniform beam of n sections the solution can be obtained from the simpler case of a beam with two different sections that can be written in the following form:

$$V_i(x_i) = c_{1i} sin\beta_i x_i + c_{2i} cos\beta_i x_i + c_{3i} sinh\beta_i x_i + c_{4i} cosh\beta_i x_i \quad (1)$$
$$\forall\, i = 1, \dots n$$

where $V_i(x_i)$ is the general mode shape solution, $c_{1i}, c_{2i}, c_{3i}, c_{4i}$ are constants determined by suitable boundary conditions and where:

$$\beta_i^4 = \frac{\omega^2 \rho_i A_i}{E_i I_i (1 + i\eta)} \quad (2)$$

The boundary condition, considering the dynamic excitation F pulsing-oscillating at frequency ω and applied at the free end of the beam belonging to the 1st section, is:

$$\frac{\partial^2 v_1(0)}{\partial x_1^2} = 0 \quad (3)$$

$$E_1 I_1 \frac{\partial^3 v_1(0)}{\partial x_1^3} = -F \sin \omega t \quad (4)$$

The continuity conditions between two adjacent sections of the beam i and $i+1$ are:

$$v_i(L_i) = v_{i+1}(0) \quad (5)$$

$$\frac{dv_i(L_i)}{dx_i} = \frac{dv_{i+1}(0)}{dx_{i+1}} \quad (6)$$

$$E_i I_i \frac{d^2 v_i(L_i)}{dx_i^2} = E_{i+1} I_{i+1} \frac{d^2 v_{i+1}(0)}{dx_{i+1}^2} \quad (7)$$

$$E_i I_i \frac{d^3 v_i(L_i)}{dx_i^3} = E_{i+1} I_{i+1} \frac{d^3 v_{i+1}(0)}{dx_{i+1}^3} \quad (8)$$

The free boundary condition at the free beam end belonging to the last n^{th} section is:

$$\frac{\partial^2 v_n(L_n)}{\partial x_n^2} = 0 \quad (9)$$

$$E_n I_n \frac{\partial^3 v_n(L_n)}{\partial x_n^3} = 0 \quad (10)$$

With reference to the previous relationships it has been decided to search the solution for a linear algebraic system of equations. This leads to a solvable linear algebraic system composed of:

- nx4 unknowns c_{ki}; k=1,2,3,4; i=1,…,n
- (n-1)x4 continuity equations
- 2+2=4 free-free boundary conditions

Further steps in this approach are based on the fundamental procedure just explained. Based on these formulae the ship structure sections were discretised in 10 parts, each with its sectional properties. This way the ship's global dynamic frequency response is obtained. Following the methodology of computing hull sectional properties in terms of area, inertia and mass distribution is discussed.

Similarly, the procedure for the generalized equation for moment has been developed. The free boundary condition at the free beam belonging to the 1st section is:

$$\frac{\partial^3 v_1(0)}{\partial x_1^3} = 0 \quad (11)$$

$$E_1 I_1 \frac{\partial^2 v_1(0)}{\partial x_1^2} = M \sin \omega t \quad (12)$$

The continuity conditions between two adjacent sections of the beam i and $i+1$ are:

$$v_i(L_i) = v_{i+1}(0) \quad (13)$$

$$\frac{\partial v_i(L_i)}{\partial x_i} = \frac{\partial v_{i+1}(0)}{\partial x_{i+1}} \quad (14)$$

$$E_i I_i \frac{\partial^3 v_i(L_i)}{\partial x_i^3} = E_{i+1} I_{i+1} \frac{\partial^3 v_{i+1}(0)}{\partial x_{i+1}^3} \quad (15)$$

$$E_i I_i \frac{\partial^2 v_i(L_i)}{\partial x_i^2} = E_{i+1} I_{i+1} \frac{\partial^2 v_{i+1}(0)}{\partial x_{i+1}^3} \quad (16)$$

The free boundary condition at the free beam end belonging to the n^{th} section i.

$$\frac{\partial^2 v_n(L_n)}{\partial x_n^2} = \frac{\partial^3 v_n(L_n)}{\partial x_n^3} = 0 \quad (17)$$

3. Cross Section Area Inertia and Mass Distribution

The new developed formula is based on available data of ships investigated by the author. For Ro/Ro Pax, Pax passenger ships the significant section and mass distribution data are mathematically calculated. The cross-section area, inertia and mass distribution are used in the next chapters as parameterized input for the new formula.. More precisely $I(x)$, $\rho(x)$ and $A(x)$ are represented by fourth-order polynomial regression for geometrical properties and fifth-order for cross section properties. In general the section area, inertia and mass can be written in the following polynomial expressions [24]:

Section Area

$$A(x) = \sum_{k=0}^{4} a_k x^k \quad (18)$$

Section Inertia
$$I(x) = \sum_{k=0}^{4} i_k x^k \quad (19)$$

Section Mass
$$\rho_m(x) = \sum_{k=0}^{5} r_k x^k \quad (20)$$

3.1 Midship Section Characteristics for Analysed ship

During the last twenty years the author performed dynamic analysis of several passenger and RO-RO passenger ships. The hull section characteristics have been calculated based on midship section drawings where scantlings are represented. These data are used for the mathematical processing. Table 3-1 presents the calculated midship section data for each ship analysed. The drawings are shown in Figure 3-1.

Table 3-1 Midship general data

Significant Ships	**SECTION**	**L [m]**	**B[m]**	**T[m]**	**Δ [t]**	**I_z [m^4]**	**A [m^2]**
Ship 1	Midship	254.8	30.8	7.3	29741	421.57	4.37
Ship 2	Midship	185.1	24.7	6.7	16480	219.41	3.61
Ship 3	Midship	280.8	32.2	7.8	43423	486.20	5.71
Ship 4	Midship	203.0	30.4	7.8	27650	472.85	4.67

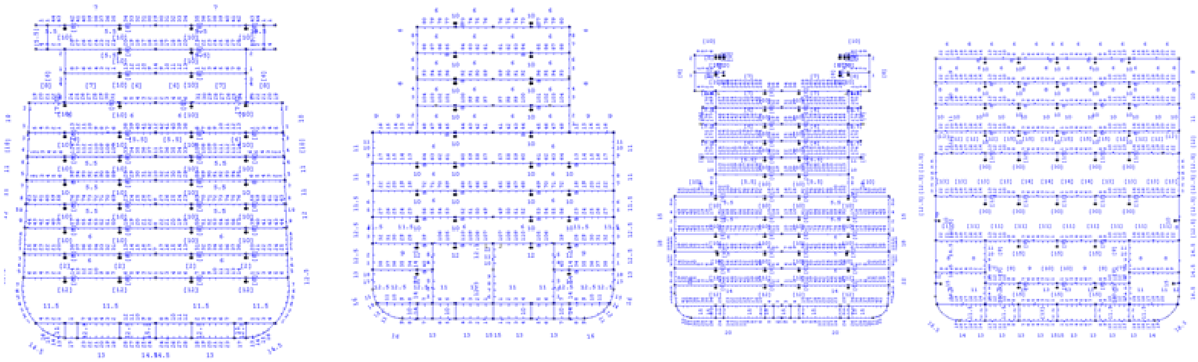

Figure 3-1 In order from left to right : Ship 1, Ship 2, Ship 3 and Ship 4 midship section

4. Calculation of Hull Cross Sectional data and Weight Distribution

It has been decided, in agreement to Naval Architecture good practice, to divide the ship hull length in ten sections. Not only in midship, but also aft and fore section, areas, inertias and mass distribution have been evaluated. The fifth order polynomial data regression is used in full load conditions based on the weight distribution data of three ships at different sections. The result can be seen on Figure 4-1.

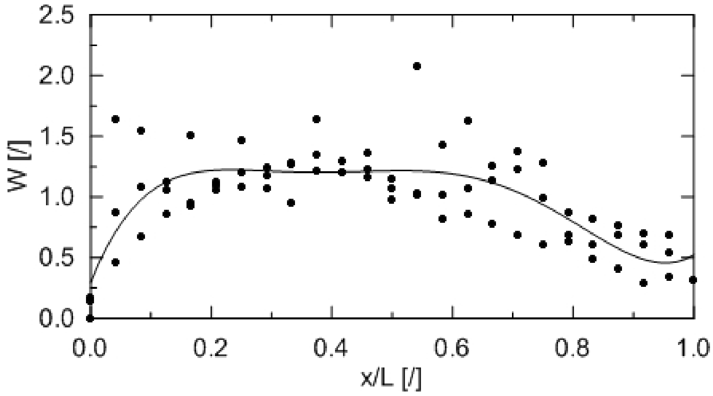

Figure 4-1 Weight distribution regression

where W[/] is the dimensionless weight:

$$W[/] = \frac{W_L}{\Delta/L} \qquad \left[\frac{t}{m} \cdot \frac{1}{t} \cdot m\right] \tag{21}$$

where W_L is the total weight per unit lenght.

The polynomial regresion is:

$$y = 30.07\,x^5 - 86.96x^4 + 91.77\,x^3 - 45.53x^2 + 10.54x + 0.33 \tag{22}$$

It has been decided to parameterize the cross section area and the moment of inertia to midship section value. The area has been represented as non-dimensional value through the following formula:

$$A^{'} = \frac{A(x)}{A_M} \tag{23}$$

with A_M is the cross section area of the midship section.

The dimensionless moment of inertia I_z' has been calculated with the following formula.

$$I_z^{'} = \frac{I_z(x)}{I_{zM}} \tag{24}$$

where I_{zM} is the moment of inertia of the midship section.

In Figure 4-2 and in Figure 4-3 the polynomial regression of the obtained data for five ships at different sections, about cross section Area A' and about moment of inertia I'_z.

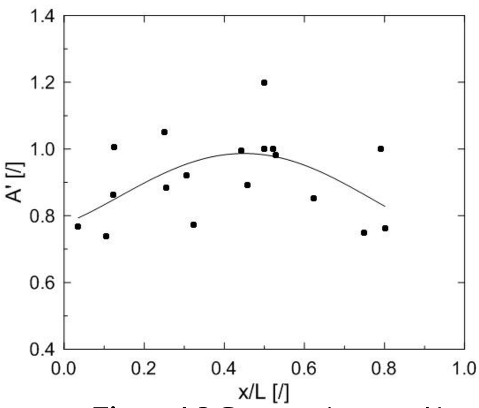

Figure 4-2 Cross section area A'

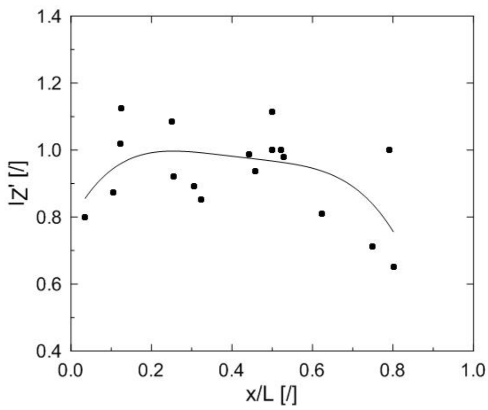

Figure 4-3 Moment of Inertia I'z

In particular the polynomial regression formula are:

$$A_i^{'} = 2.68x^4 - 4.86x^3 + 1.69x^2 + 0.45x + 0.78 \tag{25}$$

$$I_{z_i}^{'} = -8.31x^4 + 13.96x^3 - 8.77x^2 + 2.30x + 0.79 \tag{26}$$

4.1 Added Mass and Vertical Motion

For a circular cylinder in two-dimensional flow, inviscid fluid theory shows, that the added mass equals the mass of the cylinder if the cylinder has the density of the fluid. For a sphere it is one half the mass of the sphere if the sphere has the density of the fluid [25,26].

A vibrating ship acts as if the added mass, varying along its length, were attached. The magnitude of this added mass is a function of the ship underwater shape and of the mode of the vibration. The added mass per unit length is then given by:

$$A.M. = 0.5J\rho\pi b_L^2 C_V \tag{27}$$

where:

ρ = density of the fluid

C_V = added weight vertical coefficient

J = coefficient for 3-dimensional effects

b_L = half-beam at the waterline for the section being considered

For the ships under investigation the amount of added mass has been calculated and parametrized. The regression analysis used in this case is the third order polynomial.

Similarly to what has been carried out for the cross section characteristics and weight distribution on board, also the added mass has been calculated on the ship length. The following formula has been developed for the added mass parameterized to displacement and length:

$$A.M._{adimensional}\,[-] = A.M.\,\left(\frac{L}{\Delta \cdot 1000}\right) \qquad \left[\frac{kg}{m}\cdot\frac{1}{t}\cdot m\right] \tag{28}$$

Figure 4-4 shows the added mass for eight ships as a function of the ratio *x/L* for the combined and local effect mode chosen.

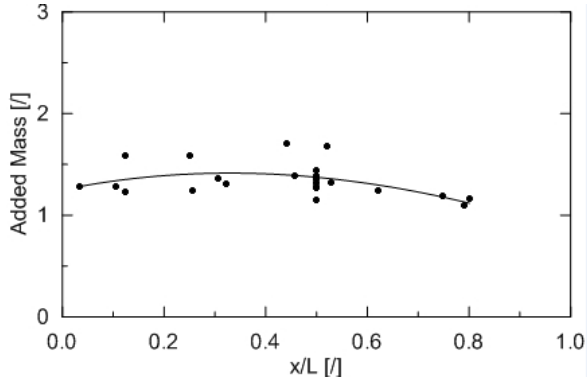

Figure 4-4 Added mass-combined global and local effect mode

The polynomial regression is:

$$y = 0.41\,x^3 - 1.85\,x^2 + 1.06\,x + 1.25 \tag{29}$$

5. Dynamic Excitation on ship

In order to perform the dynamic vibration response calculation, the propeller has been used as the dynamic excitation as it is the main source of vibrations induced to the ship. The excitations induced by the propeller are mainly divided those into the line shafting and those into the hull [27-30]. The referenced values here used have been calculated by author or obtained from model base in tests.

Diesel engine as primary source and/or Diesel generators is considered as the second source of excitations, which might be at the origin of vibrations appearing on board ship. In this paper this source will not be numerically investigated, but the excitation data are normally supplied by engine or manufacturer [31].

5.1 Propeller excitation calculation based on Regression

The non-uniform beam used to mathematically represent the ship has been excited with a dynamic force due to the propeller pressure fluctuation and shaft-line induced dynamic bending moment.

For defining the propeller excitation there are several significant parameters that can be considered. The power P [kW], the cruise speed of the ship V [kn] and displacement Δ [tonnes] have been selected. For the analysed ships the equivalent propeller excitation has been calculated based on data available to the author.

Table 5-1 Propeller Dynamic excitations

Ships	**Δ [t]**	**Power [kW]**	**Speed [kn]**	**L_{OA} [m]**	**Fn [/]**	**P/Δ**	**F [kN]**	**M [kNm]**
Ship 1	29741	21120	20	265.4	0.20	0.71	13.8	50.0
Ship 2	16480	17300	21	192.8	0.25	1.05	14.0	48.0
Ship 3	43423	67370	24	292.6	0.23	1.55	23.8	55.0
Ship 4	27650	67200	28	211.5	0.31	2.43	41.0	60.0

A regression analysis of the propeller excitation has been performed for the total vertical integrated force and shaft-line dynamic bending moment:

$$F = 2148 + 17795\,\frac{P}{\Delta} - 17755\,Fn \qquad [N] \tag{30}$$

$$M = 34940 - 1382\,\frac{P}{\Delta} + 81175\,Fn \qquad [Nm] \tag{31}$$

where P is the power, Δ the displacement, Fn the Froude number.

This way the equivalent vertical total integrated force and shaft-line bending moment have been obtained.

6. New Formula mathematical development

The new formula, based on the previously obtained data, is developed in several steps, where the total vertical vibrations response will be obtained as the product of the global, local deck's amplification and deck's position. The ship length L_R has been divided in ten sections, i=1,..,10. In the i-th ship section this form is valid:

$$\left(V_{T\,x,y,z}\right)_{k,i} = V(x)_i \;\; v_{z_{k,i}}\, v_{y_{k,i}} \qquad i = 1, \dots, 10 \quad k = 1, .., n \tag{32}$$

where n is the last deck and:

- $v_{y_{k,i}}$ is the k-th deck's local amplification factor at i-section

- $v_{z_{k,i}}$ is the variation of the vibration's with deck height
- $V(x)_i$ is the global vibration level of hull girder [mm/s]
- $\left(V_{T\,x,y,z}\right)_{k,i}$ is the total vertical vibration level [mm/s]

Here below the expression of each term of the obtained formula:

- $v_{y_{k,i}}$, the deck's local amplification factor

$$v_{y_{k,i}} = \max \delta_{s_k}(y)\frac{\omega^2}{g}A_R + 1 \quad k = 1,..,n \quad \forall\, i \tag{33}$$

where $y \in [0;\ l/2]$, and l is the beam span;

- $v_{z_{k,i}}$, the variation of the vibration's with deck height.

$$v_{z_{k,i}} = a_i z_k{}^6 + b_i z_k{}^5 + c_i z_k{}^4 + d_i z_k{}^3 + e_i z_k{}^2 + f_i z_k + g_i \tag{34}$$

$$\forall\, i \quad k = 1,..,n$$

$$\mathrm{z_k} = \frac{z_{dk}}{z_b}$$

where z_{dk} is the k-th deck height and z_b is the bottom height;

- $V(x) = V_G(x) \cdot R(x)$ is the global vibration level of hull girder [mm/s]

$$V_{G(x)_i} = \max_j \left[\begin{array}{l} a_{1+4i}\sin\left(\beta_{i+1}\, j\, \dfrac{L_R}{m \cdot n}\right) + a_{2+4i}\cos\left(\beta_{i+1}\, j\, \dfrac{L_R}{m \cdot n}\right) + \\ +a_{3+4i}\sinh\left(\beta_{i+1}\, j\, \dfrac{L_R}{m \cdot n}\right) + a_{4+4i}\cosh\left(\beta_{i+1}\, j\, \dfrac{L_R}{m \cdot n}\right) \end{array} \right] \tag{35}$$

$$\forall i \quad = 1,..,m$$

The ship length L_R has been divided in ten sections, i=1,..,10. Furthermore it has been decided to split each of the L_{10} section in ten parts, j=1,..,10. $R_{(x)_i}$ is a correction factor, parameterised to Rule length:

$$R(x)_i = -31.56\, x_i^4 + 61.79\, x_i^3 - 36.57\, x_i^2 + 4.95\, x_i + 1.66 \tag{36}$$

where x is the dimensionless length

The vector $\boldsymbol{a}$, in the expression of $V_{(x)_i}$, is the result of the product between matrix $\boldsymbol{G}$ and the dynamic excitation vector $\boldsymbol{f}$:

$$\boldsymbol{a} = \boldsymbol{G}^{-1}\,\boldsymbol{f} \tag{37}$$

where **G** is the generalized matrix and **f** the excitation vector. In particular:

$$[G] = \begin{bmatrix} \boldsymbol{B}_0 & & & & & & & \\ \boldsymbol{D_1} & \boldsymbol{S_1} & & & & & & \\ & \boldsymbol{D_2} & \boldsymbol{S_2} & & & & & \\ & & \dots & \dots & & & & \\ & & & \boldsymbol{D_i} & \boldsymbol{S_i} & & & \\ & & & & \dots & \dots & & \\ & & & & & \dots & \boldsymbol{S_n} & \\ & & & & & & \boldsymbol{D_{n-1}} & \boldsymbol{B_n} \end{bmatrix} \tag{38}$$

$$\boldsymbol{B}_0 = \begin{pmatrix} 0 & -I_1\,E\,\beta_1^2 & 0 & I_1\,E\,\beta_1^2 \\ -I_1\,E\,\beta_1^3 & 0 & I_1\,E\,\beta_1^3 & 0 \end{pmatrix}$$

$$\boldsymbol{B}_n = \begin{pmatrix} -I_n\,E\,\beta_n^2 \sin(\beta_n L_R) & -I_n\,E\,\beta_n^2 \cos(\beta_n L_R) & I_n\,E\,\beta_n^2 \sinh(\beta_n L_R) & I_n\,E\,\beta_n^2 \cosh(\beta_n L_R) \\ -I_n\,E\,\beta_n^3 \cos(\beta_n L_R) & -I_n\,E\,\beta_n^3 \sin(\beta_n L_R) & -I_n\,E\,\beta_n^3 \cosh(\beta_n L_R) & -I_n\,E\,\beta_n^3 \sinh(\beta_n L_R) \end{pmatrix}$$

$$\boldsymbol{D}_i = \begin{pmatrix} \sin(\beta_{i+1} L_R) & \cos(\beta_{i+1} L_R) & \sinh(\beta_{i+1} L_R) & \cosh(\beta_{i+1} L_R) \\ \beta_{i+1}\,cos(\beta_{i+1} L_R) & -\beta_{i+1}\,sin(\beta_{i+1} L_R) & \beta_{i+1}\,cosh(\beta_{i+1} L_R) & \beta_{i+1}\,sinh(\beta_{i+1} L_R) \\ -I_{i+1}\,E\,\beta_{i+1}^2 \sin(\beta_{i+1} L_R) & -I_{i+1}\,E\,\beta_{i+1}^2 \cos(\beta_{i+1} L_R) & I_{i+1}\,E\,\beta_{i+1}^2 \sinh(\beta_{i+1} L_R) & I_{i+1}\,E\,\beta_{i+1}^2 \cosh(\beta_{i+1} L_R) \\ = -I_{i+1}\,E\,\beta_{i+1}^3 \cos(\beta_{i+1} L_R) & -I_{i+1}\,E\,\beta_{i+1}^3 \sin(\beta_{i+1} L_R) & -I_{i+1}\,E\,\beta_{i+1}^3 \cosh(\beta_{i+1} L_R) & -I_{i+1}\,E\,\beta_{i+1}^3 \sinh(\beta_{i+1} L_R) \end{pmatrix}$$

$$\boldsymbol{S}_i = \begin{pmatrix} & & -1 & & & & -1 & \\ & -\beta_{i+2} & & & & -\beta_{i+2} & & \\ & & & -I_{i+2}\,E\,\beta_{i+2}^2 & & & & -I_{i+2}\,E\,\beta_{i+2}^2 \\ -I_{i+2}\,E\,\beta_{i+2}^3 & & & & -I_{i+2}\,E\,\beta_{i+2}^3 & & & \end{pmatrix}$$

6.1 Global Vibration Level Calibration

The calculation of the global vibration level of hull girder V_G is now described. In particular the vibration displacement has been converted by multiplying by $2\pi f$ to velocity. The hull girder global vibration has to be calculated by applying a corrector factor R(x) to the obtained analytical response levels. The corrector factor $R(x)$ was derived from the ratio between analytical and results obtained in complete FEM models (Figure 6-1).

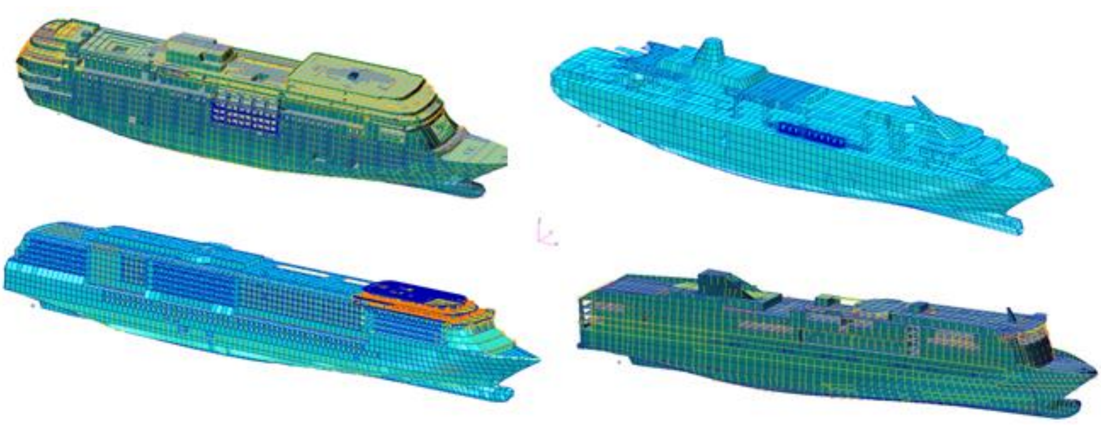

Figure 6-1 In order from left to right: Ship 1, Ship 2, Ship 3 and Ship 4Ship 3 (left) 3D FE Models

Due to the fact that the ships have been analytically discretized in ten parts, there are ten correction coefficients, one for each ship section. In Figure 6-2, Figure 6-3, Figure 6-4 and Figure 6-5 we show the FEM and corresponding analytical results.

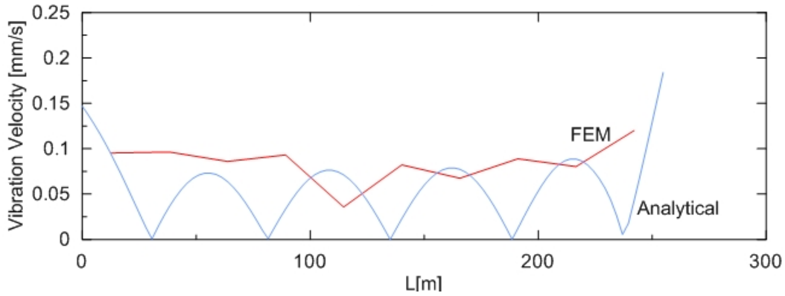

Figure 6-2 – Ship 1, Analytical-FEM velocity comparison

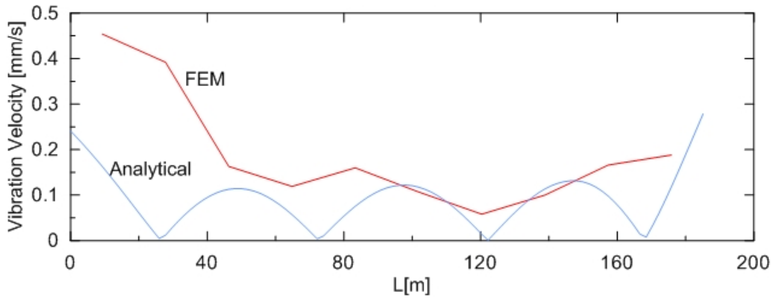

Figure 6-3 –Ship 2, Analytical-FEM velocity comparison

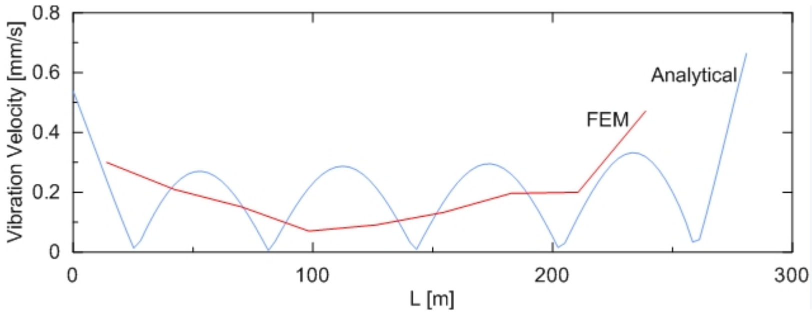

Figure 6-4 – Ship 3, Analytical-FEM velocity comparison

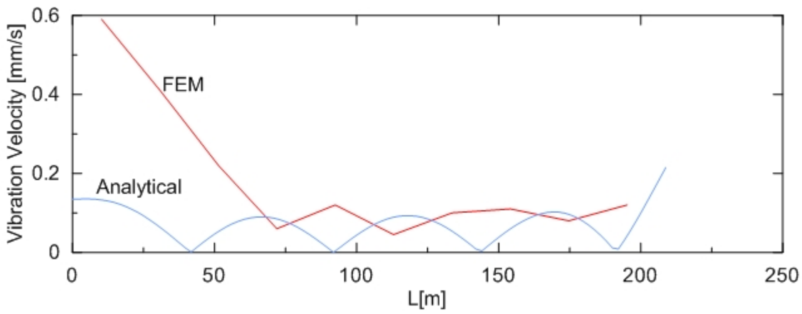

Figure 6-5 – Ship 4., Analytical-FEM velocity comparison

In Figure 6-6 the ratio between the analytical and the FEM results, i.e. the coefficient *R(x)*, is presented for each of the ten sections:

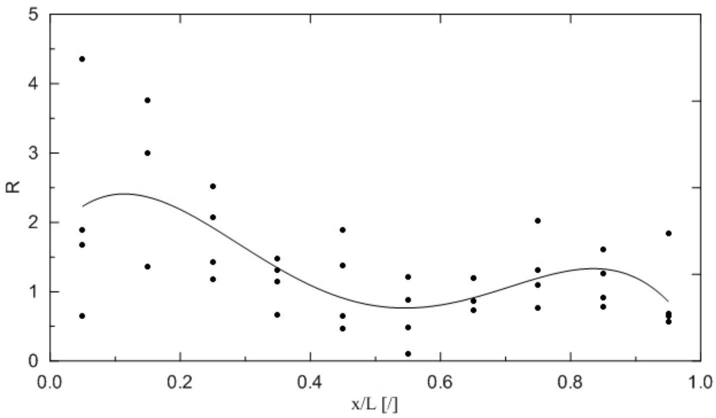

Figure 6-6 – Corrector Factor R

$R_{(x)_i}$ is the polynomial regression of the correction factor:

$$R_{(x)_i} = -31.56x^4 + 61.79x^3 - 36.57x^2 + 4.95x + 1.66 \quad (39)$$

6.2 Local Amplification Dynamic Effects of Deck Structures

It is physically reasonable that local deck effects are very significant in case the natural frequencies of deck structures are close to propellers or engines excitation frequencies. The analytical formula for calculating local amplification effects of the deck's structures will be described in this paragraph. The local amplification factors *V(y)* have been calculated based on:

- natural frequencies of deck supporting structural members;
- static deflection of deck's depending on deck weights in accordance with general arrangement;
- ratio between the natural and exciting frequency;

The following data are required for calculate local amplification *V(y)* factor calculation:

- deck structural arrangement considering spans, deck plating, deck longitudinal, deck girders and deck transverses;
- deck general arrangement in order to establish the local deck load and consequently local static deflection.

With this information in hand it is possible to calculate deck deflections and natural frequencies. Due to the fact that the ship has been discretized in ten parts, the magnification factor has to be calculated for all ship decks and for all ten ship's discretized sections. Deck structure natural frequencies have also been calculated to consider the coupled influence of deck transverses and deck longitudinal stiffeners.

The adopted beam natural frequency formula is:

$$\omega_n = c_G\, 51.2\ \ 10^2\ \sqrt{I\ A_B}\ \frac{1}{l^2} \quad (40)$$

where

c_G constant that depend on type of boundary condition;

I beam inertia [m^4];

A_B beam area [m^2];

l beam span [m].

The ratio between natural frequency of the deck transverse and natural frequency of deck stiffeners is equal to β. Considering the transversal beam and stiffeners as two springs supporting the same mass, the total coupled natural frequency is then calculated by the use of the formula:

$$f = \sqrt{f_{transversal\ beam}^2 + f_{stiffener}^2} = f_{stiffener}\sqrt{1+\beta^2} \tag{41}$$

The static deflection δ_s has been calculated with the Grashoff method.

$$\delta_s = \frac{5q_l L^4}{384\ EI} \tag{42}$$

where q_l is the distribution of the load q into two loads, one for each length of the panel.

Once the natural frequency and static deflection have been calculated it is possible to obtain the local magnification factor as *V(y)*:

$$v_{y_{k,i}} = \max \delta_{s_k}(y)\frac{\omega^2}{g}A_R + 1 \qquad k = 1,..,n \quad \forall\, i \tag{43}$$

where:

$\delta_s(y)$ static deflection of deck area [mm];

g gravity constant [mm/s^2];

ω angular velocity in terms of blade pulsing frequency;

A_R resonance factor for a one-mass system defined below

$$A_R = \frac{1}{\sqrt{(1-r^2)^2 + \left(2r\,\frac{C}{C_r}\right)^2}} \tag{44}$$

r = ratio between the excitation frequency and the natural frequency;

$\xi = C/C_r$ = critical damping ratio.

6.3 Vibration variation with Deck Height

The vibration variation with deck height will be now investigated. As can be seen from the FEM results [32,33], deck vibration levels usually globally decrease from bottom, where the main dynamic sources occur, up to the upper decks. Due to the large super extension and similarity the vertical distribution of vibrations for this type of ship is very similar. Due to this property the regression has been performed on the most significant one.

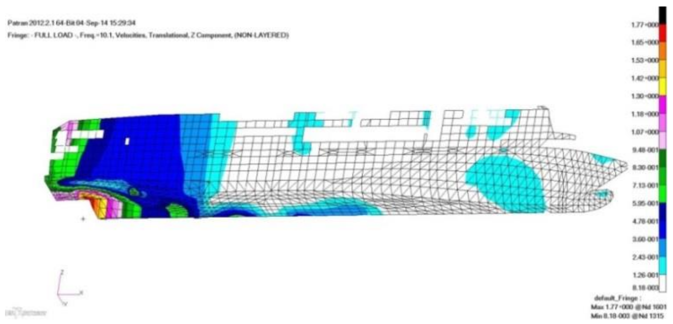

Figure 6-7 - Hull girder side shell and superstructure longitudinal bulkhead vibration velocities

For the analyzed sections, the polynomial regression has been used:

$$v_{z_{k,i}} = a_i z_k{}^6 + b_i z_k{}^5 + c_i z_k{}^4 + d_i z_k{}^3 + e_i z_k{}^2 + f_i z_k + g_i \quad \forall\, \mathrm{i} \quad k = 1, .., n \tag{45}$$

with "i" indicating the section in which the level of vibration is being considered.

In order to calculate the regression formula for each of the ten discretized section, global dynamic velocity response has been obtained from global FEM model [34] considering different ship sections and different inter deck areas, deck heights. The values reported in Table 6-1 have been taken in way of side shell area.

Table 6-1 Dimensionless values of vibration velocities for Ship 4

	X/L									
	0 .0- 0.1	**0.1 - 0.2**	**0.2 - 0.3**	**0.3 - 0.4**	**0.4 - 0.5**	**0.5 - 0.6**	**0.6 - 0.7**	**0.7 - 0.8**	**0.8 - 0.9**	**0.9 - 1.0**
Bottom	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Deck 1	0.48	0.38	0.37	0.16	0.22	0.41	0.25	0.40	0.75	0.70
Deck 2	0.47	0.39	0.36	0.09	0.22	0.43	0.24	0.25	0.75	0.70
Deck 3	0.47	0.39	0.35	0.18	0.22	0.38	0.23	0.22	0.69	0.65
Deck 4	0.47	0.40	0.35	0.08	0.22	0.32	0.22	0.21	0.69	0.65
Deck 5	0.58	0.41	0.33	0.08	0.21	0.32	0.21	0.21	0.81	0.75
Deck 6	0.63	0.42	0.33	0.08	0.21	0.32	0.20	0.20	0.81	0.75
Deck 7	0.59	0.42	0.33	0.08	0.17	0.22	0.18	0.17	0.88	0.00
Deck 8	0.50	0.52	0.35	0.13	0.21	0.22	0.21	0.21	0.94	0.00
Deck 9	0.49	0.47	0.33	0.08	0.17	0.17	0.17	0.17	0.94	0.00
Deck 10	0.49	0.47	0.35	0.00	0.21	0.21	0.21	0.21	0.94	0.00
Deck 11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.69	0.00

In order to obtain dimensionless values, valid for each ship, the vibration levels have been parameterized as a ratio between the vibration value, at the considered intermediate deck and bottom calculated vibration levels. A typical representation is show for one ship section of Ship 4, in this case for the section 0.3-0.4 (Figure 6-8).

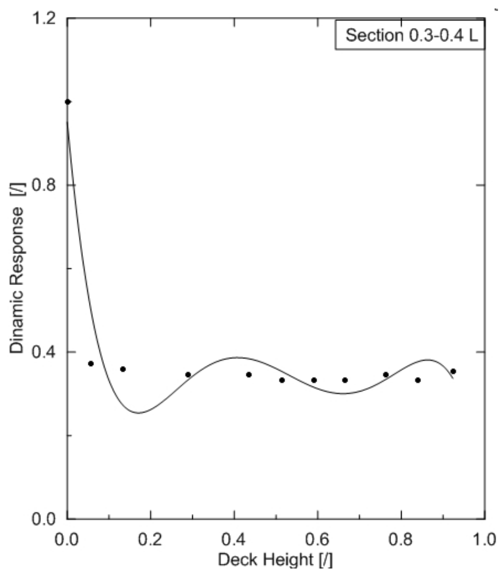

Figure 6-8 - Dynamic response variation with deck height at section 0.3-0.4L

Below are reported all the calculated *v(z)* factors for each of ten ship's discretized sections:

Section **0-0.1L** : $(v_{z_k})_1 = -20.72z_k^5 + 64.53z_k^4 - 75.01z_k^3 + 38.88z_k^2 - 8.17z_k + 0.95,$
$k = 1, \dots, n$

Section **0.1-0.2L** : $(v_{z_k})_2 = -61.59z_k^5 + 157.03z_k^4 - 146.93z_k^3 + 61.39z_k^2 - 10.80z_k + + 0.95, \quad k = 1, \dots, n$

Section **0.2-0.3L** : $(v_{z_k})_3 = -55.14z_k^5 + 144.89z_k^4 - 139.79z_k^3 + 60.08z_k^2 - 10.94z_k + +0.95, \quad k = 1, \dots, n$

Section **0.3-0.4L** : $(v_{z_k})_4 = -150.13z_k^5 + 349.07z_k^4 - 296.17z_k^3 + 110.93z_k^2 - 17.42z_k + +0.96, \quad k = 1, \dots, n$

Section **0.4-0.5L** : $(v_{z_k})_5 = -69.20z_k^5 + 182.36z_k^4 - 176.08z_k^3 + 138.92z_k^2 - - 18.06z_k + + 0.97, \quad k = 1, \dots, n$

Section **0.5-0.6L** : $(v_{z_k})_6 = -45.20z_k^5 + 117.56z_k^4 - 116.99z_k^3 + 51.34z_k^2 - 9.62z_k + +0.95, \quad k = 1, \dots, n$

Section **0.6-0.7L** : $(v_{z_k})_7 = -65.87z_k^5 + 173.62z_k^4 - 167.77z_k^3 + 71.99z_k^2 - 13.04z_k + +0.94, \quad k = 1, \dots, n$

Section **0.7-0.8L** : $(v_{z_k})_8 = -48.88z_k^5 + 131.60z_k^4 - 131.01z_k^3 + 58.86z_k^2 - 11.56z_k + +0.97, \quad k = 1, \dots, n$

Section **0.8-0.9L** : $(v_{z_k})_9 = -18.60z_k^5 + 44.61z_k^4 - 41.09z_k^3 + 18.74z_k^2 - 3.94z_k + 0.98,$
$k = 1, \dots, n$

Section **0.9-1.0L** : $(v_{z_k})_{10} = -421.18z_k^5 + 649.72z_k^4 - 358.24z_k^3 + 85.93z_k^2 - 8.81z_k + +1.00, \quad k = 1, \dots, n$

The total investigated vibrations level is obtained if the previous factors are multiplied by global hull girder response $V_G(x)$ - corrector factor *R(x)* and by local amplification factors *v(y)*

7. Validation of the new developed Formula

The new developed formula has been applied for vibration level predictions on 4 ships. The calculated vibration velocities have been compared with measurements on board.

7.1 Global Vibration Level VG(X) with correction R(x) and vibration decreasing V(z)

In Table 7-1 there are the main characteristics of the analyzed ships and in the Figure 7-1, Figure 7-2, Figure 7-3 and Figure 7-4 the dynamic vibration velocity ("constant" line) is computed and then the value is multiplied by the correction factor *R(x)* ("points).

Table 7-1 Ship 1, vibration level with and without $v_{y_{k,i}}$

Main Characteristics				
	Ship 1	**Ship 2**	**Ship 3**	**Ship 4**
L_R [m]	254.8	185.1	280.8	203.0
f [Hz]	9.5	9.2	6.9	10.0
F [N]	13800	14000	23769	41000
M [Nm]	50000	48000	55000	60000
$A_{midship}$ [m^2]	4.4	3.6	5.7	4.7
$I_{midship}$ [m^4]	421.6	219.4	486.2	472.9

Ship 1

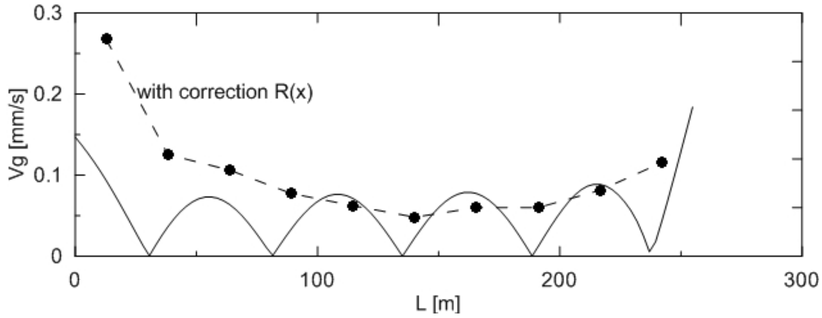

Figure 7-1 Ship1, global vibration level and absolute value with correction

Table 7-2 Ship 1, vibration level with and without $v_{y_{k,i}}$

	$V(x)_i\ v_{y_{k,i}}\ v_{z_{k,i}}\ \omega$ [mm/s]		$V(x)_i\ v_{z_{k,i}}\ \omega$ [mm/s]	
	Deck 12	**Deck 9**	**Deck 12**	**Deck 9**
0.4-0.5		0.139		0.047
0.7-0.8	0.111		0.018	

Ship 2

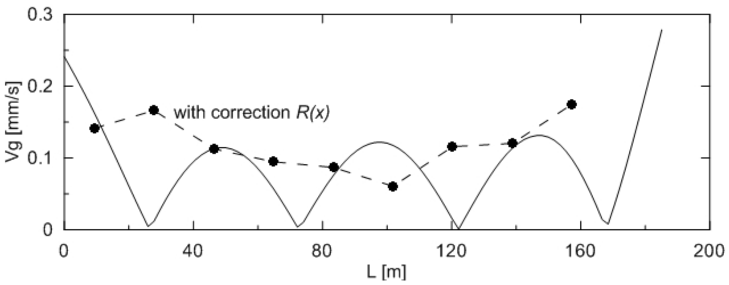

Figure 7-2 Ship 2, Global vibration level and absolute value with correction

Table 7-3 Ship 2, vibration level with and without $v_{y_{k,i}}$

	$V(x)_i\ v_{y_{k,i}}\ v_{z_{k,i}}\ \omega$ [mm/s]		$V(x)_i\ v_{z_{k,i}}\ \omega$ [mm/s]	
	Deck 6	**Deck 5**	**Deck 6**	**Deck 5**
0.0-0.1		0.779		0.195
0.1-0.2	0.144		0.054	

Ship 3

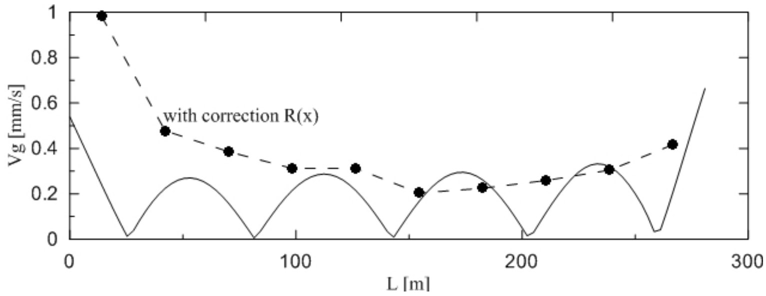

Figure 7-3 Ship 3, Global vibration level and absolute value with correction

Table 7-4 Ship 3, vibration level with and without $v_{y_{k,i}}$

	$V(x)_i\ v_{y_{k,i}}\ v_{z_{k,i}}\ \omega$ [mm/s]		$V(x)_i\ v_{z_{k,i}}\ \omega$ [mm/s]	
	Deck 9	**Deck 5**	**Deck 9**	**Deck 5**
0.4-0.5		0.135		0.016
0.8-0.9	0.143		0.018	

Ship 4

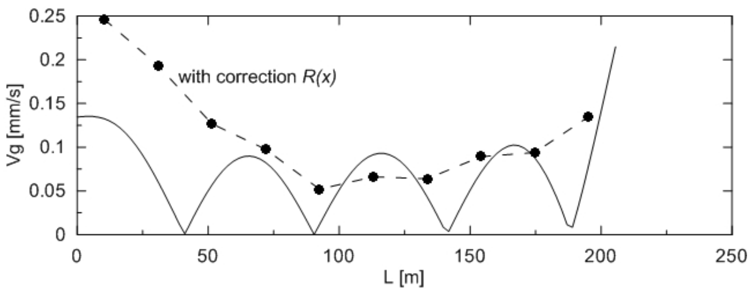

Figure 7-4 Ship 4, Global vibration level and absolute value with correction

Table 7-5 Ship 4, vibration level with and without $v_{y_{k,i}}$

	$V(x)_i\ v_{y_{k,i}}\ v_{z_{k,i}}\ \omega$ [mm/s]		$V(x)_i\ v_{z_{k,i}}\ \omega$ [mm/s]	
	Deck 4	**Deck 5**	**Deck 4**	**Deck 5**
0.0-0.1	0.904	0.528	0.137	0.152

8. Experimental Measurements on board

8.1 Description of measurements procedures

Into the Classification Societies Comfort Class Rules are in general defined the instructions for measurement procedures, condition and limits [4].

a) <u>Measuring equipment</u>

Measurement and calibration equipment are to meet the requirements of ISO 6954 and ISO 8041.

Vibrations calibrations (Figure 8-1) are to be verified at least every year, while the measuring equipment, part of measuring chain, is all together verified at least every two years.

Figure 8-1: FFT Analyser, accelerometers and calibrator in order

This verification is to be done by a national standard laboratory on a competent laboratory.

The instrumentation includes at least one transducer accelerometer (Figure 8-2) with amplifier and the Fast Fourier Transform analyser (Figure 8-2).

The Classification Societies suggests that the instrumentations have to be calibrated in situ before and after the tests.

b) <u>Sea trial conditions</u>

During the sea trials the propeller output has to correspond to the operation conditions of the technical ships specification and not less than 85% of maximum continuous rating.

The test conditions should correspond to the loading conditions defined for sea trials.

Vibration measurements have to be performed in sea and weather conditions with sea state 3 or less on the WMO sea state code. The tests have to be performed in deep water, with water depth greater than five times the mean draft.

Ship course has to be kept constant, with rudder angle less than two degrees portside or starboard for the duration of the measurements.

c) Measuring positions

Measurements are to be taken in vertical direction. In cabins, offices and other small rooms as library, meeting room the measurements are to be taken on the floor in the centre of the room. For longer rooms several measuring points may be required in general based on author experience each measuring points covering 15-20 m^2.

Vibrations are to be measured in all accommodation spaces, navigation, wheelhouse room and crew spaces.

In order to define the location and number of measuring points in general the length of the ships is divided in two parts:

- from the aft part of the ship to the front bulkhead of the casing;
    - minimum of 20% of cabins;
    - all public spaces and open daily.

  For long public areas (lounges, restaurants, etc.) measurements are to be carried out in different locations. At first tentative each measuring points covering less than 80 m^2. In case the measured levels are at limit or above the covering measuring point area should be reduced to 15-20 m^2, based on author experience. It is reasonable to divide the ship in length in two parts, because in the aft part from transom to the fore engine room bulkhead, are located all the significant sources of vibration:
    - propellers;
    - main propulsion engines;
    - diesel generators.
- from the front bulkhead of the casing to the fore end of the ship:
    - minimum of 10% of cabins;
    - all public spaces and open deck.

  For large public rooms measurements are to be carried out with each measuring point covering less than 150m^2.

d) Technical organization

The measurements are undertaken or by the Classification Societies or by an approved organization that satisfied all the criteria listed out below:

- Have instrumentation whose calibration, both before and after the measurements, can be traced back to National Standards and, hence, back to International Standards.
- Have analysis procedures capable of data reduction to the requirements and standards set out in these Rules.
- Be able to provide a written report in English with contents.

8.2 Vibration levels measurements on board during sea trials

In order to validate the developed simplified approach and the newly developed formula, several measurements on real ships have been performed in agreement with the procedures described above. Here only the most representative and more significant examples of two measurements for each ship are presented.

Measured Values - Ship 1

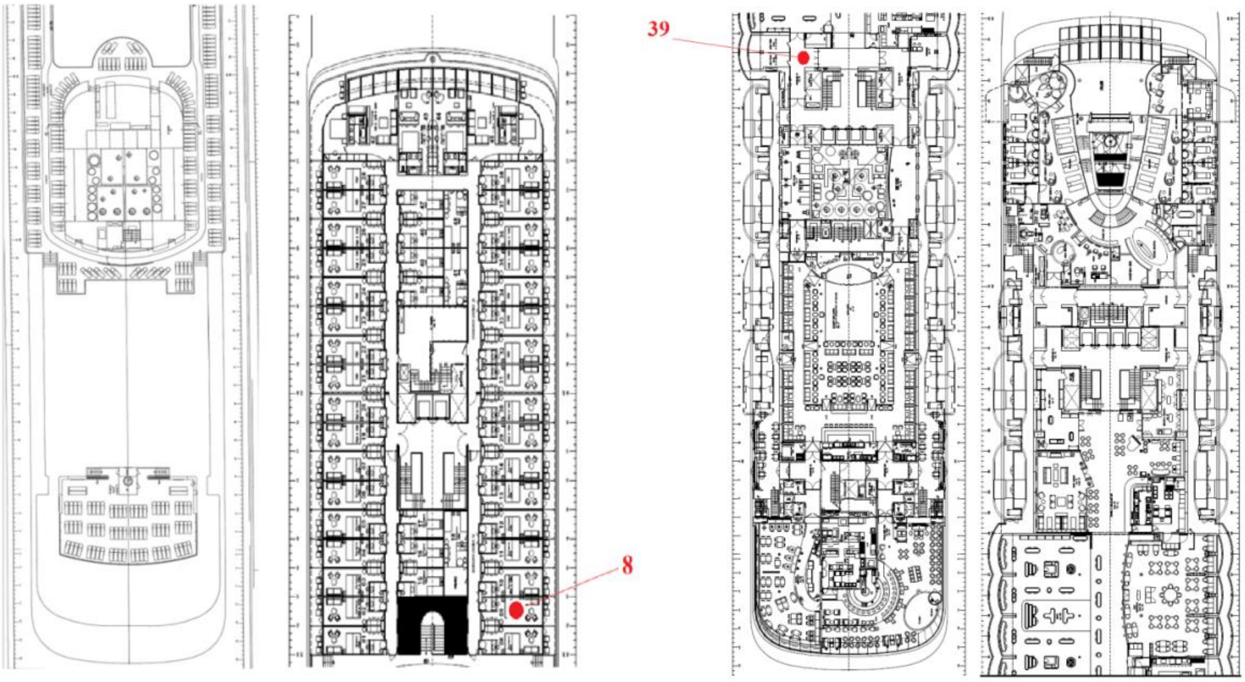

Figure 8-2 Measurement points on deck 12 (left) and on deck 9 (right)

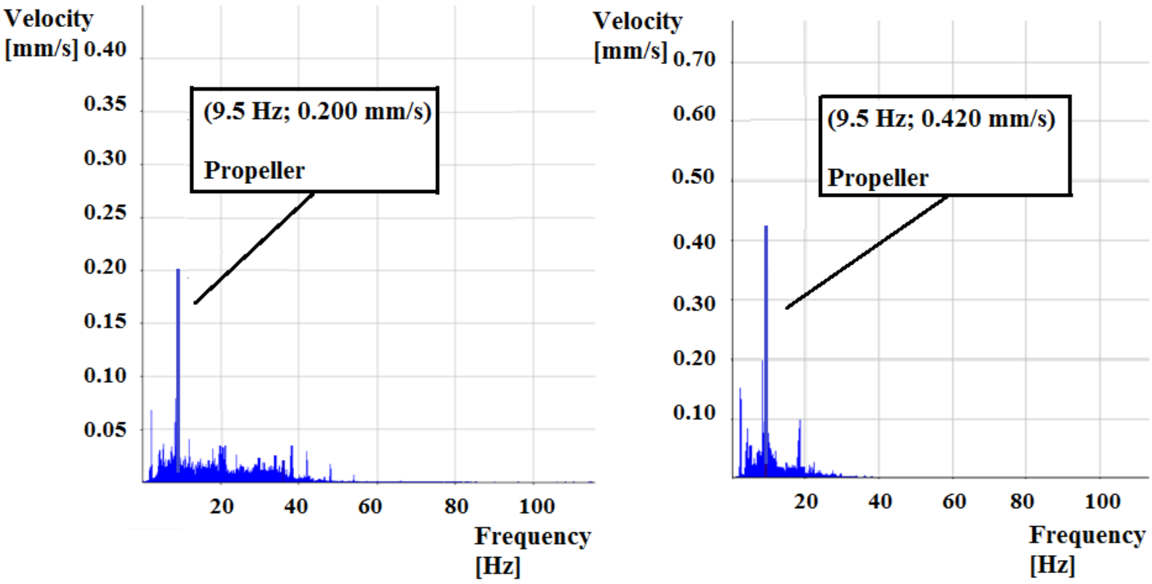

Figure 8-3 –Ship 1 deck 12 - measure 8 (left) and deck 9 - measure 39 (right)

Measured Values - Ship 2

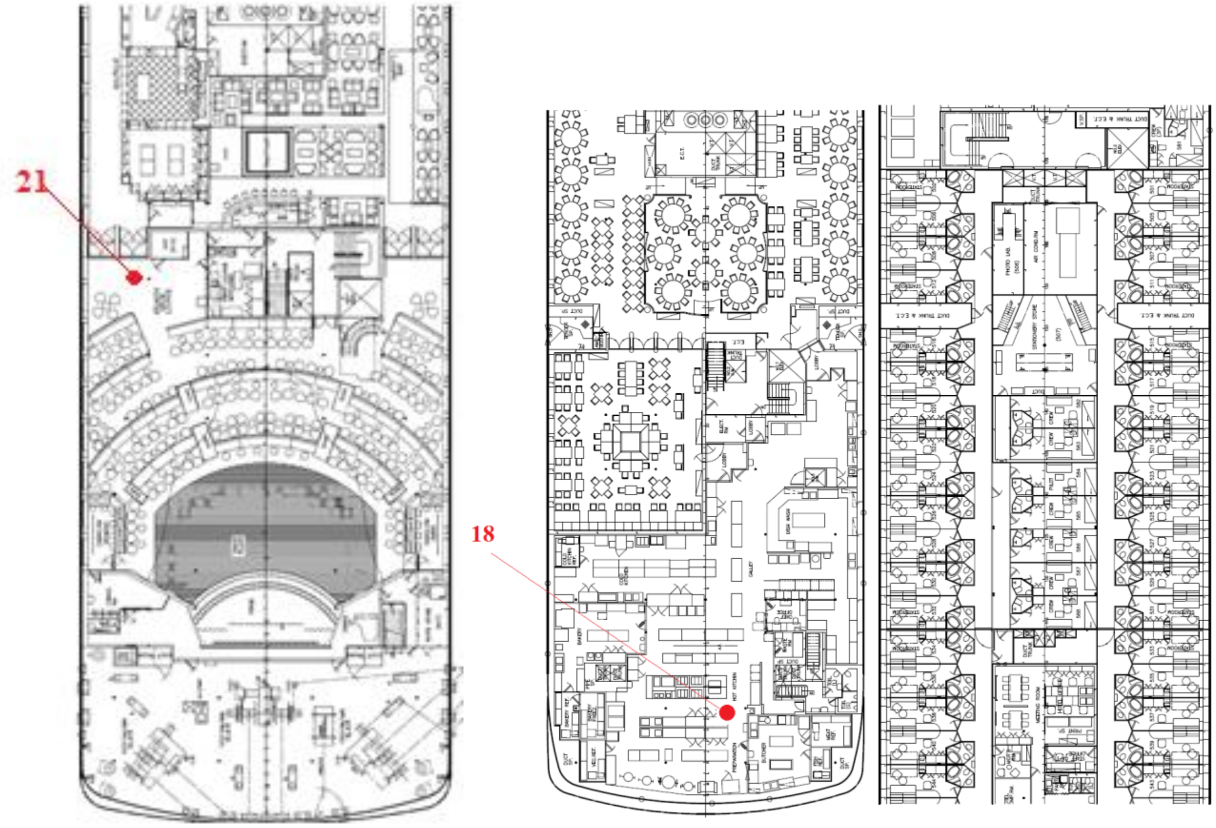

Figure 8-4 Measurement points on deck 6 (left) and 5 (right)

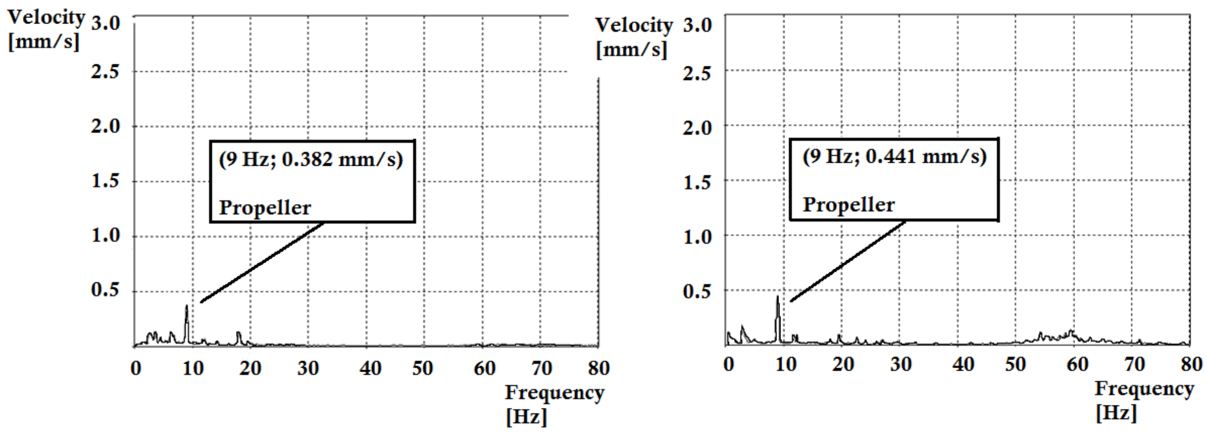

Figure 8-5 Ship 2 deck 5 - measure 021 (left) and deck 6 - measure 018 (right)

Measured Values - Ship 3

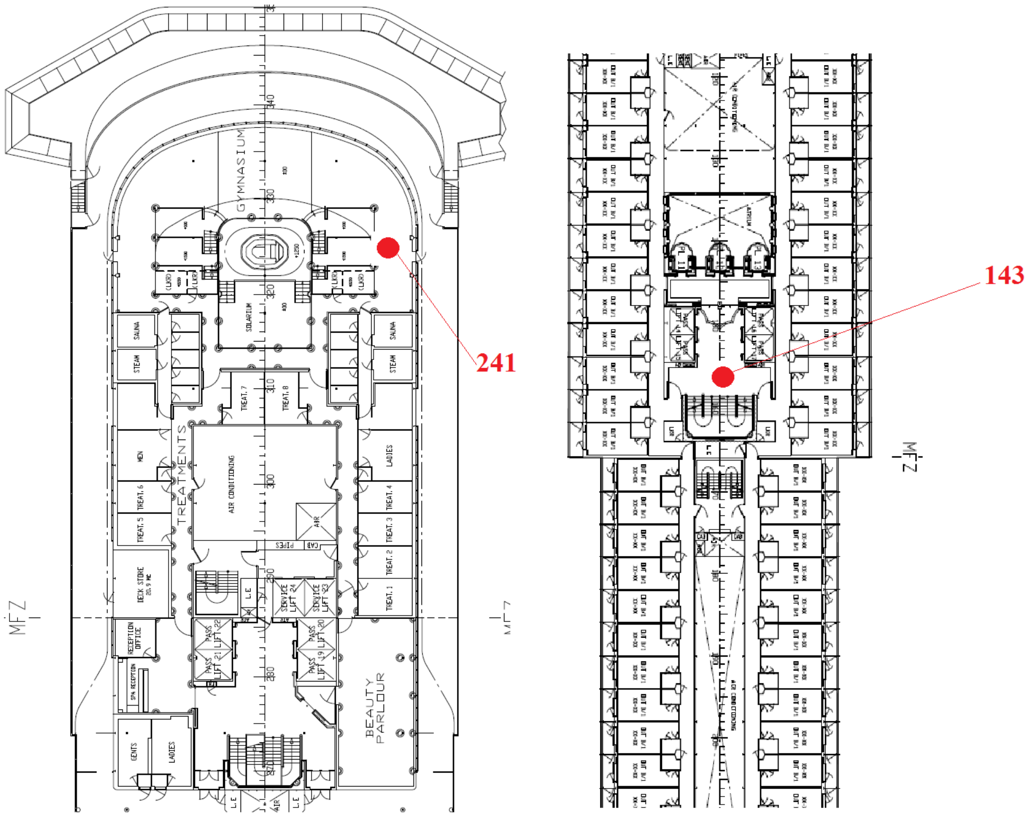

Figure 8-6 Measurement points on deck 9 (left) and on deck 5 (right)

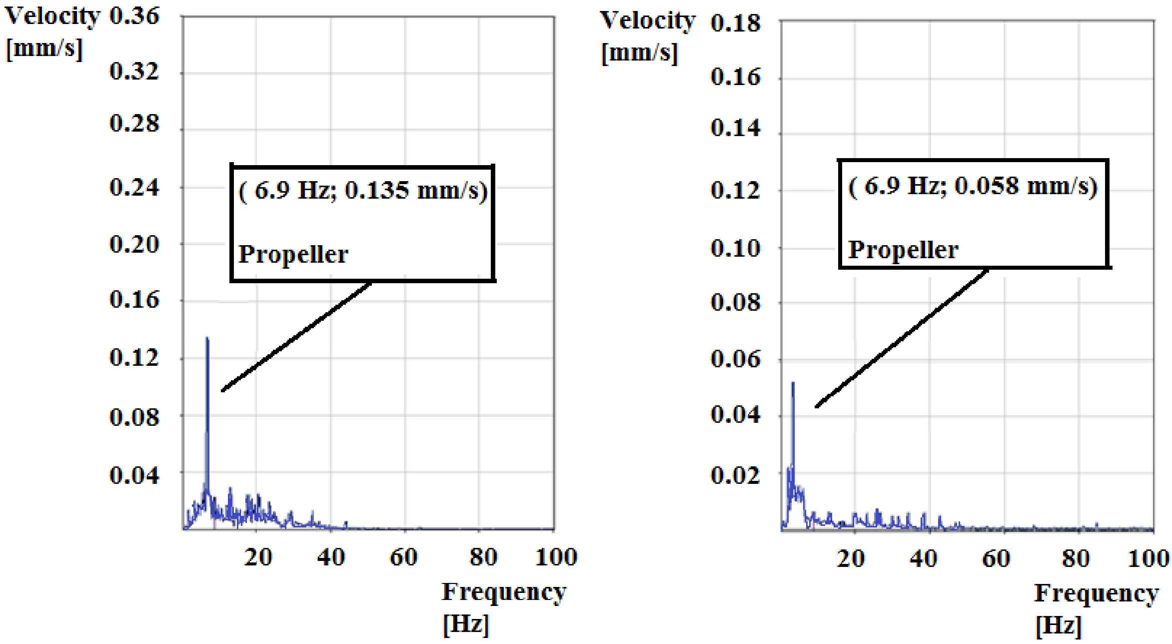

Figure 8-7 Ship 3 deck 9 - measure 241 (left) and deck 5 - measure 143(right)

Measured Values - Ship 4

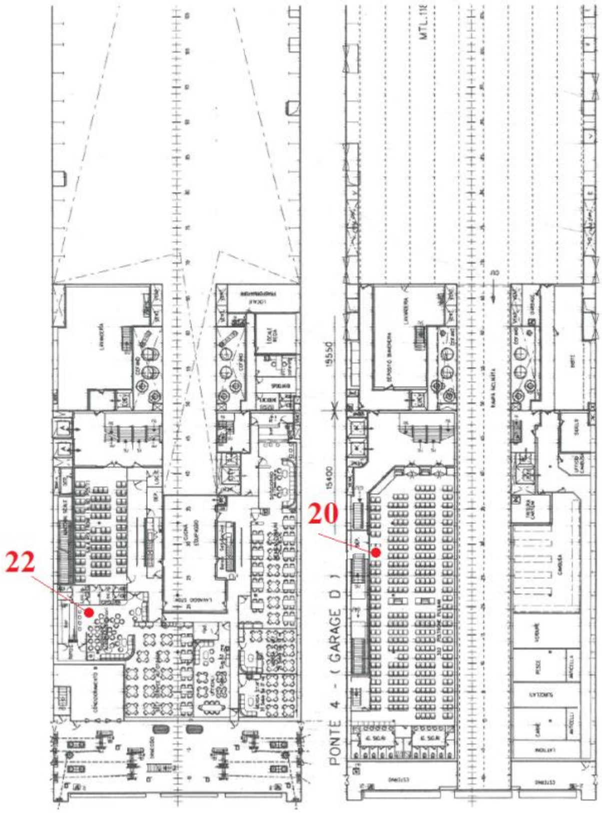

Figure 8-8 Measurement points on deck 4 and 5

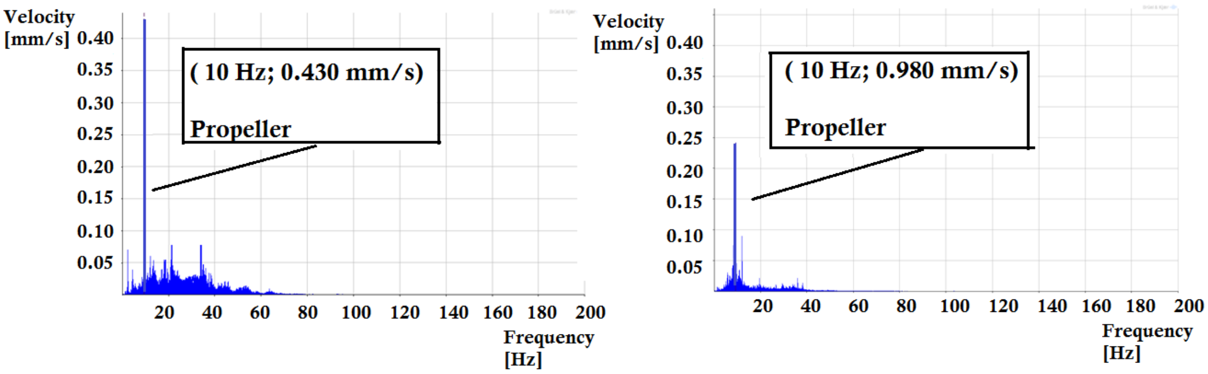

Figure 8-9 Ship 4 deck 4 - measure 20 (left) and deck 5 - measure 22 (right)

9. Comparative Analysis and Error Calculation

In order to validate the developed mathematical-physical model approach [35,36] in this section the comparison between the calculated vibration levels and the ones measured on board has been performed.

In Table 9-1 the calculated and measured data are shown and compared. The difference has been calculated, but in order to obtain a quantity able to indicate the reliability of the analytical predictions, the absolute error has been compared with the ISO6954-1984 lower bound limit, vibration level 4mm/s, see [37,38,39]. This ratio will be called Error Index 4(EI4)

$$Error\ index\ 4 = \frac{calculated - measured}{4}$$

The original particular choice of the Error index 4 is based on author experience gained during presence at several sea trials measurements. Investigating more deeply the real comfort on board, the main feature of ISO 6954-1984 is that to represent the human perception of comfort expressed as maximum 0- peak value for each frequency. The author selection of 4 mm/s is based on statement from this norm “adverse comment not probable”, which define the area below the lower limit curve in Figure 9-1.

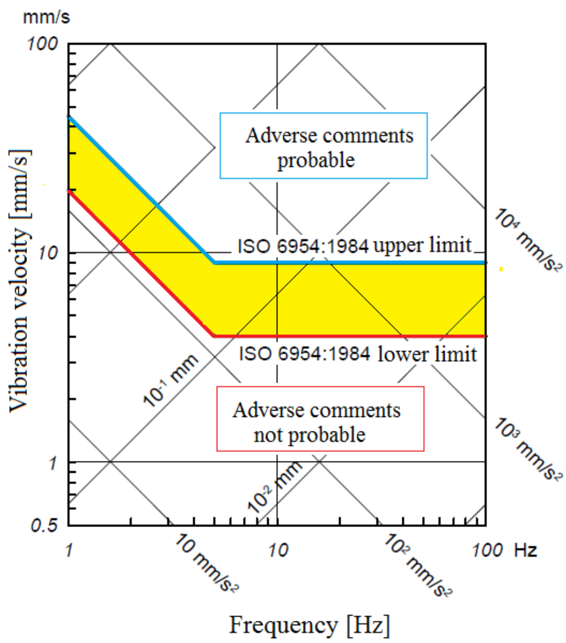

Figure 9-1: ISO 6954-1984 vibration limits

This boundary curve is very effective and of general acceptance for real defining and human perception of the basic level of comfort. This is the reason for author decision to this New original definition.

Ship 1

Table 9-1 Ship 1, measured and calculated vibration level

	x/L					
	0.7- 0.8			0.4 - 0.5		
	calculated [mm/s]	**measured [mm/s]**	**error index 4**	**calculated [mm/s]**	**measured [mm/s]**	**error index 4**
deck12	0.111	0.200	-0.022			
deck9				0.139	0.410	-0.068

Ship 2

Table 9-2 Ship 2, measured and calculated vibration level

	x/L					
	0.0 - 0.1			0.1 - 0.2		
	calculated [mm/s]	measured [mm/s]	error index 4	calculated [mm/s]	measured [mm/s]	error index 4
deck6				0.144	0.441	-0.074
deck5	0.779	0.382	0.099			

Ship 3

Table 9-3 Ship 3, measured and calculated vibration level

	x/L					
	0.4 - 0.5			0.8 - 0.9		
	calculated [mm/s]	measured [mm/s]	error index 4	calculated [mm/s]	measured [mm/s]	error index 4
deck9				0.143	0.135	0.002
deck5	0.135	0.058	0.019			

Ship 4

Table 9-4 Ship 4, measured and calculated vibration level excitation

	x/L		
	0.0 - 0.1		
	calculated [mm/s]	measured [mm/s]	error
deck5	0.528	0.430	0.024
deck4	0.904	0.980	-0.019

10. Discussion

The results obtained for all the studied ships were compared to experimental sea trial measurements. In Table 10-1 and in Figure 10-1 we show the *Error Index 4* for each measurement point. For major reasons of space in this paper there are only the two most significant points for each ship. In the whole author's data base there are a large number of measurement points. The maximum positive E*rror Index 4*, when the calculated value overestimates the measured value, is lower than 10%. The maximum negative *Error Index 4*, when calculated value underestimates the measured value, is lower than 7.5%.

Table 10-1 Error Index 4

	Error Index 4
Ship1	-0.022
	-0.068
Ship 2	0.099
	-0.074
Ship 3	0.044
	0.002
Ship 4	0.020
	-0.019

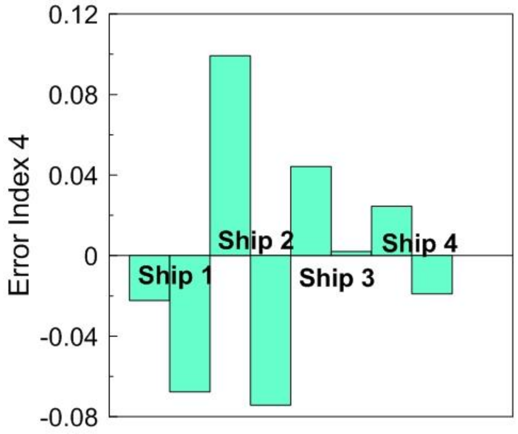

Figure 10-1 Error Index

11. Conclusions

The developed approach is based on the assumption that the ship hull can be represented as a non-uniform section beam. The basic idea for the development of the new formula is that the local vibrations on ship structures are superimposed to global hull vibrations. It has been also considered that the vibration varies with deck position in height.

Furthermore, the results obtained from FEM analyses have been used for the calculation of the coefficients necessary to improve the accuracy of results obtained with the "new developed formula".

The decrease of vibrations with height -z coordinate- was also obtained with the use of a statistical method and FEM results of an entire ship model.

The presented mathematical calculations bring to a fully consistent formula which was the primary scope of this paper. The results obtained with the use of the new formula were then compared to data from measurements on real passenger and RO-RO passenger ships. The comparison between calculated and measured vibrations values were performed for the most significant areas of different ships. The obtained results show that the maximum calculated Error Index is 10%.

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