Current Cognition of Rock Tensile Strength Testing By Brazilian Test

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Abstract

Tensile strength of intact rock materials has been determined by the indirect method more frequently than by the direct method. This paper presents a historical review of the development of the indirect method called the Brazilian test, comprising the period from 1943 to the present day. It stresses some aspects which are essential for interpreting the results of the Brazilian test due to the different degrees of stress during the testing and the direct method of determining tensile strength. The estimate of the direct tensile strength and the influence of sample saturation on the results of indirect tensile strength testing have been specifically elaborated.

Keywords

Tensile strength, indirect tensile strength, Brazilian test, rock mechanics

1. Introduction

Tensile strength of rock material is usually defined as the maximum tensile stress which can be endured by such a material. Rock material usually has a low tensile strength, which can be determined by direct and indirect methods of which the most famous is the Brazilian test. The direct testing procedure is carried out on the samples which require demanding processing conditions. Tensile stress is transferred to the ends of the samples using a cemented or glued steel plate with hooks on it (**Figure 1**). It is very important that the axis impact of the tensile matches with the axis of the sample without bending or torsion, i.e. other external impacts (**Hoek, 1964**). In order to ensure that the sample breaks at a specific point, it can be manufactured in the form of a dumbbell or dog – bone.
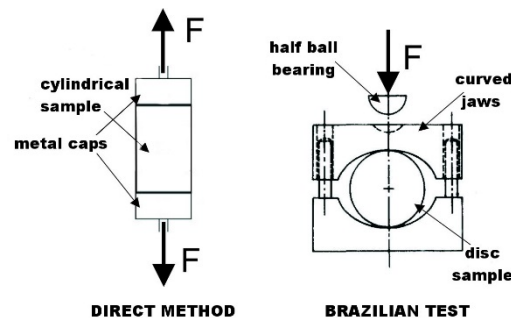

Figure 1. The principle of testing tensile strength by the direct method and the Brazilian test

Although deficiencies were observed due to the stress concentration at the ends of the sample and therefore a number of other ways of loading were designed, such as biaxial extension (**Brace, 1964**) and tension load converter (**Gorski, 1993; Klanphumeesri, 2010**), the International Society for Rock Mechanics has not changed its recommendations. These include the application of tensile stress by cemented caps on the end of the cylindrical sample. The diameter of the metal caps should not be less than 2 mm compared to the sample diameter and should be regularly glued. The ratio between the

diameter and the height of the sample should be 2.5 to 3, whereas the sides should be smooth and without roughness larger than 0.3 mm. The bases must be aligned with no irregularities larger than 0.02 mm and should not deviate from perpendicularity to the axis of the sample of 0.001 radians, which is approximately 0.05 mm for the sample diameter of 50 mm. Tensile stress is applied continuously with a constant increase, so that the fracture occurs within about 5 minutes, or grows between 0.5 and 1 MPa/s. The direct tensile strength is calculated by dividing the maximum load applied to the sample by the initial cross-sectional area (**ISRM, 1978**). Due to the complexity of direct methods, particularly in the preparation of samples and applying the load, the Brazilian test, which indirectly determines tensile strength, is more frequently used in engineering practice (**Figure 1**). The justifiability of this method is based on the experimental fact that the majority of the rocks that are in the state of the biaxial stress break due to the exceeded tensile strength, in the conditions when one principal stress is tensile, and the second compressive, the size of which does not exceed three times the tensile stress (**Briševac, 2012**).

2. Historical overview of development of Brazilian test

The Brazilian test is a popular method of indirect determination of the tensile strength of intact rock material that has been the topic of a large number of research works and debates for many years regarding its accuracy and correct use. In the last 72 years, this method has attracted many researchers and accordingly its development can be divided into three phases. The first phase began in 1943 when Carneiro proposed the test methods to obtain tensile strength of concrete (**Carneiro, 1943**). It lasted until 1978 when the International Society for Rock Mechanics issued a recommendation for its determination on the rocks (**ISRM, 1978**). Afterwards, in the period from 1979 to 1991, the second phase was characterized by the use of the standardized method of the Brazilian test. The last phase started in 1991 and continues today. It is characterized by the improvement of the original testing method (**Wong and Jong, 2013**). The same three-phase division of Wong and Li is elaborated in this paper (**Li and Wong, 2013**).

From a chronological point of view, the creator of the indirect method is Carneiro (**1943**), who introduced a test method in order to obtain the value of tensile strength of concrete and created the formula (1) to calculate tensile strength of the sample in the form of a disc which is biaxially burdened compressively by plates with a width of 0.1 diameter of samples

$$\sigma_{BTS} = \frac{2F}{\pi \cdot D \cdot t} \tag{1}$$

Where:
σ_{BTS} - indirect tensile strength determined by the Brazilian test (MPa),
F – applied load (kN),
D – diameter of the tested sample (mm),
t – thickness of the tested sample (mm).

In two months, Carneiro overtook the Japanese researchers Akazawa (**1943**), who presented a very similar method, and because of the war, there was no contact between them. This method in intact rock material was applied for the first time by Berrenbaum and Brodie (**1959**).

Assuming that the material is homogeneous, linearly elastic and isotropic, Hondros (**1959**) performed a complete solution of the stress in the case when the load was distributed over the finite arcs with diametric compression, comprising both, the discs and the cylinders.

Hobbs (**1964, 1965 and 1967**) was engaged in calculations of tensile strength on several occasions. He calculated it by diametrical compression of a disk with a central hole (ring test). The obtained results were then compared with the results of the Brazilian test. He studied the influence of lamination of rock material on the tensile strength and concluded that the variations in strength approximately match with the variations according to the Griffith failure criterion. He pointed to the existence of the relationship between the tensile and compressive strength of rock and other alternative measurements in rock mechanics.

Generalizing the Griffith failure criterion Fairhurst (**1964**) studied the fracture of the samples of the Brazilian test. It was concluded that the fracture may appear at the distance from the disc centre in the case of small angles of disc loading on the materials of a low ratio between compression and tension. Fairhurst found out that the value of the tensile stress obtained by the Brazilian test is slightly lower than the real value.

Having analyzed the strain on irregular samples exposed to the pair of concentrated loadings in the three-dimensional photoelectric experiments, followed by the mathematical analysis of the results, Hiramatsu and Oka (**1966**) presented a

new method of calculating the tensile strength for irregular samples. It was determined that the tensile strength of a rock could approximately be determined as the 0.9 part of the critical loading under which the sample broke, divided by the square of the distance between the loading points. However, the disadvantage of this method is the fact that the loading condition on such samples is neither simple nor universal but depends on the shape of the specific sample.

Colback (**1966**) found that the reason for such a small number of research works on direct tensile strength lies in the difficulties of preparing the samples and the fact that the incorrectly prepared cylindrical samples break outside the central part and thus create invalid results. Colback considered the Brazilian test as the acceptable solution for the increased number of research works. He used the corrected Griffith failure criterion theory in order to project the appearance of fracture in the disc sample of the Brazilian test. According to Colback, the fracture should start in the centre of the disc, in order to perform the correct test. Under certain circumstances, the fracture would start at the initial loading points, which would make the test invalid. He recommended the control of samples after each test, in order to determine the correctness of the fracture.

Jaeger and Hoskins (**1966**) and Hoskins (**1967**) alone compared the theoretical and experimental values determined for three different methods of measuring tensile strength, using compressive stress. It was found that the calculated value of the maximum tensile stress depends on the value of the uniaxial tensile strength of the material. Due to the high variability of tensile strength of the samples, even those of the same rock, it was recommended to test a larger number of samples.

Hudson (**1969**) compared two indirect methods, the Brazilian test and the ring test, and concluded that the tensile strength determined in such a way is a rather experimental characteristic, not the characteristic of the material.

Mellor and Hawkes (**1971**) measured tensile strength by means of the radial compression of the Brazilian disc for three types of rocks, two plastic materials, a glass material and an ice material, and the experimental results were compared with the theoretical expectations. To reduce the strain of the initial loading points, they designed the curved jaws for loading stress on the sample. The research proved that the Brazilian test is a suitable test for determining the values of uniaxial tensile strength.

Hudson and his associates (**1972**) observed that the fracture of the Brazilian disc appeared directly under the initial loading points only if flat steel plates are used in testing by means of controllable power test devices. The above stated authors recommend neither the Brazilian test nor the ring test for determining the tensile strength properties of such materials.

Barla and Innaurato (**1973**) experimentally and numerically investigated the suitability of the indirect methods of measuring tensile strength on anisotropic rock samples. They used two types of samples under different orientations along the axis of anisotropy. They proved that the experimental results can be appropriately explained by the finite element method.

Having studied the application of three-dimensional distribution of the stress on the problems with the strain and deformations in the plane, Wijk (**1978**) proved that the three-dimensional "correction" of the two-dimensional theoretical solution should be considered, even for rather thin samples in the case of the Brazilian test. These theoretical observations show that the interpretation of the indirect tensile test of rock materials is more difficult than was expected.

The team of researchers led by Yanagidani (**1978**) used strain gages for crack detection and noticed that the initial crack does not originate from the point of initiation of stress, but the zone of tensile stress. His results were important for valuation of the Brazilian test.

The International Society for Rock Mechanics (**ISRM, 1978**) adopted the standardized method of the Brazilian test to be used for indirect measuring of the tensile strength according to the formula (2). However, the method of direct measuring of the tensile strength still remains the main method used for determining the tensile strength of rock.

$$\sigma_{BTS} = 0,636 \frac{F}{D \cdot t} \qquad (2)$$

Where:
σ_{BTS} - indirect tensile strength determined by the Brazilian test (MPa),
F – applied load (kN),
D – diameter of the tested sample (mm),
t – thickness of the tested sample (mm).

Lajtai (**1980**) concluded that the tensile strength determined by the point load testing is lower than the values of tensile strength obtained by the Brazilian test and that the Brazilian test seemed to yield a more accurate definition of both the tensile strength and its variation of directions.

Sundaram and Corrales (**1980**) pointed out that assuming the same elastic characteristics of the tensile and compressive stress could result in the overestimation of values of the tensile strength of rocks obtained by the Brazilian test and emphasized the importance of considering separate elastic characteristics of the sample under tensile and compressive stress.

Pandey and Singh (**1986**) analyzed the characteristics of deformation in tensile stress and discovered that the tensile strength obtained by the Brazilian test was almost two times larger than the values obtained by the direct method. Accordingly, they claimed that tensile strength is actually an experimental characteristic, not the characteristic of the material.

Based on the study of the interdependence between the Block Punch Strength Index and the indirect tensile strength determined by the Brazilian test on the samples of breccia, calcarenite, dunite, gneiss, limestone, marble, sandstone and mudstone, a formula (3) was created to calculate the indirect tensile strength with the BP index (**Van der Schrier, 1988**).

$$\sigma_{BTS} = 0,4BPI - 0,4 \tag{3}$$

Where:

σ_{BTS} - indirect tensile strength determined by the Brazilian test (MPa),
BPI – block punch strength index (MPa)

In the statistical study carried out by Newman and Bennett (**1990**), it was confirmed that the ratio between the length and the diameter of the sample, as well as the stress rate, have a significant impact on the indirect tensile strength of sandstone. Andreev (**1991a, b**) revised the process of determining the tensile strength using the Brazilian test, concentrating on the formula and the conditions of the contact on the sample. It was determined that the fracture in the contact area of the sample does not appear if its modulus of elasticity is significantly different from the modulus of elasticity of the stress applicator. Contrary to the earlier opinion, it was concluded that the Brazilian test provides the corresponding values for the materials which perform brittle fractures. The following formula (4) for calculation of the indirect tensile strength of such materials is suggested:

$$\sigma_{BTS} = \frac{2F}{\pi \cdot D \cdot t} \cdot \left(n + \frac{3}{n} \right) \tag{4}$$

where:

σ_{BTS} - indirect tensile strength determined by the Brazilian test (MPa),
F – applied load (kN),
D – diameter of the tested sample (mm),
t – thickness of the tested sample (mm),
n – ratio between the uniaxial compressive and tensile strength.

The development of information technology enabled more demanding research works. Accordingly, simulations of fracture by means of computer software were made possible. A two-dimensional numerical simulation was created, based on the research of initiation and the growth of a crack, using the Mohr-Coulomb's failure criterion (**Malan et al. 1994**).

The information age has also facilitated the calculation of complex mathematical problems, but also more complex laboratory measurements. After conducting the testing of the indirect tensile strength of the anisotropic type of rocks (4 types of sandstone), Chen and his associates (**Chen et al, 1998**) concluded that the method based on the elastic isotropic assumptions should not be used in the analysis of anisotropic rocks. They proved that the indirect tensile strength of anisotropic rocks is not constant but depends on the angle between the plane anisotropy axis and the load on the sample. Based on the Brazilian test, they developed the method of measuring elasticity constants, involving displacement measurements using strain gages that are glued at an angle of 45 ° relative to the axis of the load.

Exadaktylos and Kaklis (**2001**) presented the explicit expressions of strains and deformations at any point of radially loaded anisotropic circular disc proved that the proposed analytical solutions can be effectively used in the feedback analyses and as a tool for determining the elasticity and strength of rock. It is presented both, in the case before the published experimental results on schists and gneisses and in the case of their testing on marble.

Based on the conducted testing of samples of 23 rock types, Sulukcu and Ulusay (**2001**) published the formula (5) for calculating the indirect tensile strength using the corrected block punch strength index and the point load strength index with the following formula (6).

$$\sigma_{BTS} = 0,86 \cdot BPI_c \tag{5}$$

$$\sigma_{BTS} = 2,3 \cdot I_{S(50)} \tag{6}$$

Where:
σ_{BTS} - indirect tensile strength determined by the Brazilian test (MPa),
BPI_c – corrected block punch strength index (MPa),
$I_{S(50)}$ – point load strength index (MPa).

Lavrov and Vervoort (**2002**) analyzed the impact of the friction force applied to the two opposite ports of stress distribution in the Brazilian test. They concluded that the distribution of stresses within the sample was slightly affected by the influence of friction in its ends.

After observing the acoustic effects caused by the test, the analysis of the Kaiser degradation effect due to increased deviation from the main strain between loading cycles in the Brazilian tests was presented. The analysis of discontinuous shifts was used to explain the experimental results (**Lavrov et al, 2002**).

Inhomogeneity of the material has a significant impact on the distribution of tensile stress along the axis of loading. In order to present this fact, the finite element method (FEM) for two-dimensional analysis of geomaterials was used. By means of this method, the inhomogeneities and microstructures of the samples were modelled (**Yue et al, 2003**).

The Chinese-Swedish researchers have designed a method that uses a flattened Brazilian disc. In one testing it would be possible to determine elasticity modulus, the indirect tensile strength (7) and the fracture toughness for brittle rocks. In order to enable the initiation of cracks in the centre of the sample, which is considered crucial for the validity of the test, the angle of loading must be larger than 20 ° (**Wang et al. 2004**).

$$\sigma_{BTS} = \frac{2F}{\pi \cdot D \cdot t} \cdot k_w \tag{7}$$

Where:
σ_{BTS} - indirect tensile strength determined by the Brazilian test (MPa),
F – applied load (kN),
D – diameter of the tested sample (mm),
t – thickness of the tested sample (mm),
k_w – coefficient which is closely related to the loading angle 2α, for $2\alpha=20º$ $k= 0,9644$; $2\alpha=30º$ $k= 0,9205$.

To determine the location of the crack, South African scientists conducted tests on the discs in which they drilled holes of different diameters and changed the position of the holes on the sample. After the experiment and simulation of the fracture they assumed that the fracture on the sample of the Brazilian test begins to spread in the vicinity of one of the sources of pressure on the disc (**Van De Steen et al., 2005**).

The Italian scientists, who experimented with various alternative ways of determining the value of tensile strength, concluded that it strictly depends on the selected test methods. They presented a critical assessment of some widely used laboratory techniques based on the experimental data obtained from the literature or their own ones and concluded that out of all the laboratory methods only the Brazilian test provides results similar to those obtained by the direct determination of tensile strength of soft rocks (**Coviello et al., 2005**).

Aydin and Basu (**2006**) observed that the weathering caused by the tropical climate weakens the microstructure of magmatic rocks. Accordingly, rock behaviour during tensile tests can be an indicator of the state of the microstructure and the weathering of rocks. Therefore, they created an indicator consisting of a 2 cm long strain gage located on the horizontal axis of the sample. In this way, they measured the size they called the Brazilian deformation index BDI on the basis of which the minimum variations caused by weathering can be obtained.

The stress analysis based on the three-dimensional finite element method 3D FEM showed the importance of the size and shape of the Brazilian disc. The conclusion was reached that the formula used in the past to calculate the indirect tensile

strength is incorrect and must be corrected according to formula (8) in relation to the ratio of the thickness / diameter, especially in the case of patterns of increased thickness (**Yu et al., 2006**).

$$\sigma_{BTS} = (0,2621 \cdot k_R + 1)\frac{2F}{\pi \cdot D \cdot t} \tag{8}$$

Where:
σ_{BTS} - indirect tensile strength determined by the Brazilian test (MPa),
F – applied load (kN),
D – diameter of the tested sample (mm),
t – thickness of the tested sample (mm),
k_R – ratio between the thickness and diameter of the test specimen.

A team of Chinese scientists has designed an experimental method for obtaining a tensile modulus of elasticity. The method used strain gages glued on the samples during the Brazilian test. The analytical formula of the tensile elasticity modulus is represented by the expression (9). They found that the ratio between the tensile elasticity modulus and the compressive elasticity modulus is from 60 to 90% for marble, sandstone and limestone (**Ye et al., 2009**).

$$E_{BTS} = A_T \cdot E_S \tag{9}$$

Where:
E_{BTS} - tensile elasticity modulus (MPa),
A_T – correction coefficient which can be calculated from formula (10),
E_S – splitting elastic modulus which can be calculated from formula (11).

$$A_T = \left(1 - \frac{D}{t} \cdot \arctan\frac{2t}{D}\right) \cdot (1-v) + \frac{2D^2 \cdot (1+v)}{4t^2 \cdot D^2} \tag{10}$$

$$E_S = \frac{0,5\sigma_t}{\varepsilon_t} \tag{11}$$

Where:
D – diameter of the tested sample (mm),
t – thickness of the tested sample (mm),
v – Poisson's ratio,
σ_t – maximum stress in test (MPa),
ε_t – the strain related to $\sigma_t/2$ in the stress–strain curve.

The analysis of the impact of the initial cracks on the distribution of stress has led to modelling by the Boundary Element Method BEM, i.e. the simulation process of the appearance and spreading of the cracks on the sample exposed to indirect tensile stress. The analyses of the models show that the stress distribution is only slightly affected by the friction between the jaw and the sample.

However, the appearance and spreading of cracks produced by the stress field is very different compared to the situation when the rock material is continuous, homogeneous, isotropic and elastic. It is considered that the analysis of the deformation of the sample along the diameter perpendicular to the load direction allows a direct determination of the tensile strength in the way that it is indicated by the point where the curve of the stress and malformation is perpendicular to the loading direction (**Lanaro et al., 2009**).

The stress analysis by the 3D FEM Method, assuming that the test materials are continuous, homogeneous and isotropic elastic, has enabled the production of a modified version of the Brazilian test.

Two special spacers are placed on the points of the initial loading, in order to reduce the stress concentration. This modification was introduced as it was considered that the original version of the Brazilian test significantly underestimates

the tensile strength of the test material. They suggested that the indirect tensile strength should be calculated by the expression (12) for brittle rocks (**Yu et al., 2009**).

$$\sigma_{BTS} = \frac{2k_S \cdot F}{\pi \cdot D \cdot t} \tag{12}$$

Where:

σ_{BTS} - indirect tensile strength determined by the Brazilian test (MPa),

F – applied load (kN),

D – diameter of the tested sample (mm),

t – thickness of the tested sample (mm),

k_S – coefficient which relates to the state of stress in the point of initiation, and its best value is 1,11.

Tavallali and Vervoort (**2010a, b**) presented the importance of the orientation layers, i.e. anisotropy in tensile strength and the crack patterns in the Brazilian test. They observed three different types of cracks in the specific stratified sandstone from the south of Belgium and called them layer activation, central fractures, and non-central fractures. They described the effect of the size and shape of grain and mineral composition on tensile strength established by the Brazilian test.

Greek scientists published the papers stating a complete explanation of stress and displacement in the Brazilian disc under the evenly distributed radial load. However, the solution was presented under the assumption that the sample material is homogeneous, isotropic and linearly elastic. They studied the effect of evenly distributed shear stress at the points of loading. They noticed that the fracture first started at the edge of the initial points and then in the middle of the sample (**Markides et al., 2010 and 2011**).

Australian scientists presented the results of the experimental research on the relationship between the strain and the malformation of the Brisbane tuff samples which were loaded in two different ways. The first one was a sinusoidal cyclic loading and the second one was a cyclic loading with the increasing mean level. They found that the indirectly measured tensile strength was reduced by 33-37% under such conditions of performing the Brazilian test (**Erarslan and Williams, 2012**). The loading impact was compared on the samples of Brisbane tuff by means of the standard jaw with various loading arc angles of the sample. Besides the experimental tests, numerical simulations were performed too. The obtained results showed that the best geometry of loading to is $2\alpha = 20° - 30°$ along the arc of the sample (**Erarslan et al., 2012**).

Markides and Kourkoulis (**2012**) analysed the components of stresses in the samples of the Brazilian disc. The samples were tested under four different types of loading: loading at one point, evenly distributed radial pressure, sinusoidal loading, parabolic loading pressure. The parabolic loading proved to be the best one. The disadvantage of such an analysis is the fact that it was carried out with the presumption of linear elasticity, ignoring the influence of shear.

Indian scientists created a formula (13) for the calculation of the indirect tensile strength for granite, schist and sandstone, in order to improve the possibilities of the BP test. According to their research, the interdependence of the indirect tensile strength determined by the Brazilian test is higher with the corrected block punch strength index, determined by the point load strength index (**Mishra and Basu, 2012**).

$$\sigma_{BTS} = 0,35 \cdot BPI_c + 3,69 \tag{13}$$

Where:

σ_{BTS} - indirect tensile strength determined by the Brazilian test (MPa),

BPI_c – corrected block punch strength index (MPa).

Kazareni (**2013**) presented a model based on the discontinuum which simulates the tensile and compressive failure of sandstone. The model reproduces the rock material in a better way, which is a dense group of irregular and distorted particles interacting at their borders. Kazareni concluded that such a way of modelling can be used to describe the relationship between microstructural and macro properties of materials.

Li and Wong (**2013**) published a review of the performed research works related to the Brazilian test and conducted a numerical analysis, having created the elastic model in the FLAC 3D program. By concentrating on the starting point of crack, they concluded that the starting point may be located near the points of loading in the moment when the tensile malformation meets the critical criterion of malformation, and that it may also be located in the centre when the tensile stress meets the maximum tensile strength criterion.

The recording with high-speed cameras was applied during the testing of indirect tensile strength under static and dynamic loading with granite samples for research purposes of the underground objects. Four ways of fracture patterns were observed: a diametrical tensile fracture, a central fracture with small wedges at the points of loading, a central fracture with a crushed zone and a fracture with the initiation from the shearly damaged frame zones. Accordingly, the authors recommended high-speed cameras to be used in the analysis of the correctness of fracture patterns as the usual optical analysis can be misleading (**Zhou et al., 2014**).

Turkish scientists have made the evaluation of indirect tensile strength by testing it with different types of jaws used to apply a pressure load on the sample. Their work shows that different materials react differently to loading and that the same type of jaw cannot be used for all the materials (**Kömürlü and Kesimal, 2015**). A disadvantage of their work may lie in the fact that only few tests were performed per individual material and the type of jaw, which are sometimes 4.

Recently, a team of the Chinese scientists presented a proposal for changes of the formula calculating the indirect tensile strength. In their opinion, Poisson's ratio was the most significant factor, including the angle of loading and the ratio of thickness and radius. They found that an increase in Poisson's ratio and a smaller angle reduces the appearance of the central initiation of cracks. Using the Griffith criterion, they modified the formula in accordance with the recommended range of values. The formula (14) has been adjusted to the previous studies, numerically and experimentally (**Lin et al., 2015**).

$$\sigma_{BTS} = -(1{,}027 + 0{,}108\nu - 0{,}014\alpha) \cdot \frac{F}{\pi \cdot r \cdot t} \tag{14}$$

Where:

σ_{BTS} - indirect tensile strength determined by the Brazilian test (MPa),

ν – Poisson's ratio,

α – half of the loading angle,

F – applied load (kN),

r – radius of the tested sample (mm),

t – thickness of the tested sample (mm).

According to the available literature, testing of tensile strength with the Brazilian test on the materials on the territory of Croatia comprises various studies conducted for the purpose of performing geotechnical structures, especially road infrastructure (**Šestanović et al., 1993, Pollak, 2002**), as well as the tests for the purpose of dimensioning underground chambers upon underground exploitation of dimension stone (**Hrženjak et al., 2008**). Indirect tensile strength was investigated under the force in the direction perpendicular to the surface layer and in the case of the force in the direction parallel with bedding planes of the A, E and F deposits of dimension stone "Korenići" in Istria (**Dobrilović et al. 2010**). The influence of saturation on the indirect tensile strength of gypsum (**Macenić, 2011**) and limestone (**Čajić, 2015**) was also investigated.

It is important to emphasize that there are differences between the results of the direct and the indirect measurements of tensile strength by the Brazilian test, which can be seen in **Table 1**, where the data was collected from the relevant literature and calculated the ratio σ_{BTS} / σ_{UTS}. However, some materials, e.g. limestone, partially show a reverse trend of the relations σ_{BTS} / $\sigma_{UTS,}$ compared to the collected data. The limestone types Indiana and Saraburi show that the Brazilian test overestimates their strength, while limestones collectively show that the test underestimates their strength.

Table 1: The ratio of tensile strength determined by the direct method and the indirect tensile strength determined by the Brazilian test

Rock type	σUTS	σBTS	σBTS/σUTS	References
Bowral trachyte	13,72	12,00	0,87	Jaeger (1967)
Trachyte	13,7	7,7	0,56	Perras and Diederichs (2014)
Vitosha syenite	20,50	21,05	1,03	Andreev (1991a)
Barre granite	13,45	14,34	1,07	Mellor and Hawkes (1971)
Granite	6,3	10,3	1,63	Perras and Diederichs (2014)
Carrara marble	6,90	8,72	1,27	Jaeger (1967)
Ufalei marble	5.90 ± 2.66	6.90 ± 1.24	1,17	Efimov (2009)
Saraburi marble	6.33 ± 0.62	8.02 ± 0.25	1,27	Fuenkajorn and Klanphumeesri (2011)
Marble	7,5	10,1	1,35	Perras and Diederichs (2014)
Gneiss	8,2	9,8	1,2	Perras and Diederichs (2014)
Quartzite	16,3	13,0	0,8	Perras and Diederichs (2014)
Schist	13,3	11,8	0,89	Perras and Diederichs (2014)
Shale	5,6	5,9	1,05	Perras and Diederichs (2014)
Grey gypsum	1,75	1,99	1,14	Andreev (1991a)
White gypsum	1,42	1,29	0,91	Andreev (1991a)
Gosford sandstone	3,59	3,72	1,04	Jaeger (1967)
Phu Phan sandstone	6.49 ± 0.22	10.68 ± 0.70	1,65	Fuenkajorn and Klanphumeesri (2011)
Sandstone	5,1	9,5	1,86	Perras i Diederichs (2014)
Gravina calcarenite	0.69 ± 0.03	0.64 ± 0.10	0,93	Coviello et al. (2005)
Indiana limestone	5,86	6,21	1,06	Mellor and Hawkes (1971)
Saraburi limestone	9.31 ± 0.65	10.90 ± 0.19	1,17	Fuenkajorn and Klanphumeesri (2011)
Limestone	7,1	6,0	0,85	Perras and Diederichs (2014)
Dolomite	5,7	8	1,4	Perras and Diederichs (2014)

σUTS – direct tensile strength; σBTS - indirect tensile strength determined by the Brazilian test

2. Estimating tensile strength with other characteristics

According to Hoek (**1966**) it is usually considered that a tensile strength is one tenth in size compared to the compressive strength and can be calculated by the expression (15).

$$\sigma_{UTS} = -\frac{\sigma_{UCS}}{10} \tag{15}$$

Where:
σ_{UTS} – direct tensile strength (MPa),
σ_{UCS} – uniaxial compressive strength (MPa).

Zhang (**2005**) suggested that the tensile strength for a particular rock material can be calculated by the formula (16). The same author stated the expression (17) for calculating tensile strength through point load strength index.

$$\sigma_{UTS} = -k_m \cdot \sigma_{UCS} \tag{16}$$

$$\sigma_{UTS} = -1,5 \cdot I_{S(50)} \tag{17}$$

Where:

σ_{UTS} – direct tensile strength (MPa),

σ_{UCS} – uniaxial compressive strength (MPa),

k_m – coefficient which is approximately from 0.03 to 0.24 depending on the type of rock,

$I_{S(50)}$ – point load strength index (MPa).

After reviewing the relevant literature Perras and Diederichs (**2014**) concluded that the largest error occurs upon estimating through uniaxial compressive strength, whereas there is a less error if the Hoek-Brown constant is used, m_i according to the expression (18).

$$\sigma_{UTS} = -\frac{\sigma_{UCS}}{m_i} \tag{18}$$

Where:

σ_{UTS} – direct tensile strength (MPa),

σ_{UCS} – uniaxial compressive strength (MPa),

m_i – Hoek-Brown constant.

Using the measured value of crack initiation (CI) to estimate the direct tensile strength according to the expression (19) provides better results than the correlation over the uniaxial compressive strength. However, such an estimate depends on the type of rock, too. The expression (19) should in the future be modified according to the material properties (**Perras and Diederichs, 2014**).

$$\sigma_{UTS} = -\frac{CI}{k_{\beta}} \tag{19}$$

Where:

σ_{UTS} – direct tensile strength (MPa),

CI – crack initiation (MPa),

k_{β} – coefficient which is 8 according to the original Griffith theory and can be as high as 12 according to the modified formulation of the Griffith theory.

Based on the literature in the field of determining the tensile strength, Swiss scientists generally concluded that the direct tensile strength is difficult to estimate on the basis of other laboratory tests. They concluded that the estimate of the direct strength through indirect tensile strength determined by the Brazilian test depends on the type of rock and recommended to be performed according to the formula (20) to improve the estimate (**Perras and Diederichs, 2014**).

$$\sigma_{UTS} = \sigma_{BTS} \cdot k_T \tag{20}$$

Where:

σ_{UTS} - direct tensile strength (MPa),

σ_{BTS} - indirect tensile strength determined by the Brazilian test (MPa),

k_T – correction coefficient which is approximately 0.9 for metamorphic, 0.8 for magmatic and 0.7 for sedimentary rocks.

3. Influence of saturation of samples on their indirect tensile strength

The issue of the influence of saturation on the tensile strength was only generally analysed in older research. Dube and Singh (**1972**) investigated the influence of humid climate on the indirect tensile strength of sandstone using the Brazilian test. It was discovered that the porous sandstones and those which have clay minerals are sensitive to the reduction of the tensile strength to the most.

Vutukuri (**1974**) conducted more extensive research, dealing with alternative methods of determining the tensile strength. He used the ring test for determining the tensile strength of limestone samples and examined the dependence of the effects of various types of liquids on the value of the tensile strength of limestone. The saturation in the period of 20 hours was carried out with: water, glycerine, ethanol, nitrobenzene, and several organic liquids. He found that the increase in the dielectric constant and surface tension of the liquid reduces the tensile strength of limestone.

Ojo and Brook (**1990**) examined the uniaxial compressive strength and the direct tensile strength of sandstone with a different degree of saturation. They found that both strengths decrease with an increasing degree of saturation, and the ratio between the uniaxial compressive strength and the direct tensile strength is larger in the dry state than in the saturated state. They stated that the decreased strength due to the saturation effect is more present in the direct tensile strength than the uniaxial compressive strength.

In view of more recent research works, the paper published by the team of Chinese scientists should be pointed out. They studied the effect of saturation of the samples with water on the indirect tensile strength of gneiss, marble and sandstone. The results showed that all of these rocks have a lower tensile strength at saturation compared to their dry state. The tests were performed by the Brazilian test and the ring test. It was concluded that the ring test shows slight differences of strength between dry and saturated samples (**You et al., 2011**).

Singaporean scientists studied the effect of saturation on the indirect tensile strength of synthetic gypsum, using high-speed cameras. The results show a rather high decrease of strength, half of the strength in the dry state after just a week of saturation. The recordings made with high-speed cameras show that the initial cracks in the majority of samples appeared in the centre, and that there is a difference between the way the fracture appeared in dry and saturated samples. Dry samples fractured after only one initiation and the crack was spreading rather quickly, whereas saturated samples had more initial cracks and slowly developed a primary tensile crack (**Wong and Jong, 2013**).

4. Discussion

The reason for a large number of the published papers regarding the Brazilian test is the fact that the indirect method, due to its constant development, has simplified the process of determining tensile strength of intact rock material. It has become possible to conduct a large number of testing procedures with a simple preparation of the samples, compared to the previous period when the preparation of samples was rather complicated and demanding. Tensile strength is an important characteristic of a specific material due to the fact that it is difficult to observe the conditions of fracture without it. Taking into consideration this characteristic, Griffith (**1921**, **1925**) introduced the criterion of strength for brittle materials. Afterwards, such a criterion was used in the rock mechanics in its original and modified forms, suggested by McClintock and Walsh (**1962**). A large number of papers on the Brazilian test comprises this criterion (Hobbs, **1964**, **1965** and **1967**; Fairhurst **1964**; Colback, **1966**; Linetal, **2015**). Although other criteria are more frequently used today, the area of tensile strength is modified according to Griffith, as it was done, e.g. by Hoek and Martin (**2014**) based on the conclusions stated in the paper by Fairhurst (**1964**).

The simple processing of samples enables a quick experimental check-up of the theoretical presumptions and their numerical analyses (Hiramatsu and Oka, **1966**; Barla and Innaurato **1973**; Sundaram and Corrales **1980**; Lavrovetal, **2002**; Yue et al., **2003**; Yu et al., **2006**; Lanaro et al.**, 2009**; Yu et al., **2009**; Markides and Kourkoulis, **2012**; Li and Wong, **2013**;). However, the sensitivity to the testing conditions requires a careful explanation of the material behaviour during the testing and finding the solutions for removal of the flaws (Hobbs, **1964**, **1965**, **1967.**; Hudson, **1969**; Hudson et al., **1972**; Yanagidani et al., **1978**; Lajtai, **1980**; Pandey and Singh, **1986**; Newman and Bennett, **1990**; **Coviello et al., 2005**; Aydin and Basu, **2006**; Tavallali and Vervoort, **2010a, b**; Erarslan and Williams, **2012**).

In view of the various results obtained on the same type of rock, the previous research works mostly comprised anisotropy (Barla and Innaurato, **1973**; Chen et al **1998**; Exadaktylos and Kaklis, **2001**; Tavallali and Vervoort **2010a,b**). In the future it is expected that more attention will be paid to other differences among the specific types of rock materials, such as the petrographic characteristics, for example.

In view of the available references (Table 1), no general trend in the ratio between the direct and indirect tensile strengths for all the types of materials can be observed. Actually, the same materials on various locations can have a different ratio between the direct and indirect tensile strengths. Due to practical engineering needs, such an issue requires more attention. Accordingly, the Brazilian test should be applied on each direct testing of tensile strength in the future, in order to determine the ratio for each tested material.

Further development of the methods for estimating tensile strength in other laboratory testings is not expected as long as it is possible to determine the indirect tensile strength in a simple way by means of the Brazilian test, which also includes

the estimate of the direct tensile strength. However, in the future it will be necessary to determine the correction coefficients according to the types of materials more precisely, as suggested in the paper by Perras and Diedrichs (**2014**). A relatively small number of papers on the effects of saturation has not provided the answer to the question how water actually affects the strength reduction. The issue of the water effect upon the indirect testing of tensile strength by the Brazilian test on the change of the cohesion or friction in the tested material has still remained open. The classification of the tested materials according to their mineralogical and petrographic characteristics, including the testing in dry and saturated conditions, can contribute to better comprehend this issue.

5. Conclusion

A large number of papers were made for the scientific purposes and they did not have any practical application. Accordingly, there is actually a rather small number of papers which may be used to improve the Brazilian test.

The standardization of the procedures using more expensive and more sophisticated equipment is not expected in the future, despite the fact that it would improve the testing procedures.

Therefore, it is necessary to determine more precisely the correction coefficients for the estimate of direct tensile strength by the Brazilian test for all types of materials, in order to obtain the optimum results.

Further research works on indirect tensile strength should be focused on the testing of the effect of saturation on various types of rock materials, including the determination of the relationship between the indirect and direct tensile strengths, due to the fact that this would have a practical purpose in engineering.

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