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# An integer programming model for assigning students to elective courses

**Ivo Beroš**[1,*] **and** **Joško Meter**[1]

[1] *Department of Applied Mathematics and Information Technology,*
*VERN' University of Applied Sciences*
*Trg bana Josipa Jelačića 3, 10000 Zagreb, Croatia*
*E-mail:* ⟨{*ivo.beros, josko.meter*}*@vern.hr*⟩

**Abstract.** This paper deals with the problem of assigning students to elective courses according to their preferences. This process of assigning students to elective courses according to their preferences often places before academic institutions numerous obstacles, the most typical being a limited number of students who can be assigned to any particular class. Furthermore, due to financial or technical reasons, the maximum number of the elective courses is determined in advance, meaning that the institution decides which courses to conduct. Therefore, the expectation that all the students will be assigned to their first choice of courses is not realistic (perfect satisfaction). This paper presents an integer programming model that maximizes the total student satisfaction in line with a number of different constraints. The measure of student satisfaction is based on a student's order of preference according to the principle: *the more a choice is met the higher the satisfaction.* Following the basic model, several versions of the models are generated to cover possible real-life situations, while taking into consideration the manner student satisfaction is measured, as well as the preference of academic institution within set technical and financial constraints. The main contribution of the paper is introducing the concept of the minimal student satisfaction level that reduces the number of students dissatisfied with the courses to which they were assigned.

**Key words**: integer programming, multi-unit assignment, elective courses

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## 1. Introduction

This paper deals with the problem of assigning students to elective courses at an academic institution in Croatia and can be set out as follows:

- The institution offers a wide range of elective courses, which are divided into modules consisting of an equal number of courses (one course can be part of only one module). In the remainder of the paper, we will use the term "course" instead of module.
- Every student must be enrolled in $K$ courses.

*Corresponding author.

 

- The timetable is devised after completion of the assignment process.
- The number of students is relatively small ($< 200$).

Conducting all the possible courses is not feasible due to requirements stemming from an institution's operations. Resolving the question as to which courses will be cancelled means defining an appropriate course elective selection system. Previously, a standard two-round system was used:

- A student selects $K$ courses.
- After the selection process, data are collected. The courses are divided into two sets: $C_I$ - courses with an insufficient number of interested students and $C_S$ - courses with a sufficient number of interested students. All the courses from the set $C_I$ are eliminated from any further processes.
- All students who selected any of the courses from $C_I$, now have to replace previously selected courses from $C_I$ with the courses from $C_S$.

This system has many obstacles: it is time-consuming, the number of students enrolled in courses they did not select in round one is relatively high, and students tend to select popular instead of desired courses, which in turn restricts possibilities of offering elective courses.

The new system consists of the following steps:

- A student selects and ranks $L$ preferable courses ($L > K$).
- After the selection process, data are summarized. Based on the solution provided by the integer programming model, students are assigned courses from their list of preferable courses.
- All the students who are not assigned to the $K$ of their preferable courses can choose the remaining courses in consent with faculty administration.

In the new system, a student can select additional courses in which they are interested and the probability is smaller that a student is enrolled in less than $K$ preferable courses. Integer programming models are constructed in consideration of the students choices as well as an educational institution's operational requirements. The solution to the model is obtained by maximizing total student satisfaction, which depends on the courses in which the student has enrolled. In general, greater student satisfaction is achieved when the students are enrolled in the courses of their first choice. An important constraint in the models is minimal student satisfaction which guarantees, if possible, that a student is assigned to courses that are higher on the list of preferable courses.

The integer programming techniques [4, 6, 7, 1] (for other techniques see surveys [12, 3]) have been used to develop the models presented in this paper given that the problem is relatively small (number of variables in the model is smaller than 1000). There are many papers that use integer programming to generate efficient solutions for a wide range of university timetable problems [5, 8, 9, 10, 2], but they are primarily concerned with maximizing student satisfaction with the timetable of elective courses in which they have already enrolled. Instead, we have developed models that take into consideration timetables but assign students to the elective courses based on to their choices as well as an educational institution's operational requirements. A problem, similar to that observed in this paper, is described in [11]. In [11], the authors deal with the problem of assigning elective courses while taking into consideration both student preferences and timetable constraints imposed by the faculty administration. The solution to the problem uses a heuristic approach because the developed integer programming model becomes impractical. Contrary to the model presented in [11], our models do not have timetable constraints, only constraints relating to minimal student satisfaction.

## 2. Assigning Students to Elective Courses

A one-round electoral system was described in the previous section. The next two subsections describe two types of models that support the electoral system. The first type includes models that presume only one group of students have enrolled into all elective courses. The second type includes models that impose additional constraints, i.e. the courses can be enrolled by either one or more groups of students, or a minimal level of student satisfaction is to be achieved. Finally, Section 3 presents an algorithm for assigning students if the models in Section 2 provide no solution.

### 2.1. The basic model

We observe that the problem of assigning students to elective courses occurs when students have to enroll in several elective courses. The assigning process has to satisfy conditions stemming from both academic and operational requirements:

- U1 - A student can select $L$ courses. The courses are referenced as *preferable courses.*
- U2 - A student has to rank the preferable courses. After the ranking, maximum weight $L$ is given to the first course, weight $L-1$ is given to the second course, and so on. The weights are referenced as a *coefficient of satisfaction.*
- U3 - A student will be assigned to $K$ courses from the list of $L$ preferable courses, where $K < L$. The sum of a student's coefficients of satisfaction for the assigned courses is designated as *student satisfaction.*
- U4 - For every course there is a maximum number of students that can be enrolled.
- U5 - For every course there is also minimum number of students that have to be enrolled.

The measure for goodness of a particular assignment is the total sum of students' satisfaction. For the sake of simplicity, we observe only the case when all the students must be enrolled in the same number of elective courses and when they can choose the same number of preferable courses.

To describe a mathematical model that satisfies conditions U1-U5, let us define parameters and variables presented in the model.

**Definition 1.** *The parameters in the mathematical model for assigning students are*

- $K$ *is the number of courses that student has to be enrolled in.*
- $L$ *is the number of preferable courses student can select.*
- *Set* $S = \{S_1, S_2, \ldots, S_M\}$ *is the set of all students. The set* $S$ *has a total of* $M$ *elements.*
- *Set* $P = \{P_1, P_2, \ldots, P_N\}$ *is the set of all offered courses. The set* $P$ *has a total of* $N$ *elements.*
- *For every course* $P_j$, $j = 1, \ldots, N$, $MP_j$ *denotes the maximum number of the students that can be enrolled in* $P_j$*. Similarly,* $mP_j$ *denotes the minimum number of students that can be enrolled in* $P_j$*.*
- *Parameters* $a_{ij}$ *are indicators of the preferable courses. If student* $S_i$ *prefers the course* $P_j$ *then* $a_{ij} = 1$*, otherwise* $a_{ij} = 0$*. The definition of* $a_{ij}$ *ensures that condition U1 is satisfied.*
- *Parameters* $c_{ij}$ *are the coefficients of satisfaction and denotes which value student* $S_i$ *attributes to a course* $P_j$*. If the course* $P_j$ *is not found among the preferable courses of student* $S_i$*, then* $c_{ij} = 0$*. The definition of* $c_{ij}$ *ensures that condition U2 is satisfied.*

*Variables in the model are binary variables* $x_{ij}$, $i = 1, \ldots, M$, $j = 1, \ldots, N$, *which denote whether student* $S_i$ *is assigned to the course* $P_j$ $(x_{ij} = 1)$ *or not* $(x_{ij} = 0)$*.*

All parameters from Definition 1 are known before students select preferable courses ($K$, $L$, $S$, $P$) or immediately after ($a_{ij}$, $c_{ij}$).

The next proposition will be used to explain how conditions U3–U5 are mathematically expressed.

**Proposition 1.** *For the parameters and variables in Definition 1, the following statements are valid:*

1. *The number of courses assigned to student* $S_i$ *are:* $\displaystyle\sum_{j=1}^{N} a_{ij}x_{ij}$.
2. *Student satisfaction for student* $S_i$*:* $\displaystyle\sum_{j=1}^{N} c_{ij}x_{ij}$.
3. *The number of students who prefer course* $P_j$*:* $\displaystyle\sum_{i=1}^{M} a_{ij}$.
4. *The number of students assigned to the course* $P_j$*:* $\displaystyle\sum_{i=1}^{M} a_{ij}x_{ij}$.

5. *The sum of coefficients of satisfaction for all students assigned to the course* $P_j$*:* $\sum_{i=1}^{M} c_{ij}x_{ij}$*.*

**Proof**. We will prove only the first statement. From the definition of the variables $x_{ij}$ it is obvious that the number of courses to which student $S_i$ assigned is defined by $\sum_{j=1}^{N} x_{ij}$. But, $S_i$ can be assigned only to preferable courses, i.e. $x_{ij}$ can be 1 only when $a_{ij}$ is also equal to 1, so we obtain

$$\sum_{j=1}^{N} x_{ij} = \sum_{j=1, a_{ij}=1}^{N} x_{ij} = \sum_{j=1}^{N} a_{ij}x_{ij}.$$

□

From Proposition 1, the condition U3 can be expressed as

$$\sum_{j=1}^{N} a_{ij}x_{ij} = K, \quad i = 1, \ldots, M. \tag{1}$$

Unfortunately, condition U3 in a real situation can be impracticable because there is a possibility that a student chooses at least $L - K + 1$ courses where the number of interested students is smaller than the minimal number of students that must be enrolled in the course.Hence, condition U3 is replaced by

U3' - A student will be assigned to no more than $K$ courses from the list of preferable courses, which can be expressed as

$$\sum_{j=1}^{N} a_{ij}x_{ij} \leq K, \quad i = 1, \ldots, M. \tag{2}$$

In practice, it means that students who are not assigned to $K$ courses from the list of preferable courses will be assigned to another course after meeting with the faculty administration.

Similarly, we consider conditions U4 and U5. From Proposition 1, it follows that condition U4 is given by

$$\sum_{i=1}^{M} a_{ij}x_{ij} \leq MP_j, \quad j = 1, \ldots, N, \tag{3}$$

and the condition U5 is given by

$$\sum_{i=1}^{M} a_{ij}x_{ij} \geq mP_j, \quad j = 1, \ldots, N. \tag{4}$$

The goal function is the total sum of student satisfaction. According to Proposition 1, this sum is given by

$$\sum_{i=1}^{M}\sum_{j=1}^{N} c_{ij}x_{ij} \tag{5}$$

Now we can formulate a basic model for assigning students to elective courses and considering it as a problem of integer programming, where the total sum of student satisfaction is maximized subject to (2–4):

$$(PR)\quad \begin{cases} \sum_{i=1}^{M}\sum_{j=1}^{N} c_{ij}x_{ij} \to \max \\ \sum_{j=1}^{N} a_{ij}x_{ij} \le K, \quad i=1,\ldots,M; \\ \sum_{i=1}^{M} a_{ij}x_{ij} \le MP_j, \quad j=1,\ldots,N; \\ \sum_{i=1}^{M} a_{ij}x_{ij} \ge mP_j, \quad j=1,\ldots,N; \\ x_{ij} = 0 \text{ or } 1,\ i=1,\ldots,M,\ j=1,\ldots,N. \end{cases}$$

For the sake of simplicity, the formulation of problem (PR) is given as a problem with $M \cdot N$ variables, but the number of variables in the problem is only $M \cdot L$.

## 2.2. The models with additional conditions

Model (PR) is a basic model for assigning students to elective courses. It is the appropriate model for the case when only one group of students is enrolled into every single course, but if there are courses into which more than one group can enroll, hence we have to expand the model for additional conditions. Actually, we observe three types of conditions:

U6 - The number of groups is limited. Students enrolled into a particular course can be divided in one or more groups.

U7 - Student satisfaction has to be higher than any arbitrary constant. We refer to this condition as a *minimal satisfaction condition*. The term *dissatisfied student* means that the minimal satisfaction condition has not been met for the student.

U8 - The average student satisfaction assigned to a course must be higher than any other arbitrary constant.

From a business point of view, the group is the basic unit of costs. It requires a classroom (with limited capacity) and a teacher, hence the number of classes must be restricted to cut expenses. A typical problem with groups is the case when the number of enrolled students is slightly higher than the maximum number of students in a group, hence the faculty administration has to decide on whether there will be one or two groups in the course. For example, the maximum number of students in one group is 20, whereas 24 students are enrolled in the course. Is it better to form two smaller groups or to transfer the four students to other courses?

Before conducting further analysis of the previously mentioned conditions, we will define additional parameters and variables.

**Definition 2.** *The parameters required for analyzing conditions U6–U8 are:*

- $r_j$ *- maximum number of groups in which course* $P_j$ *can be taught.* $r_j$ *can be zero.*
- $g_j$ *- the upper bound of the number of students enrolled in one group for the course* $P_j$*. For* $k$ *groups, the upper bound is* $mP_j^k = k \cdot g_j$*.*
- $mP_j^k$ *- the lower bound of the number of students enrolled in course* $P_j$ *required to teach* $P_j$ *in* $k$ *groups. Note that* $mP_j^1 = mP_j$*.*
- $G$ *- the maximum total number of groups.*
- $B^S$ *- the minimum student satisfaction.*
- $B^C$ *- the minimum average satisfaction for the courses.*

*Binary variables* $y_{jk}$*,* $j = 1, \ldots, N$*,* $k = 1, \ldots, r_j$*, are used to describe the number of groups for a specific course. If the course* $P_j$ *is taught in* $k$ *groups, then* $y_{jk} = 1$*, otherwise* $y_{jk} = 0$*.*

From Definition 2, it is evident that

$$y_{j1} + y_{j2} + \cdots + y_{jk_j} \leq 1, \; j = 1, \ldots, N. \tag{6}$$

If $y_{j1} + y_{j2} + \cdots + y_{jk_j} = 0$, then all the variables $y_{j1}, y_{j2}, \ldots, y_{jk_j}$ are zero and there are no students assigned to course $P_j$.

**Proposition 2.** *Course* $P_j$ *is taught in* $k$ *groups if and only if* $\sum_{l=1}^{r_j} l\, y_{jl} = k$

**Proof**. If the course $P_j$ is taught in $k$ groups, then $y_{jk} = 1$, and the other $y_{jl}$ are zero, which therefore is $\sum_{l=1}^{r_j} l\, y_{jl} = k y_{jk} = k$. On the other hand, if $\sum_{l=1}^{r_j} l\, y_{jl} = k$, from (6) follows that only $y_{jk}$ can be one, so course $P_j$ is taught in $k$ groups. □

In the model (PR), the conditions U4 and U5 are given by (3) and (4), but these conditions are not suitable for the case where a course can be taught in more than one group, hence we have to replace (3) and (4) with more appropriate conditions.

If the number of students enrolled in course $P_j$ is suitable for $k$ groups then that number has to be between the corresponding lower and upper bounds:

$$mP_j^k \leq \sum_{i=1}^{M} a_{ij} x_{ij} \leq k\, g_j. \tag{7}$$

Using definition of $y_{jk}$ and Proposition 2, (7) can be written as

$$\sum_{l=1}^{r_j} y_{jl} mP_j^l \leq \sum_{i=1}^{M} a_{ij} x_{ij} \leq \sum_{l=1}^{r_j} l\, y_{jl}\, g_j, \tag{8}$$

and we can replace (3) with

$$\sum_{i=1}^{M} a_{ij}x_{ij} - \sum_{l=1}^{r_j} l\, y_{jl}\, g_j \leq 0, \quad j = 1, \ldots, N, \tag{9}$$

and (4)

$$\sum_{i=1}^{M} a_{ij}x_{ij} - \sum_{l=1}^{r_j} y_{jl} m P_j^l \geq 0, \quad j = 1, \ldots, N. \tag{10}$$

According to Proposition 2, the total number of groups is $\sum_{j=1}^{N}\sum_{l=1}^{r_j} l\, y_{jl}$, hence condition U6 is given by

$$\sum_{j=1}^{N}\sum_{l=1}^{r_j} l\, y_{jl} \leq G. \tag{11}$$

Finally, we define model (PRG) as a model for assigning students to a limited number of groups that satisfy conditions U1–U6.

$$(PRG)\begin{cases} \sum_{i=1}^{M}\sum_{j=1}^{N} c_{ij}x_{ij} \to \max \\ \sum_{j=1}^{N} a_{ij}x_{ij} \leq K, \quad i = 1, \ldots, M; \\ \sum_{i=1}^{M} a_{ij}x_{ij} - \sum_{l=1}^{r_j} l y_{jl} g_j \leq 0,\ j = 1, \ldots, N, \\ \sum_{i=1}^{M} a_{ij}x_{ij} - \sum_{l=1}^{r_j} y_{jl} m P_j^l \geq 0,\ j = 1, \ldots, N, \\ \sum_{j=1}^{N}\sum_{l=1}^{r_j} l\, y_{jl} \leq G, \\ \sum_{l=1}^{r_j} y_{jl} \leq 1,\ j = 1, \ldots, N, \\ x_{ij} = 0 \text{ or } 1,\ i = 1, \ldots, M,\ j = 1, \ldots, N, \\ y_{jk} = 0 \text{ or } 1, \quad j = 1, \ldots, N,\ k = 0, \ldots, r_j. \end{cases}$$

Models (PR) and (PRG) allow students to be assigned to courses which are their last choices, although the courses, which are their first choices, are still available. To avoid this case, an additional condition U7 is added to the models (PR) and (PRG). Condition U7 ensures that student satisfaction for every student is at least $B^S$, and is given by

$$\sum_{j=1}^{N} c_{ij}x_{ij} \geq B^S, \quad i = 1, \ldots, M, \tag{12}$$

The choice of $B^S$ is arbitrary, but for large $B^S$, there is a high probability that the model for assigning students does not provide a solution.

| | Additional conditions | | |
|---|---|---|---|
| Basic models | U7 | U8 | U7 and U8 |
| (PR) | (PR*) | (PR1) | (PR1*) |
| (PRG) | (PRG*) | (PRG1) | (PRG1*) |

Table 1: *The names of assigning models with different additional conditions.*

Condition U8 is introduced to ensure that all the courses are enrolled by equally motivated students. The basic idea is that: if the coefficients of satisfaction are higher, students are more motivated and teaching results are better. For the course $P_j$, we define the average satisfaction of students by

$$\frac{\sum_{i=1}^{M} c_{ij}x_{ij}}{\sum_{i=1}^{M} a_{ij}x_{ij}},$$

where the denominator is number of students enrolled in $P_j$, and the nominator is the sum of the coefficients of satisfaction for these students. From

$$\frac{\sum_{i=1}^{M} c_{ij}x_{ij}}{\sum_{i=1}^{M} a_{ij}x_{ij}} \geq B^C \Leftrightarrow \sum_{i=1}^{M} c_{ij}x_{ij} \geq B^C \sum_{i=1}^{M} a_{ij}x_{ij}$$

it follows that condition U8 is given by

$$\sum_{i=1}^{M} \big(c_{ij} - B^P a_{ij}\big)x_{ij} \geq 0, \quad j = 1, \ldots, N. \tag{13}$$

Conditions U7 and U8 can be combined with both models (PR) and (PRG). The names of the models with different additional conditions are given in the Table 1.

As all variables in the aforementioned models can have only the values 0 or 1, the target function is obviously upper bounded. If we are able to find a feasible solution, we have proved the existence of a solution. Unfortunately, we are only able to provide some of the necessary conditions for the existence of a solution, however the sufficient conditions are unknown.

From the nature of the problem, it becomes evident that the solution is not unique.

## 3. Application of the models

In this section, we apply the assigning models to a real-life problem. The main parameters of the observed problem are

- The number of students is $M = 166$, the number of elective courses is $N = 21$.
- Every student has to choose and rank $L = 4$ preferable courses, and is assigned to the $K = 2$ courses.
- The coefficient of satisfaction is 4 for the course of first choice, 3 for the second one, 2 for the third choice and 1 for the last choice.

- All students enrolled in course $P_j$, can be divided into no mora than two groups, i.e., $r_j = 2$ for $j = 1, \ldots, N$.

- The lower and upper bounds for the number of students in the $k$ groups and maximum number of students in the one group are the same for all courses:

  $$g_j = 28,\ MP_j^1 = g_j,\ MP_j^2 = 2g_j,\ mP_j^1 = mP^1,\ mP_j^2 = mP^1 + g_j, j = 1, \ldots, N.$$

  The lower bound for the number of students in one group, $mP^1$, can have values of 10, 11 and 12.

- The maximum number of groups is $G = 16$.

Table 2 contains data on student preferences. The number of interested students and their average coefficient of satisfaction are given for each course.

| Course | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ | $P_{11}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 9 | 10 | 73 | 28 | 67 | 3 | 86 | 41 | 21 | 34 | 89 |
| B | 2,33 | 2,70 | 2,77 | 1,96 | 2,16 | 1,33 | 2,76 | 2,44 | 2,10 | 2,32 | 2,83 |
| Course | $P_{12}$ | $P_{13}$ | $P_{14}$ | $P_{15}$ | $P_{16}$ | $P_{17}$ | $P_{18}$ | $P_{19}$ | $P_{20}$ | $P_{21}$ | |
| A | 10 | 14 | 48 | 9 | 14 | 26 | 19 | 36 | 12 | 15 | |
| B | 2,20 | 2,36 | 2,67 | 2,11 | 2,29 | 1,88 | 1,84 | 2,78 | 2,50 | 3,07 | |

Table 2: *The collected data about students interest in particular course. A - Number of students interested in course, B - average coefficients of satisfaction of students interested in course.*

Although the minimum student satisfaction $B^S$ is not listed as an original parameter to the problem, the number of dissatisfied students is smaller when the condition of minimum student satisfaction is included. We set $B^S = 5$.

The problem is solved by the use of models (PRG), (PRG1) and (APRG$^*$), which is an adapted version of model (PRG$^*$). Model (PRG$^*$), for the number of groups $G = 16$ and $B^S = 5$, has no solution if $mP^1 \geq 10$, so we develop an adapted model (APRG$^*$). The first step in the model (APRG$^*$) is to find a solution to the model (PRG$^*$) for the smallest possible number of groups $G^*$ and let $P^*$ be the set of the all included courses. Let $g$ be the difference between $G^*$ and $G$. Subsequently, for all subsets of $P^*$ with $g$ elements, we solve the model (PRG$^*$) subject to further limitations:

- Only courses from $P^* \setminus T$ are allowed, where $T$ is any subset of $P^*$ with $g$ elements.

- If a student $S_i$ can not achieve minimum satisfaction in the courses from $P^* \setminus T$, we do not apply the minimum satisfaction condition for $S_i$.

Let $R_T$ be the solution to (PRG$^*$) for a particular $T \subset P^*$. Then the solution to (APRG$^*$) is the best of solutions $R_T$. The details of model (APRG$^*$) are shown in Figure 1.

Solve the model (PRG$^*$) for the smallest possible number of groups $G^*$. Let $P^* = \{P_{j_1}, \ldots, P_{j_{G^*}}\}$ is as set of all courses included in the solution, and let $P_E^* = P \setminus P^*$ as the set of omitted courses.
$g \leftarrow G^* - G$. // number of groups to discard
$T_g$ = set of the all subsets of $P^*$ with $g$ elements.
$i \leftarrow 0$
**for all** $T \in T_g$ **do**
  Let $S_E$ represent the set of all students where the minimum satisfaction condition is unattainable when courses $P_E = P_E^* \cup T$ are omitted.
  **for all** $P_s \in P_E$ **do**
    $mP_s \leftarrow 0$ // exclude $P_s$ from solution
    $MP_s \leftarrow 0$
  **end for**
  **for all** $S_r \in S_E$ **do**
    $B_r^S \leftarrow 0$ // for student $S_r$ do not apply the minimum satisfaction condition
  **end for**
  $i \leftarrow i + 1$
  $R_i$ is the solution to the model (PRG$^*$), but without courses from $P_E$, and without the condition of minimum satisfaction for all students from $S_E$.
**end for**
The final solution is the best of the solutions $R_i$.

Figure 1: *Algorithm for the model (APRG$^*$).*

The solution to the problem for the parameter $mP^1$ and various models are given in the Table 3. Given in Table 4 is the structure of the solutions, i.e., which courses are included in the particular solution. In the all solutions, except for solutions 6 and 7, students are assigned to 16 groups. Furthermore, for all solutions, students enrolled in courses $P_3$, $P_7$ and $P_{11}$ are divided into two groups, but other courses included in the solutions have only one group. For all the models, a higher $mP^1$ increases the number of students with a student satisfaction less than $B^S$.

Model (PRG) is the basic model for the case when students can be enrolled in one or more groups, hence the associated solutions have the highest value of the target/goal function compared to models (PRG1), (PRG$^*$) and (APRG$^*$) for the same parameters. The presented solutions have only one student who is assigned to one course and the number of students with a student satisfaction smaller than $B^S$ is also small.

Using the model (PRG1) somehow complicates matter because the minimum average satisfaction $B^C$ has to be chosen carefully, but there is no obvious choice as is the case for minimum student satisfaction $B^S$ and the best approach is to experiment with different $B^C$. In this problem, for solutions 4 and 7, $B^C$ is set to 2.80, for solution 5, $B^C$ is 2.60, and for solution 6, $B^C$ is 2.75. It is interesting to compare solutions to the models (PRG) and (PRG1) for $mP^1 = 10$ (solutions 1 and 4) and $mP^1 = 11$ (solutions 2 and 3). In the case when $mP^1 = 10$, both solutions have same value of goal the function, but the solution to model (PRG1) looks better because there are no students assigned to only one course and there are also fewer students where the student satisfaction is smaller than $B^S$. In the case when $mP^2 = 11$, the solutions have a similar behaviour as in the case of $mP^1 = 10$. For the case where $mP^1 = 12$, two solutions are presented which differ in the choice of $B^C$. When $B^C = 2.60$ (solution 6), the solution includes 15 groups, but when $B^C = 2.80$ (solution 7), the solution include only 14 groups. As expected, decreasing the number of groups causes an increasing number of dissatisfied students.

| A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| 1 | PRG | 10 | 1096 | 165 | 1 | 4 | 2,70 |
| 2 | PRG | 11 | 1095 | 165 | 1 | 4 | 2,64 |
| 3 | PRG | 12 | 1089 | 165 | 1 | 5 | 2,50 |
| 4 | PRG1 | 10 | 1096 | 166 | 0 | 3 | 2,80 |
| 5 | PRG1 | 11 | 1095 | 166 | 0 | 3 | 2,64 |
| 6 | PRG1 | 12 | 1077 | 161 | 5 | 8 | 2,75 |
| 7 | PRG1 | 12 | 1060 | 159 | 7 | 11 | 2,83 |
| 8 | APRG* | 10 | 1091 | 165 | 1 | 2 | 2,70 |
| 9 | APRG* | 10 | 1089 | 166 | 0 | 2 | 2,64 |
| 10 | APRG* | 11 | 1088 | 165 | 1 | 2 | 2,55 |
| 11 | APRG* | 11 | 1085 | 166 | 0 | 2 | 2,36 |
| 12 | APRG* | 12 | 1084 | 165 | 1 | 2 | 2,50 |
| 13 | APRG* | 12 | 1080 | 166 | 0 | 2 | 2,25 |
| 14 | APRG* | 12 | 1057 | 166 | 0 | 1 | 2,25 |

Table 3: *Solutions to the problem. Description of the columns: A - Solution number, B - Model, C - $mP^1$, D - The value of the goal function, E - number of the students assigned to the two courses, F - number of the students assigned to the one course, G - number of the students with student satisfaction less than $B^S$, H - The minimum average satisfaction for any course included in the solution.*

From the definition of the model (APRG*), it is evident that the solutions to the model is one of the solutions to many auxiliary problems, specifically the one with the greatest value of the goal function. On the other hand, we can observe other measures, like the number of students assigned to the two courses or the number of dissatisfied students. Therefore Table 3 presents two or more solutions for the model (APRG*) using the same parameters - the solution with a maximum value of the goal function, and the solution where all students are assigned to the two courses (if there are multiple such solutions, the solution with the maximum goal function is chosen). Generally, solutions for the model (APRG*) have a smaller value for the goal function than the solution for the model (PRG) based on the same $mP^1$, however the number of dissatisfied students is smaller. The last presented solution is interesting because the number of dissatisfied students is only one, but there is a significant difference in the value of the goal function between that solution and solution 13, where the number of dissatisfied students is two. The conclusion is that the attempt to decrease the number of dissatisfied students results in increasing the number of students whose satisfaction is below the maximum.

| | Solutions | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Course** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| $P_2$ | | | | | | | | | + | | | | | |
| $P_3$ | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| $P_4$ | + | + | + | + | + | + | + | + | + | | | | | |
| $P_5$ | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| $P_7$ | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| $P_8$ | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| $P_9$ | | | | | | + | | + | | + | + | + | + | |
| $P_{10}$ | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| $P_{11}$ | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| $P_{13}$ | + | | + | | | | | | | + | + | + | + | + |
| $P_{14}$ | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| $P_{16}$ | + | + | + | + | + | | | + | + | + | | + | | + |
| $P_{17}$ | + | + | + | + | + | + | + | | | + | + | + | + | + |
| $P_{18}$ | | | | | | | | | | | + | | + | + |
| $P_{19}$ | + | + | + | + | + | + | + | + | + | + | + | + | + | + |
| $P_{20}$ | | + | | + | + | | | + | + | | | | | |
| $P_{21}$ | + | + | + | + | + | + | + | + | + | + | + | + | + | + |

Table 4: *The structure of solution. The vertical lines divide solutions from the various models.*

All calculations were done using the software program *Mathematica* and the associated solver `LinearProgramming`, which can solve integer programming problems. The time to find the solution for the model (based on a particular choice of parameters) is between 20 seconds and 4 minutes, so it seems that there is no reason to develop a special algorithm for solving the problem.

## 4. Conclusion

An approach was developed to the problem of assigning students to elective courses at academic institution. As the timetable was devised after the assignment process, the problem is less demanding than the classical course scheduling problem, hence we directed our attention to maximizing student satisfaction. In the standard two-round system, the main problem is that all the students cannot be assigned to their desired courses due to physical (too many students) or financial (insufficient number of students) constraints. In the new system, we allowed students to enroll into more desired courses based on preference rankings. The integer programming model was built according to operational demands where student preferences were built into the model's objective function and the additional constraints (minimum student satisfaction), which assured a better choice of the courses when possible. The former two-round system is now replaced with the one-round system, with a shortened time span and students enrolled into the courses they prefer. The models were not tested in the situation with large number of variables.

Additionally, the models can be easily extended to satisfy the demands of "giving better students priority" or the fact that "some courses are important and have to be included in the solution" by modifying the goal function, coefficients of satisfaction and existing constraints. The main contribution of the new model is the introduction of the concept of minimum student satisfaction, which reduces the number of students unsatisfied with their assigned courses.

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