XIAO-SHAN LU, Ph.D.
E-mail: shan_xl@163.com
School of Transportation Engineering,
Hefei University of Technology
No.193, Tunxi Road, Baohe District, Hefei 230009, China
REN-YONG GUO, Professor, Ph.D.,
Corresponding author
E-mail: buaa_guorenyong@126.com,
csguorenyong@imu.edu.cn
College of Computer Science, Inner Mongolia University
No. 235, Daxue West Road, Saihan District,
Hohhot 010021, China

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A BI-OBJECTIVE MODEL FOR SITING PARK-AND-RIDE FACILITIES WITH SPATIAL EQUITY CONSTRAINTS

ABSTRACT

A bi-objective programming model (BP) with spatial equity constraints is proposed to site park-and-ride (P&R) facilities in traffic networks. Both the number and locations of P&R facilities are determined. The maximal coverage and minimal resource utilization criteria, which are generally conflicting, are simultaneously considered to reveal the trade-off between the quality and cost of coverage. Furthermore, the concept of passenger flow volume per cost is defined and several properties of the model solutions are analyzed. Finally, this model is applied to site P&R facilities in Anaheim, California. Application results show the trade-offs associated with passenger flow volume, cost and passenger flow volume per cost, and the effects of spatial equity constraints on the spatial deployment of P&R facilities.

KEY WORDS

traffic network; park-and-ride; bi-objective programming; spatial equity;

1. INTRODUCTION

Continued reliance on private automobiles is not likely to be sustainable for urban development, due to limited natural resources, environmental pollution and road congestion issues. Providing reliable and efficient public transportation is an effective way to overcome car dependency and alleviate urban congestion. One of the options considered entails the use of park-and-ride (P&R), a combinational mode of private cars and public transportations, allowing commuters to use their own cars in the beginning of their trip but switch to public transit at some point later in the same trip. P&R facilities can help alleviate traffic congestion and other adverse external effects of private vehicle travel. Studies in the United Kingdom and North America have concluded that P&R facilities are effective in reducing congestion [1, 2]. This importance is reflected as well in transportation planning of other cities like Beijing, where 26 large-sized P&R facilities are planned to be built in the vicinity of the subway lines during Beijing's Twelfth Five-Year Plan period [3].

There is no doubt that the P&R facilities should be well planned to continue to play a role in effective transit systems. One essential aspect when planning for P&R services is finding appropriate geographical locations for the facilities to provide services. Planning and locating P&R facilities is a complex activity that must take multiple factors into account, including potential demands, characteristics of potential users, impacts on the surrounding neighborhoods, and the overall economic performance in terms of benefits and costs [4]. From an economic perspective, there are at least two important concerns when siting P&R facilities, i.e., covering as many potential users as possible, and spending as little as possible. The former concern is important as this provides another travel alternative to private automobiles. Generally, a user is regarded as covered if they are within a maximum acceptable distance from a facility, but certainly demand decreases as the distance from services increases. Thus, covering as many users as possible within a coverage distance of a facility is a major concern. In addition, due to the heavy investment of transport infrastructure projects and the limited budget, the cost becomes another major concern.

Existing literature related to the P&R facility location problem can be generally categorized into three classes, i.e., decision support system [5, 6], equilibrium analysis [7, 8] and mathematical programming [9-13]. Horner and Grubesic [5] and Faghri et al. [6]

developed various hybrid knowledge-based expert system and geographic information system tools to determine the optimal location of a P&R facility. These systems provide multiple criteria to investigate alternative solutions, such as close to the central business district (CBD), the competition around the parking lot, and near the subway station. Their studies focus on the system development and do not explicitly use the optimization model to describe the problem. Through the equilibrium analyses, Wang et al. [7] optimized the location and parking charge of a P&R facility in a linear city under the assumption that the traffic demand is uniformly distributed along the corridor. Liu et al. [8] developed a competitive railway or highway system with multiple P&R services in a corridor in which commuters choose between the drive only alternative and the P&R facilities located continuously along the corridor to characterize the equilibrium mode choice. Equilibrium analysis is limited to a particular travel corridor.

Sargious and Janarthanan [9] proposed a method for locating P&R facilities such that the total cost of the system is minimized. Honer and Groves [10] proposed a network flow-based model for location analyses of rail P&R. Farhana and Murray [11] proposed a multi-objective spatial optimization model to account for P&R application-specific objectives. These objectives cover as much potential demand as possible, using the existing P&R facilities as much as possible and as close as possible to major roadways. Fan et al. [12] used a bi-level programming model to locate P&R facilities considering the interaction between decision makers and commuters. Commuters are assumed to follow a stochastic user equilibrium behaviour. Aros-Vera et al. [13] proposed a mixed linear programming formulation to determine the location of a certain number of P&R facilities so that their usage is maximized. The facilities are modelled as hubs. Nevertheless, these works, mentioned above, do not address one important issue, i.e., spatial equity.

Bröcker et al. [14] pointed out that the goal of all transport infrastructure projects should be evaluated from social welfare perspective, which is reflected in two aspects, efficiency and social equity. Although the demand coverage issue has been studied a lot in the previous traffic literature [15], so far as we know, most models cannot address the trade-off associated with differing levels of importance for coverage and cost objectives, while taking spatial equity into consideration. It may cause some P&R facilities clustering with spatial equity constraint missing, and the spatial distribution of public resources becomes unfair. Recently, Lu and Huang [16] studied the P&R location problem with spatial equity constraints, but their earlier work ignored several important properties of the model solution, which will be developed further in this paper.

From cost-benefit perspective of public transport infrastructure investment, incorporating the spatial equity factor, this paper proposes a bi-objective programming model for the P&R facility location problem. In the model, the objectives are to maximize the number of P&R users and to minimize the cost of constructing P&R facilities simultaneously. Using a linear weighted technique, the bi-objective model is transformed into a single objective model. The concept of passenger flow volume per cost is introduced. Some properties of the model are analyzed and investigated. Finally, this model is applied to the transportation network in Anaheim, California.

2. MODEL DESCRIPTION

In this section, the proposed model is described in detail. Before doing this, the notations involved in the model are given as follows. Let i and I be the index and index set of areas, respectively, J is the index set of potential facility sites, a_i the demand in area i, c_j the cost of locating a P&R facility on site j and a non-negative integer, $\hat{a}_{ij}$ the demand in area i anticipated to use P&R facility j based on relative proximity, d_{ij} the distance between area i and potential facility site j, E the maximum acceptable service standard (distance), and α the minimum acceptable distance between two P&R facilities. Location variable x_j is defined such that x_j=1 if a facility is located at site j and x_j=0 otherwise. Allocation variable y_{ij} is defined such that y_{ij}=1 if the demand in zone i is served by a facility at site j and y_{ij}=0 otherwise.

Based on the assumption of user rationality, Holguín-Veras et al. [17] studied the catchment area of a given P&R site and the optimal P&R facility location in a linear and two-dimensional cities, respectively. They found that an ellipse could approximate the catchment area of a P&R facility in a two-dimensional city. Here, the following piecewise distance decay function is used to modify the demand attracted

$$\hat{a}_{ij} = \begin{cases} a_i e^{-\beta d_{ij}}, & \text{if } d_{ij} \le E, \\ 0, & \text{otherwise}, \end{cases} \tag{1}$$

where β is the distance decay parameter, and β>0. The larger parameter β is, the more quickly the attracted demand $\hat{a}_{ij}$ decays with distance d_{ij}. Other types of functions can also be utilized to represent the distance decay, such as the power function [18]. It is worth mentioning that our model is different from the classical set covering model and maximal covering model. In the two classical models, although there exists a criterion of coverage, namely the distance threshold, E, it is supposed that all the residents within the threshold distance will patronize the facility. That is to say, the coverage function in the two classical models is binary: either the demand is covered or not covered. Thus, the two models cannot express other distance decay forms, for example, the decrease of the demand with distance in a piecewise or non-linear manner.

From the perspective of P&R facility investors, they hope to make as much profit as possible, while paying as little as possible. Profit is composed of two parts: parking fee and transit fare. Suppose the parking fee and the transit fare are fixed. Let the sum of the parking fee and transit fare be p. Then the total revenue is $p\sum_{i\in I}\sum_{j\in J}\hat{a}_{ij}y_{ij}$. Since parameter p is a constant, the objective of maximizing the revenue could be simplified setting p=1 and maximizing the coverage $\sum_{i\in I}\sum_{j\in J}\hat{a}_{ij}y_{ij}$.

With the above notations, this proposed model is formulated as:

$$\text{(BP)}\quad \max_{y_{ij}} \quad Q(y)=\sum_{i\in I}\sum_{j\in J}\hat{a}_{ij}y_{ij}, \tag{2}$$

$$\min_{x_j} \quad C(x)=\sum_{j\in J}c_jx_j, \tag{3}$$

subject to

$$y_{ij}\le x_j, \quad \forall i\in I, \forall j\in J, \tag{4}$$

$$\sum_j y_{ij}\le 1, \quad \forall i\in I, \tag{5}$$

$$x_k+x_l\le 1, \quad \text{if } d_{kl}<\alpha, \forall k,l\in J, k\ne l, \tag{6}$$

$$x_j, y_{ij}\in\{0,1\}, \quad \forall i\in I, \forall j\in J. \tag{7}$$

In model (BP), objective (2) maximizes the passenger flow volume using P&R facilities and objective (3) minimizes the cost of constructing the P&R facilities. If the construction cost at each potential site is identical, objective (3) could be simplified setting c_j=1 and minimizing the number of P&R facilities selected, i.e.

$$\min_{x_j} C(x)=\sum_{j\in J}x_j. \tag{8}$$

Constraints (4) require the demand is assigned to a facility only if it is open. Constraints (5) ensure the demand of each area is allocated to no more than one site. This implies the user rationality assumption, namely, if the demand is inside the threshold distances of multiple facilities, they will choose the nearest one. Constraints (6) are the spatial equity constraints. Here, we define the spatial equity issue in the sense that several P&R facilities may cluster if there are no spatial distance constraints. The constraints ensure the distance of any two P&R facilities is not less than a certain value. In other words, the spatial clustering phenomenon could be avoided and the spatial distribution of P&R facilities in a city could reach a balance to some extent. Note that it is different from that in the context of road pricing. The spatial equity in road pricing literature is defined as the difference of the generalized travel costs of drivers travelling between several origin-destination (O-D) pairs. The spatial equity may be significantly different when tolls are charged at some selected links [19]. Constraints (7) are the integrality constraints.

3. MODEL ANALYSIS

Let S denote the feasible region of the model (BP), namely, S is the set of all (x,y) that satisfy equations (4) to (7). The model can be transformed into a single objective problem by the linearly weighted method, i.e.,

$$\max \quad \omega Q(y)-(1-\omega)C(x), \tag{9}$$

$$\text{s.t.} \quad (x,y)\in S, \tag{10}$$

where ω is a constant, and $\omega\in(0,1]$. If $\omega\ne 0$, then the model could be rewritten in another form

$$(\text{SP}_\lambda)\quad f(\lambda)=\max\ Q(y)-\lambda C(x), \tag{11}$$

$$\text{s.t.} \quad (x,y)\in S, \tag{12}$$

where $\lambda=(1-\omega)/\omega\ge 0$.

Let $z(x,y)=\left(Q(y),-C(x)\right)$ denote the objective function vector of the model (BP). Then the model (BP) can be formulated as follows: $\max z(x,y)$, *s.t.* $(x,y)\in S$.

Definition 1. Given a feasible point of (BP), $(x^*,y^*)\in S$, if there is no $(x,y)\in S$, such that $z(x,y)\succ z(x^*,y^*)$, then (x^*,y^*) is an efficient solution. Here, $z(x,y)\succ z(x^*,y^*)$ is equivalent to $\left(Q(y),-C(x)\right)\ge\left(Q(y^*),-C(x^*)\right)$, but there exists $Q(y)>Q(y^*)$ or $C(x)<C(x^*)$, $(x,y)\in S$.

Definition 2. Given a feasible point of (BP), $(x^*,y^*)\in S$, if there is no $(x,y)\in S$, such that $z(x,y)>z(x^*,y^*)$, then (x^*,y^*) is a weakly efficient solution.

Definition 3. $Q(y)/C(x)$ is called passenger flow volume per cost (PFVC), where $Q(y)$ is the total passenger flow volume (PFV) using P&R facilities, and $C(x)$ is the total cost for building the P&R facilities.

We put forward the concept of PFVC, which could be regarded as an index of evaluating the return of investment on transport infrastructure investment, like P&R facilities. Generally speaking, marginal profit is used to evaluate the rate of investment return in economics. Since the profit comes from ticket revenue in this model and ticket revenue is proportional to PFV, we could adopt the PFVC to investigate the level of the rate of investment return.

Theorem 1. Given $\lambda\ge 0$, an optimal solution of the single objective problem (SP_λ) must be a weakly efficient solution of the model (BP). Specially, when $\lambda>0$, an optimal solution of the single objective problem (SP_λ) must be an efficient solution of the model (BP).

Proof. Let (x^*,y^*) be an optimal solution of (SP_λ), but not a weakly efficient solution of the model (BP). According to Definition 2, there exists $(x',y')\in S$, such that $z(x',y')>z(x^*,y^*)$. Given $\lambda\ge 0$, we have $\left(1,\lambda\right)\cdot z(x',y')>\left(1,\lambda\right)\cdot z(x^*,y^*)$. This implies that (x^*,y^*) is not an optimal solution of (SP_λ), which leads to a contradiction. Specially, when $\lambda>0$, the theorem can be proved similarly. The proof is completed.

The model (SP_λ) is closely related with the following problem (F_1)

$$(F_1) \qquad \max \quad \frac{Q(y)}{C(x)}, \tag{13}$$

$$\text{s.t.} \qquad (x,y)\in S\,. \tag{14}$$

Firstly, the relationship of model solutions between models (SP_λ) and (F_1) is given by Lemma 1. Furthermore, we investigate the properties of the model (SP_λ).

Lemma 1. If λ^* is the optimal value of the objective function of the problem (F_1), then we have

i) $f(\lambda)$ is a convex, piecewise linear and decreasing function with respect to $\lambda\geq 0$;

ii) If $f(\lambda)=0$, then $\lambda=\lambda^*$;

iii) The optimal solution of (SP_{λ^*}) is identical to that of (F_1).

For the proof of Lemma 1, readers may refer to Ref. [20]. Lemma 1 will be helpful for the proof of Property 1 to 3.

Property 1. If $(x^*(\lambda), y^*(\lambda))$ is the optimal solution of (SP_λ), then

i) $Q(y^*(\lambda))$ and $C(x^*(\lambda))$ are decreasing with $\lambda\geq 0$;

ii) $Q(y^*(\lambda))/C(x^*(\lambda))$ is increasing with $\lambda\in[0,\lambda^*]$, and $\lambda\leq Q(y^*(\lambda))/C(x^*(\lambda))\leq\lambda^*$; it is decreasing with $\lambda\in[\lambda^*,+\infty)$, and $Q(y^*(\lambda))/C(x^*(\lambda))\leq\lambda^*\leq\lambda$.

Proof.

i) For $0\leq\lambda_1<\lambda_2$, from the optimality of $f(\lambda_1)$ and $f(\lambda_2)$, we have

$$Q(y^*(\lambda_1))-\lambda_1 C(x^*(\lambda_1))\geq Q(y^*(\lambda_2))-\lambda_1 C(x^*(\lambda_2)) \tag{15}$$

$$Q(y^*(\lambda_2))-\lambda_2 C(x^*(\lambda_2))\geq Q(y^*(\lambda_1))-\lambda_2 C(x^*(\lambda_1)) \tag{16}$$

Rearranging inequality (15),

$$Q(y^*(\lambda_1))-Q(y^*(\lambda_2))\geq\lambda_1\left[C(x^*(\lambda_1))-C(x^*(\lambda_2))\right] \tag{17}$$

Adding (16) to (15) yields $(\lambda_2-\lambda_1)\left[C(x^*(\lambda_1))-C(x^*(\lambda_2))\right]\geq 0$. Since $\lambda_2-\lambda_1>0$, we obtain $C(x^*(\lambda_1))\geq C(x^*(\lambda_2))$. From (17) it follows $Q(y^*(\lambda_1))\geq Q(y^*(\lambda_2))$. Thus, both $Q(y^*(\lambda))$ and $C(x^*(\lambda))$ are decreasing functions of $\lambda\geq 0$.

ii) According to (i) and (ii) of Lemma 1, when $\lambda\leq\lambda^*$, we have $f(\lambda)\geq 0$, i.e. $Q(y^*(\lambda))/C(x^*(\lambda))\geq\lambda$. Thus, we obtain $\lambda\leq Q(y^*(\lambda))/C(x^*(\lambda))\leq\lambda^*$. When $\lambda\geq\lambda^*$, we have $f(\lambda)\leq 0$, i.e. $Q(y^*(\lambda))/C(x^*(\lambda))\leq\lambda$. By the definition of λ^*, we have $Q(y^*(\lambda))/C(x^*(\lambda))\leq\lambda^*\leq\lambda$. When $0\leq\lambda_1<\lambda_2\leq\lambda^*$, we obtain the following inequality by Eq. (16) $Q(y^*(\lambda_1))-Q(y^*(\lambda_2))\leq\lambda_2\left[C(x^*(\lambda_1))-C(x^*(\lambda_2))\right]$ $\leq\dfrac{Q(y^*(\lambda_2))}{C(x^*(\lambda_2))}\left[C(x^*(\lambda_1))-C(x^*(\lambda_2))\right]$. Arranging that inequality, we thus have $\dfrac{Q(y^*(\lambda_1))}{C(x^*(\lambda_1))}\leq\dfrac{Q(y^*(\lambda_2))}{C(x^*(\lambda_2))}$. Therefore, $Q(y^*(\lambda))/C(x^*(\lambda))$ is increasing with $\lambda\in[0,\lambda^*]$. When $\lambda^*\leq\lambda_1<\lambda_2$, the result could be proved similarly. The proof is completed.

From Property 1 (ii), we know that the PFVC first increases and then decreases with $\lambda\in[0,+\infty)$. When $\lambda=\lambda^*$, the PFVC takes the maximum λ^*. When $\lambda\in[0,\lambda^*]$, λ and λ^* are the lower bound and upper bound of the PFVC, respectively. When $\lambda\in[\lambda^*,+\infty)$, λ^* is the upper bound of the PFVC, what is the lower bound? Property 2 gives the answer as below.

Property 2. There exists a constant $\lambda_0\geq\lambda^*$, when $\lambda>\lambda_0$, we have

i) $Q(y^*(\lambda))=Q(y^*(\lambda_0))$, $C(x^*(\lambda))=C(x^*(\lambda_0))$, $Q(y^*(\lambda))/C(x^*(\lambda))=Q(y^*(\lambda_0))/C(x^*(\lambda_0))$;

ii) The optimal solution of (SP_λ) is identical to that of (SP_{λ_0}).

Proof.

i) Firstly we prove a relaxed version, namely, there exists a constant $\lambda_0>0$ satisfying (i). This property is proved by the contradiction method.

Suppose that there is no such constant $\lambda_0>0$ satisfying $C(x^*(\lambda))=C(x^*(\lambda_0))$ for any $\lambda>\lambda_0$. Then there exists $\lambda_1>\lambda_0$ such that $C(x^*(\lambda_1))\neq C(x^*(\lambda_0))$. According to Property 1 (i), we have $C(x^*(\lambda_1))<C(x^*(\lambda_0))$. In a similar manner, there exists $\lambda_2>\lambda_1$ such that $C(x^*(\lambda_2))<C(x^*(\lambda_1))$.

In this way, an infinite sequence is generated, i.e., $x^*(\lambda_n)$, $n=0,1,2\cdots$, such that $C(x^*(\lambda_n))$ is strictly decreasing. Since c_j is an integer, we thus have $C(x^*(\lambda_n))\leq C(x^*(\lambda_{n-1}))-1\leq\cdots\leq C(x^*(\lambda_0))-n$. So we directly conclude that $\lim\limits_{n\to\infty}C(x^*(\lambda_n))=-\infty$, which contradicts with $C(x^*(\lambda_n))\geq 0$.

Therefore, we get $C(x^*(\lambda))=C(x^*(\lambda_0))$. For $\lambda>\lambda_0$, from the optimality of $f(\lambda)$ and $f(\lambda_0)$, we have $Q(y^*(\lambda))-\lambda C(x^*(\lambda))\geq Q(y^*(\lambda_0))-\lambda C(x^*(\lambda_0))$, $Q(y^*(\lambda_0))-\lambda_0 C(x^*(\lambda_0))\geq Q(y^*(\lambda))-\lambda_0 C(x^*(\lambda))$.

Combining the above two inequalities leads to $Q(y^*(\lambda))=Q(y^*(\lambda_0))$. Then it follows $Q(y^*(\lambda))/C(x^*(\lambda))=Q(y^*(\lambda_0))/C(x^*(\lambda_0))$.

That is to say, PFVC keeps unchanged with $\lambda\in[\lambda_0,+\infty)$. Recalling (i) and (ii) of Property 1, $Q(y^*(\lambda))$ and $C(x^*(\lambda))$ are decreasing with $\lambda\geq 0$, thereby $Q(y^*(\lambda_0))$ and $C(x^*(\lambda_0))$ are the minimum with $\lambda\in[0,+\infty)$. Since $Q(y^*(\lambda))/C(x^*(\lambda))$ first increases and then decreases with $\lambda\in[0,+\infty)$ and $Q(y^*(\lambda^*))/C(x^*(\lambda^*))$ takes the maximum λ^*, we know that $Q(y^*(\lambda_0))/C(x^*(\lambda_0))$ is the lower bound of PFVC with $\lambda\in[\lambda^*,+\infty)$. Obviously, we obtain $\lambda_0\geq\lambda^*$.

ii) From (i) above, when $\lambda>\lambda_0$, $Q(y^*(\lambda))-\lambda C(x^*(\lambda))=Q(y^*(\lambda_0))-\lambda C(x^*(\lambda_0))$. So the optimal solution of (SP_{λ_0}) is also that of (SP_λ) and vice versa. The proof is completed.

Property 2(i) gives the lower bound of PFVC when $\lambda\in[\lambda^*,+\infty)$. So far, we have obtained the upper and lower bounds of PFVC with $\lambda\geq 0$.

From Property 1 and 2, we know that PFVC first increases, then decreases and then remains unchanged with $\lambda \geq 0$. We wonder whether there are some characteristics of the solution when the highest PFVC is achieved?

Property 3. The optimal solution of ($SP_{\lambda*}$) is given by a single open P&R facility that offers the highest PFVC.

Proof. According to (iii) of Lemma 1, we know that the optimal solution of ($SP_{\lambda*}$) is identical to that of (F_1). With only one facility located at j, let the objective function value of problem (F_1) be $z(j)$, and $z(j) = Q_j / C_j$, where Q_j and C_j are the corresponding passenger flow volume and building cost. Repeat for $j = 1, ..., |J|$, and reorder the indices so that $z(1) \geq z(2) \geq \cdots \geq z(|J|)$. Next, consider the problem with sites 1 and 2 open. Let $z(1,2) = (Q_1 + Q_2)/(C_1 + C_2)$ denote the objective function value of problem (F_1), where Q_1, Q_2 and C_1, C_2 are respective coverage contributions and investments. It is clear that $z(1) \geq \max\left\{\frac{Q_1}{C_1}, \frac{Q_2}{C_2}\right\}$. In addition, we know that $\max\left\{\frac{Q_1}{C_1}, \frac{Q_2}{C_2}\right\} \geq \frac{Q_1 + Q_2}{C_1 + C_2}$. Thus, we have $z(1) \geq z(1,2)$. Using an inductive approach, we could show that max PFVC cannot exceed $z(1)$ for any combination of open sites. The proof is completed.

4. APPLICATION EXAMPLE

The above bi-objective programming model is applied to site P&R facilities in Anaheim, California, which is shown in *Figure 1*. The Anaheim network consists of 38 areas, and each of them is represented by the symbol '○' with a number. These positions are also selected as the potential sites for P&R facilities. Travel demand data of each area are obtained from the homepage of traffic network testing problem [21], see *Table 1*. The distance information could be obtained from *Figure 1*. The construction cost data of locating P&R facilities assumed in this paper are shown in *Table 2*. We use the Lingo software to solve the model (SP_λ). The consuming time is so little that it can be negligible. Here, parameter E is set large enough so that the area demand is always covered by P&R facilities, but it declines with the distance. Other parameter values are set as α=3(km) and β=0.2. Note that we set α here to show the effect of spatial equity constraints. Since the change of α does not lead to general conclusions, we will not do sensitivity analysis about parameter α.

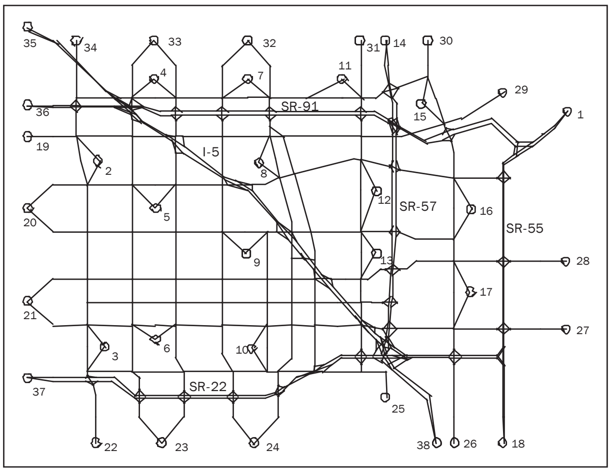

Figure 1 – Anaheim network

Table 1 – Input data of the area demand

i	a_i (person)	i	a_i (person)
1	7,075	20	504
2	9,663	21	2,642
3	7,669	22	1,524
4	12,174	23	1,523
5	2,587	24	376
6	6,577	25	8,554
7	7,137	26	2,975
8	722	27	548
9	2,238	28	2,083
10	149	29	1,145
11	486	30	2,936
12	488	31	3,639
13	37	32	2,058
14	125	33	1,783
15	407	34	5,322
16	249	35	1,965
17	648	36	933
18	2,869	37	338
19	1,038	38	1,512

Table 2 – Input data of the construction cost of P&R facilities

j	c_j (10^4 \$)	j	c_j (10^4 \$)
1	220	20	430
2	620	21	420
3	500	22	360
4	520	23	370
5	560	24	370
6	480	25	360
7	570	26	300
8	600	27	414
9	544	28	428
10	492	29	390
11	526	30	396
12	460	31	378
13	442	32	352
14	380	33	350
15	360	34	360
16	410	35	360
17	410	36	396
18	332	37	240
19	456	38	290

The location results of P&R facilities are shown in *Table 3* when parameter λ takes different values. As can be seen from *Table 3*, when $\lambda \geq 0$, $f(\lambda)$ is a decreasing function. In this case, when $\lambda^*=134.3$, we have $f(\lambda^*)=0$; when $\lambda=\lambda^*=134.3$, the optimal solution $(x^*(\lambda^*), y^*(\lambda^*))$ of the model ($SP_{\lambda*}$) is the same with that of model (F_1). At the same time, $Q(y^*)/C(x^*)$ reaches the maximal value 134.3. We can also observe that $Q(y^*)/C(x^*)$ first increases, then decreases and then keeps unchanged with $\lambda \in [0,+\infty)$. When $\lambda \in [0,134.3]$, the inequality $Q(y^*)/C(x^*) \geq \lambda$ always holds. When $\lambda \in [134.3,+\infty)$, $Q(y^*(\lambda))/C(x^*(\lambda)) \geq Q(y^*(\lambda_0))/C(x^*(\lambda_0))$, where λ_0 can be set to 150. When $\lambda>\lambda_0$, $Q(y^*)$, $C(x^*)$ and the optimal location of (SP_λ) remains unchanged. The upper bound of the PFVC is $\lambda^*=134.3$ with $\lambda \in [0,+\infty)$. These results support Lemma 1, Property 1 and 2 in Section 3. In addition, the optimal location of ($SP_{\lambda*}$) is given by a single open P&R facility 37 offering the highest PFVC, which verifies Property 3.

Figure 2 shows the effective solutions of the model (BP) and the trade-offs related with PFV, PFVC and cost. These effective solutions (Pareto optimal solutions) form the Pareto frontier, represented by the curve marked by a dash line. It reflects the trade-off between the coverage goal and the cost goal. As can be seen from *Figure 2*, the two goals are conflicting. The rightmost point of *Figure 2* yields the largest volume, and also most cost. From the slope of the Pareto frontier, the right two solutions may not be favored by the decision makers, since a large number of construction costs only brings a small increase in volume.

Then, the trade-offs associated with PFV, PFVC and cost should be carefully investigated. Since the PFVC can be used as an index of evaluating the return of investment, it is necessary to take the PFVC into the decision process. The trade-off between PFVC and cost, is represented by the curve marked with a real line, see *Figure 2*. Along with the increase in coverage and construction cost, the PFVC first increases and then decreases. Although the three points on the left side of the two figures are close to or reach the maximal PFVC, their location results are only one parking lot to build in the Anaheim network, which is impossible in reality. Thus, the decision makers could select from the rest four options, which corresponds to cases λ=3, 6, 12, and 25, respectively.

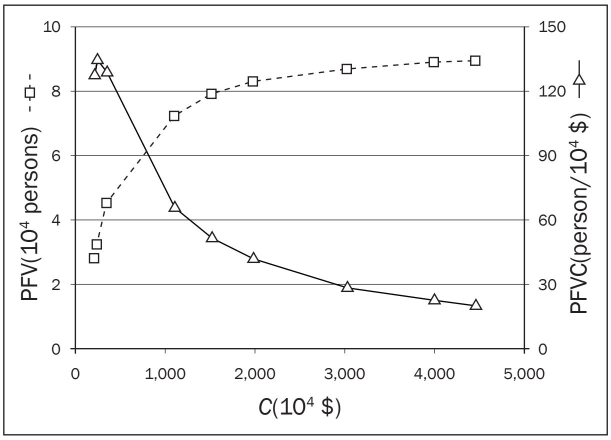

Figure 2 – Effective solutions of the model (BP) and the trade-offs related with PFV, PFVC and cost

Figure 3 displays the locations for P&R facilities when λ=6. *Figure 3(a)* is the result by the model without spatial equity constraints. A total of 10 P&R facilities are located, whereas spatial clustering phenomenon is generated to some degree, as highlighted by the dashed ellipses. *Figure 3(b)* is the result of our model; there are 5 transfer facilities selected, the optimal deployment tends to be more spread-out due to the spatial equity constraints.

Table 3 – Effects of Parameter λ on the locations of P&R facilities

λ	Location	$Q(y^*)$	$C(x^*)$	$Q(y^*)/C(x^*)$	$f(\lambda)$
0	1,2,3,7,9,12,25,28,30,35	89,253.5	4,458	20.0	89,253.5
1	1,2,3,7,9,25,28,30,35	89,013.4	3,998	22.3	85,015.4
3	1,2,3,7,25,30,35	86,616.3	3,026	28.6	77,538.3
6	1,3,4,25,31	82,826.5	1,978	41.9	70,958.5
12	1,3,4,38	78,992.5	1,530	51.6	60,632.5
25	1,33,37,38	72,225.7	1,100	65.7	44,725.7
50	33	44,960.5	350	128.5	27,460.5
128	37	32,221.3	240	134.3*	1,501.3
134.3*	37	32,221.3	240	134.3*	0
150	1	28,011	220	127.3	-4,989
160	1	28,011	220	127.3	-7,189

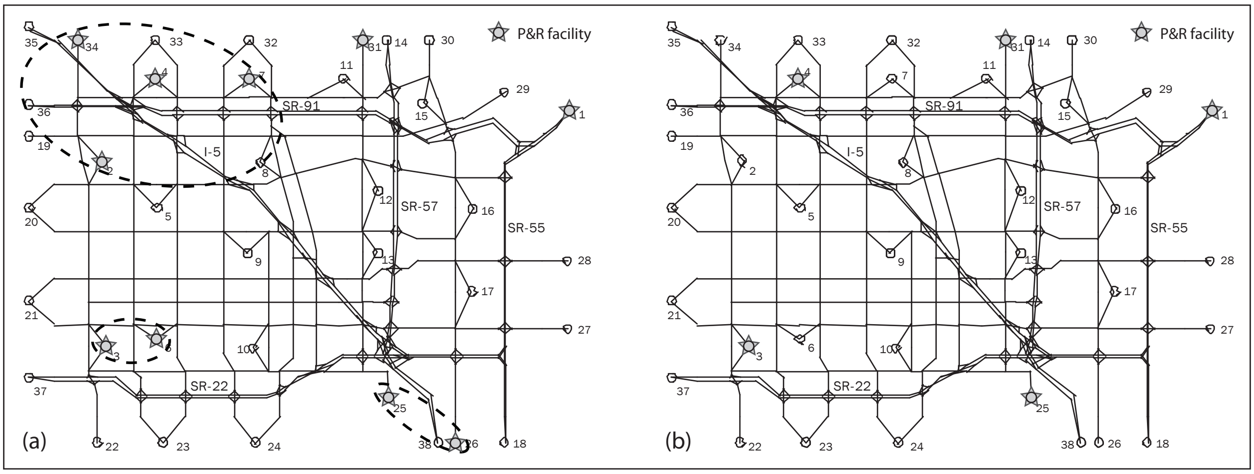

Figure 3 – Location results of P&R facilities (λ=6)
(a) Without the spatial equity constraints (6) *(b) With the spatial equity constraints (6)*

5. CONCLUSION

P&R services are an integral element of public transportation system. Locating P&R facilities is an important step when planning for such services. Coverage and cost issues are two major concerns from economic perspective. This paper develops a bi-objective programming model to account for P&R application specific objectives. The objectives are conflicting but integrated into one model. Moreover, the developed model addresses the issues of broad importance in location analysis in general, such as spatial equity and distance decay. This model is transformed into a single objective problem via the linearly weighted method. The concept of the passenger flow volume per cost is defined. Properties of solutions of the model are studied. Finally, this model is applied to the traffic network in Anaheim, California.

It should be pointed out that spatial equity in this paper is considered from the point of view that it may lead to inequity if the facilities are too close to each other. However, it does not mean that the greater the distance, the more fair the deployment. Further research about spatial equity of P&R facilities should be discussed in the future. In addition, demand allocation is based on the distance decay function in our paper. It is meaningful to take travellers' route choice behaviour into consideration. Next, it is assumed that the capacities of P&R facilities are infinite in our model. In fact, the actual space of P&R facility has great impact on their spatial deployment and the travellers' transferring choice [22]. It would be insightful to explore such capacitated P&R facility location models. Finally, the travel demand may fluctuate year by year. It is also interesting to study the P&R facility location problem under demand uncertainty [23].

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卢晓珊
合肥工业大学
交通运输工程学院
中国 合肥，230009
郭仁拥
内蒙古大学
计算机学院
中国 呼和浩特，010021

摘要

空间公平性约束下停车换乘设施选址的双目标模型

一个带有空间公平性约束的双目标规划模型被建立，用以研究交通网络中停车换乘设施的选址问题。决策变量是换乘设施的数量和位置。最大客流覆盖和最少建设成本这两个相互冲突的目标被同时考虑，用以权衡设施覆盖的质量和成本。进一步地，单位成本客流量的概念被提出，几个有关模型的解的性质被分析。最后，该模型被应用于加州Anaheim市的换乘设施布局问题中。算例结果表明了客流量，成本和单位成本客流量三者之间的权衡，以及空间公平性约束对停车换乘设施空间布局的影响。

关键词

交通网络；停车换乘设施；双目标规划；空间公平性

REFERENCES

[1] Bolger D, Colquhoun D, Morrall J. Planning and design of park-and-ride facilities for the Calgary light rail transit system. Transportation Research Record. 1992;1361:141-148.

[2] Roberts A, Parker A, Surl R, Gallop K. Park-and-ride: new case studies. Proceedings of Institution of Civil Engineers – Transport. 1996;117:234-235.

[3] Lu XS, Huang HJ, Liu TL. Pricing and hierarchical logit-based mode choice models in a multimodal corridor with trip-chain costs. Systems Engineering Procedia. 2011;2:231-242.

[4] AASHTO. Guide for park-and-ride facilities. American Association of State Highway and Transportation Officials; 2004.

[5] Horner MW, Grubesic TH. A-GIS-based planning approach to locating urban rail terminals. Transportation. 2001;28(1):55-77.

[6] Faghri A, Lang A, Hamad K, Henck H. Integrated knowledge-based geographic information system for determining optimal location of park-and-ride facilities. Journal of Urban Planning and Development-ASCE. 2002;128(1):18-41.

[7] Wang JYT, Yang H, Lindsey R. Locating and pricing park-and-ride facilities in a linear monocentric city with deterministic mode choice. Transportation Research Part B. 2004;38(8):709-731.

[8] Liu TL, Huang HJ, Yang H, Zhang XN. Continuum modeling of park-and-ride services in a linear monocentric city with deterministic mode choice. Transportation Research Part B. 2009;43(6):692-707.

[9] Sargious MA, Janarthanan N. Forcasting demand for the park-and-ride mode and determining the optimal location of stations. Canadian Journal of Civil Engineering. 1983;10(4):695-702.

[10] Horner M, Groves S. Network flow-based strategies for identifying rail park-and-ride facility locations. Socio-Economic Planning Sciences. 2007;41(3):255-268.

[11] Farhana B, Murray AT. Siting park-and-ride facilities using a multi-objective spatial optimization model. Computers & Operations Research. 2008;35(2):445-456.

[12] Fan W, Khan MB, Ma J, Jiang X. Bilevel programming model for locating park-and-ride facilities. Journal of Urban Planning and Development. 2004;140(3). Available from: http://dx.doi.org/10.1061/(ASCE)UP.1943-5444.0000178

[13] Aros-Vera F, Marianov V, Mitchell JE. p-hub approach for the optimal park-and-ride facility location problem. European Journal of Operational Research. 2013;226(2):277-285.

[14] Bröcker J, Korzhenevych A, Schürmann C. Assessing spatial equity and efficiency impacts of transport infrastructure projects. Transportation Research Part B. 2010;44:795-811.

[15] Lu XS, Huang HJ, Long JC. Camera location optimization for traffic surveillance in urban road networks with multiple user classes. International Journal of System Science. 2013;44(12):2211-2222.

[16] Lu XS, Huang HJ. Bi-objective progrmming model for P&R facility location with spatial equity constraints [in Chinese]. Systems Engineering – Theory & Practice. 2014;34(9):2379-2385.

[17] Holguín-Veras J, Yushimito WF, Aros-Vera F, Reilly JJ. User rationality and optimal park-and-ride location under potential demand maximization. Transportation Research part B. 2012;46(8):949-970.

[18] Fotheringham S, O'Kelly M. Spatial interaction models: formulations and applications. Amsterdam: Kluwer Academic; 1989.

[19] Yang H, Zhang XN. Multiclass network toll design problem with social and spatial equity constraints. Journal of Transportation Engineering. 2002;128(5):420-428.

[20] Barros I. Discrete and fractional programming techniques for location models. Netherlands: Kluwer Academic Publishers; 1998.

[21] Hillel G. Transportation Network Test Problems. 2014 Mar 15. Available from: http://www.bgu.ac.il/~bargera/tntp/

[22] Wu LY, Zhang XS, Zhang JL. Capacitated facility location problem with general setup cost. Computers & Operations Research. 2006;33(5):1226-1241.

[23] Li ZC, Lam WHK, Wong SC, Zhu DL, Huang HJ. Modeling park-and-ride services in a multimodal transport network with elastic demand. Transportation Research Record. 2007;1994:101-109.