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PRIMJENA NUMERIČKE I ANALITIČKE METODE GRANIČNIH ELEMENATA U ANALIZI REBRASTIH PLOČA

NUMERICAL AND ANALYTICAL BOUNDARY ELEMENT METHOD APPLICATION IN RIBBED SLAB ANALYSIS

Mykola Surianinov, Oleksandr Chuchmai, Oleksii Shyliaiev

Pregledni rad

Sažetak: *U članku je provedena analiza rebrastih ploča uz pomoć numeričke i analitičke metode graničnih elemenata. Naveden je detaljni redoslijed rješenja problema, te je riješen konkretni primjer.*

Ključne riječi: *glatke ploče, metoda graničnih elemenata, poprečne grede, rebraste ploče, savijanje*

Review article

Abstract: *Article is dedicated to analysis of ribbed slabs using the numerical and analytical boundary element method. Program realization of problem solution if performed. Test example of calculation is given.*

Keywords: *boundary element method, coffered ceilings, cross beams, deflection, smooth plate.*

1. INTRODUCTION

Ribbed slabs are widely applicable in many branches of industry: civil engineering, mechanical engineering, aviation, shipbuilding etc. It is an advanced type of ceiling in the civil engineering – coffered ceilings, which can be described as plate supported with system of crossed beam-ribs, which are located in the bottom. In this construction, concrete is removed from the tensile region of cross-section. There are retained only ribs in which tensile reinforcement is placed. As a result, the considerable material economy can be achieved beside ceilings with solid cross-section. On the other hand, overlapping bays can be increased significantly.

Classical methods of structural mechanics (work method or deflection method) and numerical methods are generally used for analysis of such systems. The most common method is the finite element method [1, 2]. Application of numerical and analytical boundary element method (NA BEM) is highly efficient [3-5].

2. MAIN SECTION

Let us look at the fragment of coffered ceiling as shown in the figure 1.

The main idea of proposed approach consists in following. Construction parts with ribs parallel to axis *ox* (call this direction lateral), are considered as a smooth plates with thickness $h_1 = h + h_{rib}$ and width $b = b_{rib}$, where h - plate thickness, h_{rib}, b_{rib} - rib cross-sections dimensions (which are not necessarilly the same).

For this modules (module is a unit of sampling in BEM) the theory of smooth plates analysis,

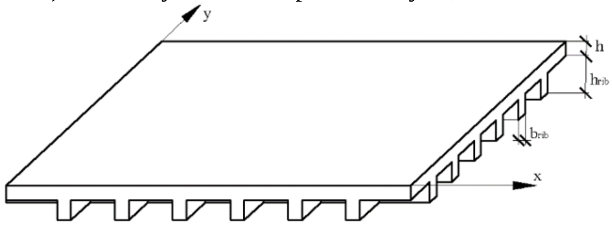


Figure 1. Fragment of coffered ceiling

stated in [3, 4], with according expression of fundamental functions, Green's function and load vector is valid. In the figure 2 such modules are indicated with even numbers. Other modules (odd numbers in the figure 2) are represented as plates supported with ribs in longitudinal direction, i.e. parallel to axis *oy*.

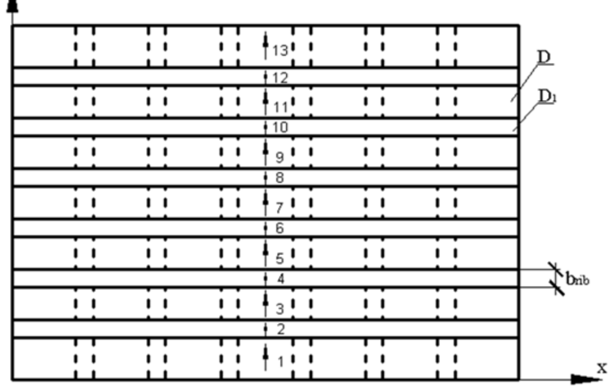


Figure 2. System discretization to one-dimensional modules

Let us take a closer look at NA BEM application to these modules.

Differential equation of plate bending flexure in this case becomes [6]

$$\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2 \partial y^2}+\frac{\partial^4 W}{\partial y^4}=\frac{\bar{q}}{D}, \quad (1)$$

where $W = W(x,y)$ – plate deflection; $\bar{q} = \bar{q}(x,y)$ – free term of an equation, which takes into account not only external loads, but also presence of support ribs in longitudinal direction, as shown in figure 3.

We apply for (1) the Kantorovich-Vlasov method, i.e. deflection of plate median surface we will evaluate as:

$$W(x,\, y)=W_1(y)X_1(x)+ \\ +W_2(y)X_2(x)+...+W_k(y)X_k(x). \quad (2)$$

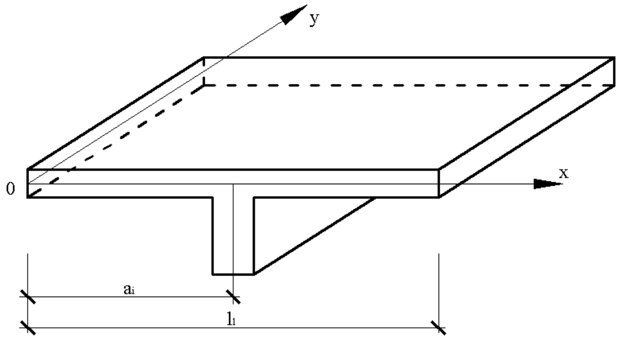


Figure 3. Plate with a rib in longitudinal direction

Truncate the series (2) to one term and take deflection variation only in y direction. It is equal to assume the design model as the plate with infinite number of degree of freedom in one direction and one degree of freedom in another direction. Then plate median surface point deflection will be:

$$W(x,\, y)=W(y)X(x). \quad (3)$$

The function of lateral distribution of deflections $X(x)$ should be selected so that it most accurately describes the shape of plate swept surface in the direction of axis x. Beam bending flexure curves, which have the same with plate conditions of support in the direction of axis x, fully satisfies this requirement.

There are two ways to select deflections function $X(x)$ – static and dynamic. In static method, the beam deflection is defined by applied static load.

In this case $X_1(x)$ will be:

$$X_1(x)=\left(\frac{x}{l_1}-2\frac{x^3}{l_1^3}+\frac{x^4}{l_1^4}\right)+M(0)\left(8\frac{x}{l_1}-12\frac{x^2}{l_1^2}+4\frac{x^3}{l_1^3}\right)+ \\ +M(l_1)\left(4\frac{x}{l_1}-4\frac{x^3}{l_1^3}\right); \\ M(0)=M(l_1)=-\frac{l_1^2}{12}.$$

Applied load should be so that symmetric and skew symmetric forms of deflection curves sequentially alternates.

In dynamic method, beam natural mode shapes define its deflections.

In this case $X_1(x)$, for example, will be:

$$X_1(x)=\sin(\omega_1 x/l_1)-sh(\omega_1 x/l_1)- \\ -\alpha_*[\cos(\omega_1 x/l_1)-ch(\omega_1 x/l_1)]; \\ \alpha_*=\frac{\sin\omega_1-sh\omega_1}{\cos\omega_1-ch\omega_1};\ \omega_1=4.73004075\,.$$

Load $\bar{q} = \bar{q}(x,y)$ has the most general view in the case that supporting ribs has both solid and thin-walled cross-section:

$$\bar{q}(x,y)=q(x,y)-\sum_{i=1}^{n}EI_xW^{IV}(y)X(a_i)\delta(x-a_i)- \\ -\sum_{i=1}^{n}\frac{GA}{k_1}W''(y)X(a_i)\delta(x-a_i)- \\ -\sum_{i=1}^{n}\left[EI_\omega W^{IV}(y)X'(a_i)-GI_K W''(y)X'(a_i)\right]\delta'(x-a_i), \quad (4)$$

where EI_x, EI_ω, EI_k – rib stiffness at flexure and torque;

k_1 – factor, which depends on cross-section shape;

a_i – coordinate of i-rib location.

$k_1 = 1.2$ for rectangle cross-section. Factor k_1 is defined by equation [7] in general case.

$$k_1=\frac{A_V(z)}{B_x^2(z)}\int_A\frac{S_{EyA}^2}{Gb^2}dA,$$

where A – cross-section area; $A_V=\int_A GdA$;

$B_x(z)=\int_A Ey^2dA$; $S_{EyA}=\int_A ExdA$.

Delta function $\delta(x\text{-}a_i)$ and its derivative $\delta'(x\text{-}a_i)$ are used in (4) when rib is very narrow-width and can be considered as a line; in other case rib existence is considered by difference of Heaviside unit functions.

Let us return to the equation (3). We substitute it into (1) and into known from the plate flexure theory expressions of static parameters (bending moments and adjusted shear forces). Multiplying both sides of each expression by $X(x)$ and integrating in limits [0; l_1], we will get Cauchy problem for one-dimensional model of rectangle plate, supported with longitudinal ribs, flexure:

$$W^{IV}(y)-2r^2W''(y)+s^4W(y)=\frac{\bar{q}(y)}{D} \quad (5)$$

at initial conditions

$$DW(0);\ D\theta(0)=DW'(0); \\ M(0)=-D\bar{A}\left[W''(0)-\mu r^2W(0)\right]; \\ Q(0)=-D\bar{A}\left[W'''(0)-(2-\mu)r^2W'(0)\right], \quad (6)$$

where

$$r^2 = -\overline{B}/\overline{A};\; s^4 = C/\overline{A};$$

$$\overline{q}(y) = \int_0^{l_1} \overline{q}(x,y)X(x)dx/\overline{A}; \tag{7}$$

$$\overline{A} = A + \frac{1}{D}\sum_{i=1}^{n} EI_x X^2(a_i) + \frac{1}{D}\sum_{i=1}^{n} EI_\omega \left[X'(a_i)\right]^2; \tag{8}$$

$$\overline{B} = B + \frac{1}{2D}\sum_{i=1}^{n} EI_k \left[X'(a_i)\right]^2 + \frac{1}{2D}\sum_{i=1}^{n} \frac{GA}{k_1} X^2(a_i); \tag{9}$$

$$A = \int_0^{l_1} X^2(x)dx;\; B = \int_0^{l_1} X''(x)X(x)dx;$$
$$C = \int_0^{l_1} X^{IV}(x)X(x)dx. \tag{10}$$

Differential equation (5) by its structure is similar to the equation, that describes flexure of plates without rib supports, but form of expressions (7)-(9) argues that in (5) ribs parameters are taken into account. With reference to realization of boundary element method algorithm it means, that rib parameters will be taken into account in fundamental functions expressions.

Kinematic and static parameters for supported plate will be in general the same as for the plate without ribs, but in numerical expression they will be different. Because deflection *W(y)*, contained in following equations, will differ from deflection of according „smooth“ plate.

$$\theta_x(x,y) = W(y)X'(x);\; \theta_y(x,y) = W'(y)X(x); \tag{11}$$

$$M_x(x,y) = -D[W(y)X''(x) + \mu W''(y)X(x)];$$
$$M_y(x,y) = -D[W''(y)X(x) + \mu W(y)X''(x)]; \tag{12}$$

$$H_x(x,y) = -H_y(x,y) = -D(1-\mu)W'(y)X'(x); \tag{13}$$

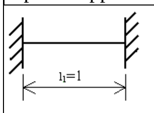
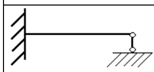
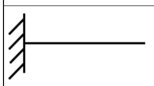
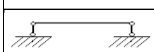
$$V_x(x,y) = -D[W_y(y)X'''(x) + W''(y)X'(x)]; \tag{14}$$

$$V_y(x,y) = -D[W'''(y)X(x) + W'(y)X''(x)], \tag{15}$$

where $\theta_x(x,y)$, $\theta_y(x,y)$ – normals angles of deflection in direction of axes *x* and *y*; $M_x(x,y)$, $M_y(x,y)$ – bending moments; $H_x(x,y)$, $H_y(x,y)$ – torque moments; $V_x(x,y)$, $V_y(x,y)$ – shear forces.

Factors (7)-(10) values are calculated in program Scilab [8] and are shown in table 1 (in condition that stiffness rib dimensions are *b*x*h* = 0.1 x 0.1 m).

Table 1. Factors numerical values

Conditions of plate support	Factors		
	$\overline{A}$	$\overline{B}$	$\overline{C}$
	4,7510773e+00	-1,27447255e+01	518,5521
	3,9617707e+00	-1,05574402e+01	237,3516
	3,0717246e+00	3,49815534e+00	22,9401
	1,9218750e+00	-4,9348022e+00	48,7045

Cauchy problem solving (5)-(6) could be expressed accordingly to the algorithm of boundary element method

$$\begin{pmatrix} DW(y) \\ D\theta(y) \\ M(y) \\ Q(y) \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & -A_{13} & -A_{14} \\ A_{21} & A_{22} & -A_{23} & -A_{24} \\ -A_{31} & -A_{32} & A_{33} & A_{34} \\ -A_{41} & -A_{42} & A_{43} & A_{44} \end{pmatrix} \times$$

$$\times \begin{pmatrix} DW(0) \\ D\theta(0) \\ D\theta(0) \\ Q(0) \end{pmatrix} + \int_0^y \begin{pmatrix} A_{14}(y-\xi) \\ A_{13}(y-\xi) \\ -A_{12}(y-\xi) \\ -A_{11}(y-\xi) \end{pmatrix} \overline{q}(\xi)d\xi \tag{16}$$

Thus at Kantorovich-Vlasov method application, solution of main differential equation is reducing to defining of deflection (4), where function *X(x)* is given and function *W(y)*is defining from (16) in the form:

$$DW(y) = A_{11}\cdot DW(0) + A_{12}\cdot D\theta(0) - A_{13}\cdot M(0) -$$
$$- A_{14}\cdot Q(0) + \int_0^y A_{14}(y-\xi)q(\xi)d\xi. \tag{17}$$

Solution of equation (5) depend on roots of according characteristics equation, which can be expressed as:

$$k_{1-4} = \pm\sqrt{r^2 \pm \sqrt{r^4 - s^4}}. \tag{18}$$

Fundamental function form is defining by *r* and *s* relation, which depend on boundary conditions on longitudinal edges of plate and stiffness ribs parameters. Herewith there should be six cases:

1. $|s| > |r|$, at that

$$k_{1-4} = \pm\alpha \pm i\beta,$$

where

$$\alpha = \sqrt{\frac{s^2 + r^2}{2}};\; \beta = \sqrt{\frac{s^2 - r^2}{2}}.$$

Deflection *W(y)* will be written in form:

$$W(y) = C_1\Phi_1 + C_2\Phi_2 + C_3\Phi_3 + C_4\Phi_4, \tag{19}$$

where Φ_1, Φ_2, Φ_3, Φ_4, - hyperbolic-trigonometric functions:

$$\left.\begin{aligned} \Phi_1 &= ch\alpha y \sin\beta y; \\ \Phi_2 &= ch\alpha y \cos\beta y; \\ \Phi_3 &= sh\alpha y \cos\beta y; \\ \Phi_4 &= sh\alpha y \sin\beta y. \end{aligned}\right\} \tag{20}$$

2. $s = r$, at that characteristic equation roots will be real and divisible:

$$k_{1-4} = \pm\sqrt{r^2}.$$

Deflection *W(y)* will be written in form (19) again:

$$W(y) = C_1\Phi_1 + C_2\Phi_2 + C_3\Phi_3 + C_4\Phi_4,$$

but now $\Phi_1 = ychry$; $\Phi_2 = chry$; $\Phi_3 = shry$; $\Phi_4 = yshry$.

3. $|s| < |r|$, roots will be real and different:

$$\lambda_1 = \sqrt{r^2 + \sqrt{r^4 - s^4}}, \quad \lambda_2 = \sqrt{r^2 - \sqrt{r^4 - s^4}}.$$

Than integrated deflection:

$$W(y) = C_1\Phi_1 + C_2\Phi_2 + C_3\Phi_3 + C_4\Phi_4,$$

where

$$\Phi_1 = sh\lambda_2 y;\ \Phi_2 = ch\lambda_1 y;\ \Phi_3 = sh\lambda_1 y;$$
$$\Phi_4 = ch\lambda_2 y.$$

4. Roots of equation (18) are real and divisible:

$$k_{1,2} = 0;\ k_{3,4} = \pm r_1;\ r_1 = -2\overline{B}/\overline{A}.$$

In this case in expansion

$W(y) = C_1\Phi_1 + C_2\Phi_2 + C_3\Phi_3 + C_4\Phi_4$ we have

$$\Phi_1 = shr_1 y;\ \Phi_2 = 1;\ \Phi_3 = y;\ \Phi_4 = chr_1 y.$$

5. Roots of equation (18) are real and imaginary

$$k_{1,2} = \pm i\beta;\ k_{3,4} = \pm\alpha.$$

In this case

$$\Phi_1 = \cos\beta y;\ \Phi_2 = \sin\beta y;\ \Phi_3 = ch\alpha y;\ \Phi_4 = sh\alpha y.$$

6. Roots of equation (18) are imaginary

$$k_{1,2} = \pm\alpha i;\ k_{3,4} = \pm\beta i;$$

In this case:

$$\Phi_1 = \cos\alpha y;\ \Phi_2 = \sin\alpha y;\ \Phi_3 = \cos\beta y;$$
$$\Phi_4 = \sin\beta y$$

3. CONCLUSION

Thus full solution of the problem consists of six variants of roots of according characteristics equation; there are formulated six systems of fundamental orthonormalized functions (total number of 96) for them, Green's function is generated, six variants of load vector are formulated.

Program realization of developed algorithms is performed in software package Scilab. Ability of boundary conditions variation, rib number and their cross-section dimensions change, character of external load variation is foreseen.

Here on the figure 4 as a test example we show the calculation results of uniformly loaded on all area ribbed slab fragment with rigid restraint on all contour.

Deflection and bending moment at the center of construction are calculated. Calculation results are shown in the table 2, where values of deflection and bending moment at the center of plate calculated with finite element method in program ANSYS [8] are shown too.

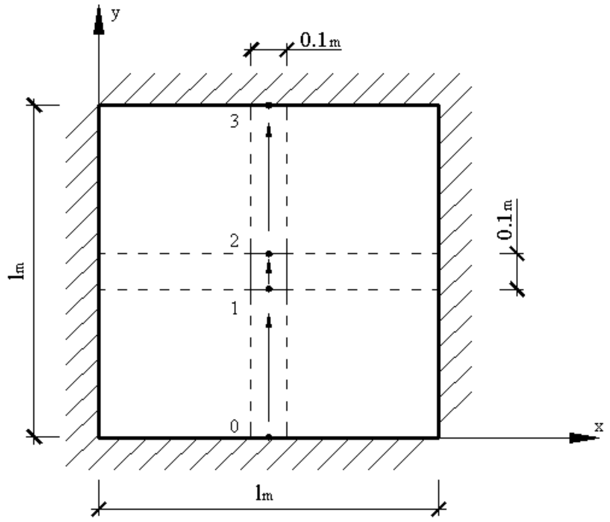


Figure 4. Test example

Table 2. Comparison of results, obtained by two methods

Value	NA BEM	FEM, ANSYS	Inaccuracy, %
Deflection, m	-0.1406e-06	-0.1357e-06	3.48
Bending moment, kNm	0.922e-02	0,884e-02	4.12

As we can see, boundary element method shows practically the same results with finite element method.

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