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FRACTAL PARAMETRIC MODELS OF URBAN SPACES

Mirjana Devetaković, Ivana Petruševski, Ivana Ćirović, Ljiljana Petruševski

Original scientific paper
This study proposes, elaborates and supports with a software solution, a parametric "random curds" model of urban spaces. The aim is integrating nature geometry into the built environment. We apply fractal mathematical "random curds" model with various interpretation of "live" and "dead" cells and propose new "random curds" model with various states of "live" cells. As a way of selecting fit solutions, we propose genetic algorithm method.

Keywords: *built environment; geometry of nature; random fractals; urban space*

Fraktalni parametrijski modeli urbanih prostora

Izvorni znanstveni članak
U radu se pomoću softverskog rješenja predlaže, elaborira i podržava parametrijski "random curds" model urbanih prostora. Cilj je integriranje geometrije prirode u izgrađeni okoliš. Primjenjujemo fraktalni matematički "random curds" model s različitom interpretacijom "živih" i "mrtvih" ćelija i predlažemo novi "random curds" model s različitim stanjima "živih" stanica. Kao način odabira odgovarajućih rješenja, predlažemo metodu genetskog algoritma.

Ključne riječi: *geometrija prirode; izgrađeni okoliš; slučajni fraktali; urbani prostor*

1 Introduction

The most exciting scientific discoveries in the last two decades, such as fractals, cellular automata, complexity theory, evolutionary biology and artificial intelligence, give us an idea on the way humans communicate with their environment. Organisms, buildings, towns and cities all share the same general rules for governing a complex hierarchical system.

All matter, biological – living, and non-living is self-organized, and makes a coherent structure in its self-organization. In the process of evolution, the human mind has adapted to complex systems and complex patterns of the natural world, so that the patterns we observe around us affect our internal functioning as human beings. According to some authors [1, 2, 3], people are physiologically and psychologically connected to structures characterized by organized complexity. This connection is of vital importance in our lives and it contributes to physical and mental well-being of people. A conclusion that follows from this is that the built environment should meet the key requirement, to be similar to nature. Therefore, we need to introduce more of nature into our immediate environment and create a built environment, by integrating the "geometry of nature".

The goal is to make the built environment an integral part of the hierarchical self-organized system, which, given the current development of technology and materials is not yet possible, but generating architectural and urban structure by using the principles and forms of fractal geometry as the "geometry of nature" is certainly a step in the right direction.

Natural forms look irregular and show similar structure on different scales. Fractal geometry, unlike Euclidean geometry, offers much better methods for describing complex structure of natural objects with complex structure. Statistical similarity and uneven distribution presented in natural forms cannot be simulated with smooth forms of Euclidean geometry. Instead, we need a new approach to complexity in order to handle the irregularities of the structure itself. Mathematically speaking, a fractal structure occurs in an infinite iterative process, i.e. recursions, in which a simple mathematical procedure, a simple constructive procedure is repeated, acting in every iteration and affecting the outcome of the previous iteration. The presence of scaling in the constructive procedure ensures self-similarity or self-affinity of fractals, same geometric forms in all scales. Increasingly small parts of the fractals represent smaller copies of the entire fractal object. However, there are no mathematically defined fractal forms in nature, generated through an infinite number of steps and with an infinite number of scales, they are visible as a result of several iterations, but their self-similarity and self-affinity is recognizable.

In addition, natural fractal forms are not geometrically and deterministically defined, we can recognize their statistical self-similarity or statistical self-affinity. They belong to the group of the so-called random or stochastic fractals. Constructive procedures for such fractals are deterministic, but the random variable is presented in each stage of the process (meaning statistically random variable with expected, mean values and deviations from those values).

In this paper, we proposed a software-realized parametric fractal model of urban spaces, a model of fractal distribution of physical structure, applicable in the first steps in design of new settlements and new urban blocks [4]. The model is fractal in terms of natural fractals, it belongs to the class of random fractals and was realized in two to three iterations, which is sufficient to recognize fractality and achieve the specified goal. To generate fractals, we use the procedure of "curdling" and get a stochastic fractal "random curds" as the mathematical model of random clustering of matter in nature. The first one to suggest the application of this model in architectural and urban compositions as a way to generate natural rhythm in architecture was Bovill [5, 6]. The mathematical model is based on the division of the observed part of space (1D - line, 2D - plane, 3D - space) to cells. The two possible states of cells: "live" and "dead" have proved to be the limiting factor in the application of this mathematical model, because we need a larger number of different states. One of the potential methods of application is to overlap several "random curds" of fractal forms, which is an acceptable solution in some cases [7, 8]. However, in the case of urban morphology, i.e. different interpretations of "live" cells overlapped with "random curds" of fractal forms, we can have content overlay within the same cell, which represents a problem. These contents should be mutually coordinated, which is often impossible.

We achieved the diversity of cell states and the possibility of potential applications in another way here, by appropriate interpretation of "live" and "dead" cells through iterations, and by further generalization of the process itself, i.e. introduction of the term "random curds" with more states.

A stochastic approach and model parameterization provide a huge number of variations that allow model adaption and the selection of the optimal variation, chosen by different criteria.

2 Theoretical framework

The "random curds" fractal and its generation procedure (curdling) have stemmed from the effort to mimic reality by purely geometric means. The procedure was proposed by Mandelbrot [9], and he stated clusters of stars and galaxies as an example. Similarly to cellular automata [10], contruction requires a raster, i.e. grid that divides the observed part of the line, plane or space into cells. Theoretical discussions usually mention a rectangular grid, although this is not necessary. In general, a grid can have different shapes: square, hexagonal, triangular. Moreover, in practical application, it can be adapted to the geometry of the terrain and cells do not need to be of the same shape or size.

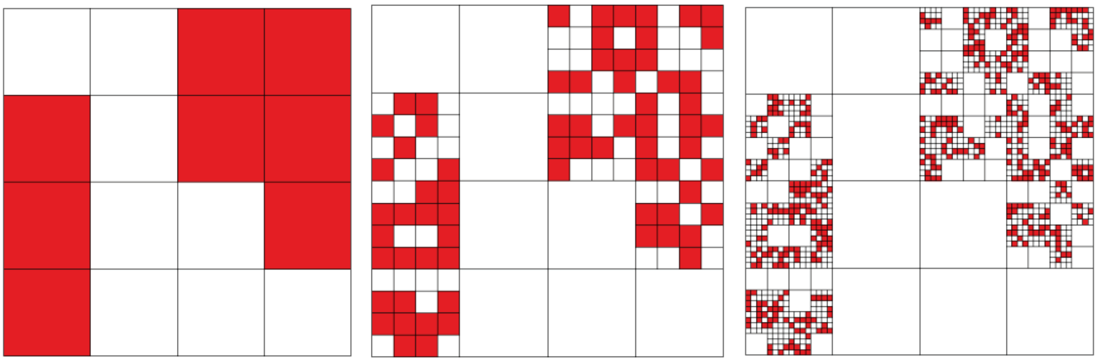

Figure 1 The first, second and third iteration with probability of survival $p = 0{,}6$ in the first two iterations, and probability $p = 0{,}4$ in the third iteration

Each cell can be in one of two possible states: "live" or "dead" ("survives – does not survive""full - empty" or "black - white", or "exists – does not exist", etc.). In their initial state, all cells are "live", and a cell "survives" with probability $p\ (0 \le p \le 1)$. The procedure occurs in iterations. With every next iteration, each "surviving" cell is divided into new cells again, in the same way as before. These cells are initially "live" in that iteration. Each one of these new cells within that iteration survives with probability p and the procedure is repeated again (Fig. 1).

The probability of "survival" is fixed within the iteration for all cells, and it can change through iterations. When it comes to implementation, we will get a good result and the recognizability of clustering after the second or third iteration, except that it should be noted that the obtained result is not unambiguous (Fig. 2).

Figure 2 Examples of four different structures (3rd iteration) with the same probabilities of survival $p = 0{,}6$ in the first and second iteration, and probability $p = 0{,}4$ in the third iteration

Stochastic selection of "survival" provides a large number of theoretically equal solutions, which is an advantage for the implementation and selection of an optimal solution, chosen by some additional criteria. In addition, a self-organized and complex hierarchical system, which is suitable as such for generating a "good" urban structure [11] is recognizable, where a built environment is formed by "incorporating" the "geometry of nature" [12].

Following the ideas of cellular automata and Markov chains, we proposed and further discussed "random curds" with several states. The procedure for generation is the same, "dead" cells cannot change their state, cells survive with total probability p, where they change to one of the finite number of states with probabilities $p_1, p_2, \dots, p_n (p_1 + p_2 + \dots + p_n = p)$ (Fig. 3), or they change from state i to state j with probability p_{ij}, where $\sum_{j=1}^{n} p_{ij} = 1$.

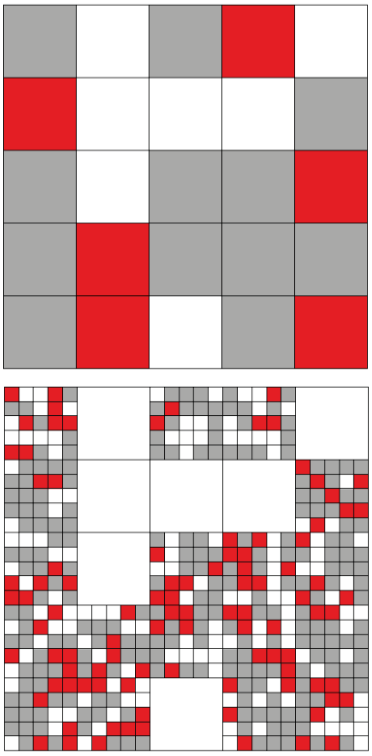

Figure 3 "Random curds" with two states, and the total probability of "survival" $p = 0,5$, where the probability of changing to state highlighted in red is $p_1 = 0,3$ and the probability of changing to state highlighted in grey is $p_1 = 0,2$ - the first and second iteration

3 Fractal parametric models

In this paper, we discuss fractal parametric models of urban spaces, designed for generating urban blocks and wider urban areas, cities, city blocks, or structure composed of several urban blocks. Following the ideas of modern theoretical approaches [11] of the city as a complex hierarchical system, we come to the fractal model concept, and in addition, following the concept of biophilia [1, 2, 3, 12, 13], fractal concept is directed toward stochastic fractals, with the intention of realizing models of built environment with integrated "geometry of nature". The fact that there are no mathematically defined fractals generated in an infinite number of iterations and characterized by self-similarity through an infinite number of proportions in nature speaks in favour of the probability of achieving the defined goal. Fractality in nature can be reduced to several scales, but statistical self-similarity or self-affinity is visible, almost explicit. Similar to the geometry of nature, it is reasonable to expect we can get a good result after two or three iterations.

The input parametric values in the model are whole numbers m and n of the initial raster of the $m \times n$ type. This parameterization enables adaptation and application of the model to any terrain, whereas the selection of m and n values adapts the model to reality, providing cells of adequate size for the planned activities.

In addition, the input parameters are the probabilities of cell "survival" in each iteration. Parameterization of the probabilities provides a high level of model generality, diversity of its implementation and adaptation to the real needs of modeling in each case.

We will illustrate the basic concept of the "random curds" model proposed in this paper through the following three examples: different interpretation of "live" and "dead" cells through iterations.

3.1 "Random curds" models of wider urban areas

This model is based on different interpretations of "surviving" and "non-surviving" cells through three iterations. Essentially, "surviving" and "non-surviving" cells are spaces with different purposes. For example, in the first iteration, "surviving" cells are lots chosen for construction - urban blocks, whereas "non-surviving" cells are lots where no construction is planned (we will not discuss the content of these lots for now). In the second iteration, each "surviving" cell is again divided to cells, some of which will "survive" and some will not. We can interpret "live" cells as lots designed for construction, and "dead" as free surfaces within blocks (we will not discuss the content of these lots either). In the third iteration, "live" cells from the second iteration - lots within the block where construction is planned, are divided again to yet more cells, some of which survive, and some do not. "Surviving" cells will be interpreted as areas containing buildings, and those cells that did not "survive", as free surfaces in the immediate vicinity of objects that provide the required spacing between buildings and represent additional free surfaces within the block. As a result, we have a built environment with uneven distribution in this model, created by matter clustering, similar to the clustering of matter found in nature (Figs. 2 and 4).

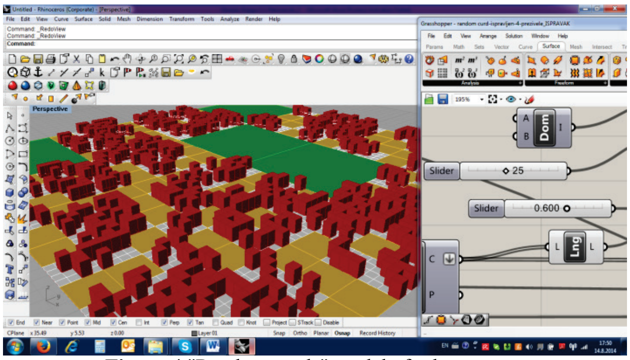

Figure 4 "Random curds" model of urban spaces

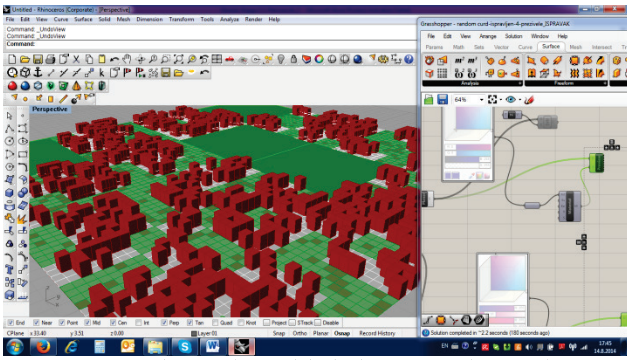

Figure 5 "Random curds" model of urban spaces - intervention on "dead"cells

An example of an intervention on all "dead"cells of the second iteration is shown in Fig. 5. Different hues of green represent "live" and "dead" cells with different purposes, for example, grassy surfaces and positions of trees.

3.2 "Random curds" models of an urban block

In this section, we will additionally explain the key role of the interpretation of "live" and "dead" cells. The proposed model of matter clustering, realized through three iterations will be applied to an urban block here. In the first iteration, "live"cells are lots intended for construction (residential or mixed-use), and "dead" cells are free surfaces that can subsequently obtain additional content: green spaces, school, kindergarten, recreation facilities. In the second iteration, each "live"cell is divided to new cells again. Surviving cells in that iteration are lots that will contain objects, while non-surviving cells will represent free surfaces that provide spacing between objects, and free surfaces within those lots. In the third iteration, surviving cells are lots containing objects, and those that did not survive can be interpreted as free surfaces in the immediate vicinity of an object, or within an object - patios, atriums, inner courtyards, skylights, etc (Fig. 6).

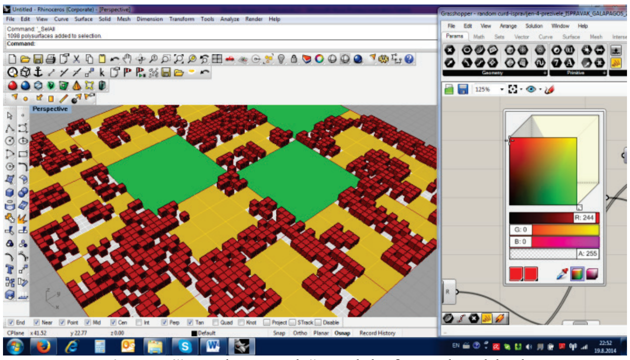

Figure 6"Random curds" model of an urban block

3.3 "Random curds" models of an urban block with more than two states

Unlike the previous one, this urban block model was realized in two iterations. Each of the "live" cells can be in one of two states, i.e. there are two types of "live" cells. In the first iteration, the cell "survives" as a cell of one of two possible types, with parametric possibilities $p_1$and p_2. The possibility that the cell will not survive is in that case $1-(p_1+p_2)$, where $0 \leq p_1+p_2 \leq 1$. In the second iteration, each of the two types of "live" cells is divided to new cells again. It is considered that these new cells are initially of the same type as the cell they originate from, and then they "survive", retaining or changing their state, with possibilities $p_{11,}\, p_{12}$ for cells of the first type and with possibilities $p_{21,}\, p_{22}$ for cells of the second type.

The area of the urban block is divided into cells – lots. Cells that do not "survive" are highlighted in grey in both iterations (Fig. 7). "Live" cells in the first iteration are lots where construction of two different types is planned (e.g. high-rise or low-rise buildings), and these construction types represent two different states of the "live"cells. In the second iteration, each of these "live" cells is divided to smaller cells, some of which do not survive (grey), and those that do also have two types: they represent the land containing an object, and additional public areas (e.g. paved surfaces, urban furniture, etc.). In Fig. 7, both types of the surviving cells in the lots intended for high-rise buildings are highlighted in red, and the lots with low-rise buildings are highlighted in yellow, except that cells that represent the land containing objects are covered by a cube in the same color, which represents the object in the model. Cells that did not survive can, for example, be used as green spaces, except that we should keep in mind man's need for the presence of the complex "geometry of nature". A lawn does not leave the impression of a complex hierarchical structure. It will not disrupt the complex hierarchical structure of a built environment, but it will not contribute to the feeling of a beneficial environment. The presence of trees and various plants with complex geometric structure and distribution would, similarly as in nature, represent an additional benefit. Introducing natural surroundings and the complex geometry of nature, together with the integrated "geometry of nature" into the built environment is a good direction for creating a quality environment for people [12].

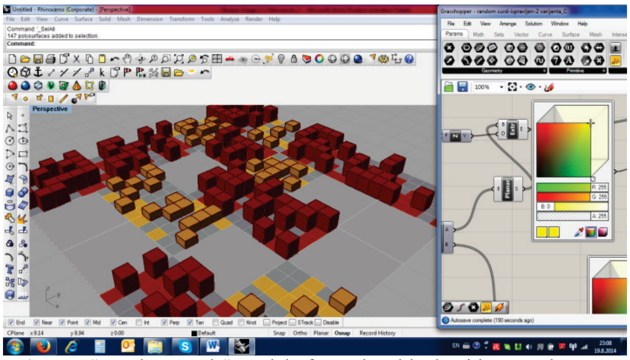

Figure 7"Random curds" model of an urban block with more than two states

4 Solution selection - genetic algorithms

The stochastic principle and a variation of parametric values of probabilities ensure a huge number of variations within the model, which are hard to grasp even with the help of computer browsing. This is why we propose a selection of more suitable variations, chosen by a criterion, with the use of genetic algorithms.

All models are realized within the Grasshopper graphical algorithm editor for Rhino, and in that environment, we can use the Galapagos graphical editor, which enables the realization of the genetic algorithm with one's own choice of a fitness function. The possibility of choosing suitable variations by the application of genetic algorithms and the model itself were tested through two experiments within the "random curds" model with more than two states. Both experiments represented a search of variations with occupancy index of 30 %. In the first experiment, the range of probabilities of cell survival in the first iteration was from 60 % to 80 %, where the range was equal for each of the states, from 30 % to 40 %. In the second iteration, the range of cell survival was from 40 % to 60 %, and it was also equal for each state, from 20 % to 30 %. We get a more significant number of suitable variations after several iterations (Fig. 8).

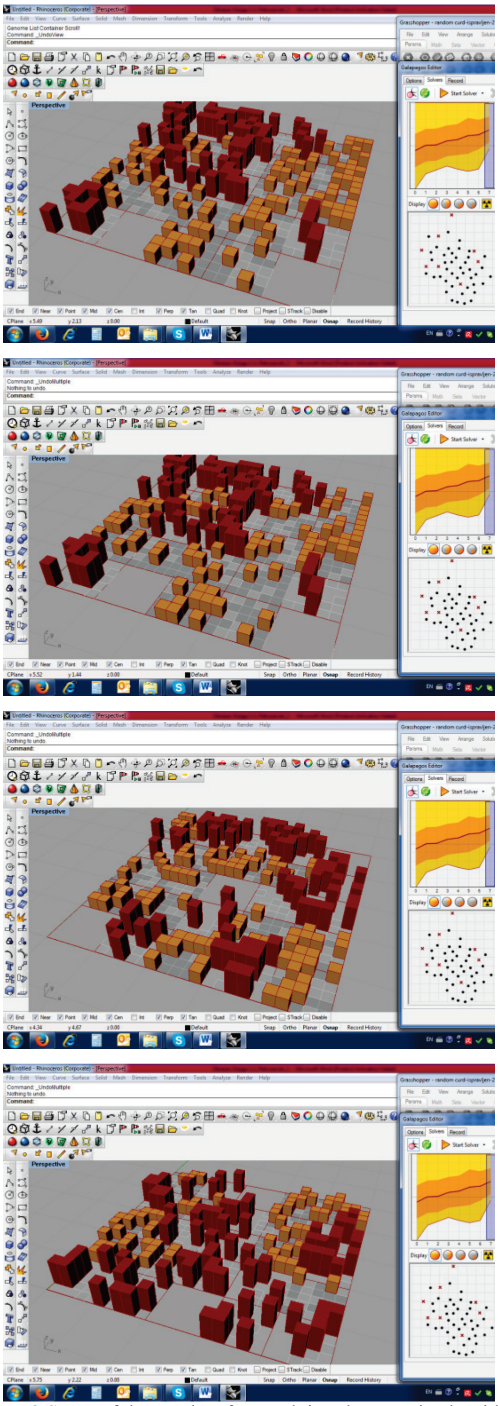

Figure 8 Some of the results after applying the genetic algorithm - occupancy 30 %

In the second experiment, we corrected the probabilities of survival from the first iteration. We increased the probabilities of survival of the cells intended for low-rise buildings (their range is from 40 % to 50 %) and reduced the probabilities of survival of the cells intended for high-rise buildings (their range is from 20 % to 30 %), then executed the genetic algorithm again, with the same requirement that occupancy equals 30 %. We obtained expected results, with a smaller presence of high-rise buildings (Fig. 9).

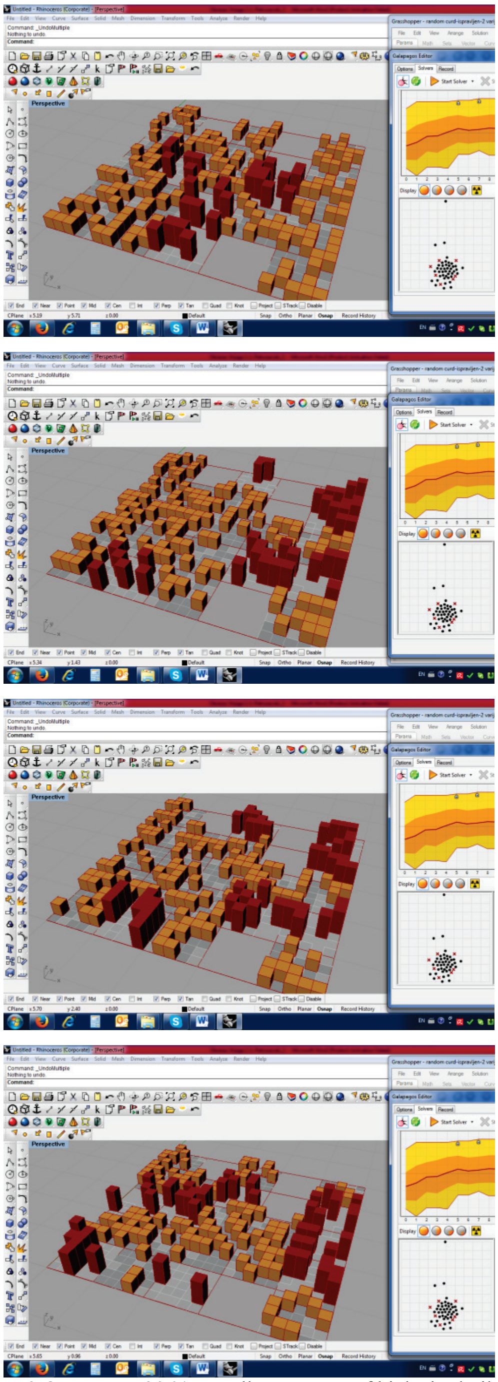

Figure 9 Occupancy 30 % - smaller presence of high-rise buildings

5 Conclusion

The primary objective of this paper was to generate a model of self-organized complex hierarchical system with the aspiration to become the basis for the formation of a built environment with "integrated geometry of nature". The model is stochastic and fractal, modelled after natural forms. We used the "random curds" fractal, which originated from the attempt to "mimic reality by purely geometric means" [9]. The procedure of curdling was used to generate the fractal, and the result is the stochastic fractal "random curds" as the mathematical model of random clustering of matter in nature. We expanded the concept of the "random curds" fractal with several states, as the mathematical model of random clustering of several types of matter in nature.

The model has a high level of generality, and the main topic of this paper, fractal parametric models of urban spaces can be treated as illustrative examples of its application, which contribute to the understanding of the model and open new possibilities for its various further uses.

The stochastic principle and a variation of parametric values of probabilities ensure a huge number of variations within the model, which are hard to grasp even with the help of computer browsing. This is why we propose a selection of more suitable variations, chosen by a criterion with the use of genetic algorithms. The obtained number of suitable variations chosen by a defined criterion represents an additional benefit, and the final selection of an acceptable variation is up to the project designer.

Aknowledgements

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Authors’ addresses

Mirjana Devetaković, PhD, Assist. Professor
University of Belgrade, Faculty of Architecture
Bulevar kralja Aleksandra 73/2
11000 Belgrade, Serbia
E-mail: mirjana.devetakovic@gmail.com

Ivana Petruševski, PhDCand.
University of Belgrade, Faculty of Architecture
Bulevar kralja Aleksandra 73/2
11000 Belgrade, Serbia
E-mail: ivanapetrusevski@gmail.com

Ivana Ćirović, PhD, SeniorLecturer
Business Technical College of Vocational Studies
Trg Svetog Save 34
31000 Užice, Serbia
E-mail: ivana.cirovic1@gmail.com

Ljiljana Petruševski, PhD, Professor
University of Belgrade, Faculty of Architecture
Bulevar kralja Aleksandra 73/2
11000 Belgrade, Serbia
E-mail: ljpetrusevski@gmail.com