

# TRACKING GATE ALGORITHM FOR GENERAL NONLINEAR SYSTEMS WITH TARGET CLASS INFORMATION

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## Abstract:

Multitarget tracking in clutter usually involves data association. The traditional method to handle this problem is to construct a tracking gate for predicting the position of the target being tracked, which leads to great uncertainties of measurements-to-tracks association with the unknown class of targets. This paper proposes a new tracking gate algorithm for general nonlinear systems, where the target class information is integrated into our algorithm. Firstly, a joint probability density description of the target state and target class is given, by which the tracking gates for each target class in general nonlinear system are developed. Then, a simulation with ground formation target tracking is carried out to examine our algorithm. Compared with the traditional tracking gate, the results demonstrate that our algorithm has significantly improved the probabilities of the measurements-to-tracks association.

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## 1 Introduction

In order to solve the problem of maneuvering target tracking in clutter, the data association needs to be considered. The most commonly used method for data association is the tracking gate, by which the valid measurements are differentiated with the invalid one and passed to following processes. Various tracking gate algorithms have been investigated for data association [1-3]. For example, Bar-Shalom et al. [2] have proved the ellipsoidal gate is optimal in the sense of minimal volume for a given in-gate probability of target-originated measurement with Gaussian assumptions. In probabilistic data association filter (PDAF), the state estimate and state covariance are updated in

terms of weighted sum of the valid measurements. Therefore, the optimal ellipsoidal gate is a natural choice in the PDAF as well as other methods which incorporate PDAF with interacting multiple model [4, 5]. Apart from those linear tracking systems, some effective tracking gate algorithms for nonlinear system were proposed in [6-8].

In recent years, more and more researchers have begun to integrate the target class information into tracking process. For example, a Bayesian fusion algorithm was used to combine identity for target tracking [9]. In [10], the class-dependent kinematic model is constructed for simultaneous target tracking and classification. This paper studies how to integrate the target class or identity information into the tracking gate algorithm.

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## 2 Related work

Two commonly used gating algorithms are the rectangular gate and the ellipsoidal gate. The rectangular gate is relatively simple, and is only defined by the predicted target position and innovation. Compared with the rectangular gate, the ellipsoidal gate considers the distribution of target measurements, and is suitable for more practical applications.

Let  $\mathbf{z}_{k+1}$  denote the measurement at time  $k+1$ ;  $\mathbf{z}_{1:k} = \{\mathbf{z}_1, \dots, \mathbf{z}_k\}$  are the measurement sequences up to the time  $k$ ;  $\mathbf{v}_{k+1} = \mathbf{z} - \hat{\mathbf{z}}_{k|k+1}$  is the measurement residual or innovation;  $\phi_{i,k+1}^{1/2}$  is a normalizing factor and  $|v_{i,k+1}|/\phi_{i,k+1}^{1/2}$  is a standard Gaussian variable. A rectangular gate in an  $n$ -dimensional measurement space is denoted by:

$$G_{k+1}^R = \{\mathbf{z} \in \mathfrak{R}^n : |v_{i,k+1}|/\phi_{i,k+1}^{1/2} \leq \gamma_{\alpha/n}, i=1, \dots, n\}, \quad (1)$$

where  $v_{i,k+1}$  is the  $i$ th component of the innovation  $\mathbf{v}_{k+1}$ ;  $\gamma_{\alpha/n}$  is called the rectangular gate threshold. Since  $\gamma_{\alpha/n}$  is an upper  $\alpha/n$  quantile of a standard Gaussian function, the overall coverage probability can be calculated by Bonferroni's inequality:

$$\begin{aligned} P_G &= \Pr\{\mathbf{z}_{k+1} \in G_{k+1}^R\} \\ &= \Pr\left\{\bigcup_{i=1}^n (|v_{i,k+1}|/\phi_{i,k+1}^{1/2} \leq \gamma_{\alpha/n})\right\} \\ &= 1 - \sum_{i=1}^n \Pr\{|v_{i,k+1}|/\phi_{i,k+1}^{1/2} > \gamma_{\alpha/n}\} \\ &= 1 - n \cdot \alpha/n = 1 - \alpha \end{aligned} \quad (2)$$

Assuming that the true measurement conditioned by the past observation is a Gaussian variable with a probability density function given by:

$$p(z_{k+1} | \mathbf{z}_{1:k}) = N(z_{k+1}; \hat{z}_{k+1|k}, \mathbf{S}_{k+1}), \quad (3)$$

where  $\hat{z}_{k+1|k}$  is the predicted measurement;  $\mathbf{S}_{k+1}$  is the innovation covariance; Symbol  $N$  presumes that the random variable is Gaussian distributed with the corresponding mean and variance. The ellipsoidal gate is given by:

$$G_{k+1}^E = \{\mathbf{z} \in \mathfrak{R}^n : \mathbf{v}'_{k+1} \mathbf{S}_{k+1}^{-1} \mathbf{v}_{k+1} \leq \gamma\}, \quad (4)$$

where  $\gamma$  is the gate threshold. The random variable:

$$\varepsilon_{k+1} = \mathbf{v}'_{k+1} \mathbf{S}_{k+1}^{-1} \mathbf{v}_{k+1} \sim \chi^2(n), \quad (5)$$

is called the squared normalized innovation which is a chi-square distributed random variable with  $n$  degrees of freedom. Given that coverage probability is  $1-\alpha$ , the threshold  $\gamma$  is the upper  $\alpha$  quantile of chi-square distribution  $\chi^2(n)$ , satisfying:

$$\Pr\{\mathbf{z}_{k+1} \in G_{k+1}^E | \mathbf{z}_{1:k}\} = \Pr\{\varepsilon_{k+1} \leq \gamma | \mathbf{z}_{1:k}\} = 1 - \alpha. \quad (6)$$

## 3 General nonlinear system model

This section discusses how to construct the tracking gate for general nonlinear system. If the target attribute or the feature information is available, the target class can be considered as a discrete random variable and presented by a class probability mass function:

$$p(x_k, c = i | \mathbf{z}_{1:k}), \quad (7)$$

where  $\mathbf{x}_k$  is state vector;  $c$  is one of known target classes;  $\mathbf{z}_{1:k} = \{\mathbf{z}_{1:k}^x, \mathbf{z}_{1:k}^c\}$  are the measurement sequences up to  $k$  from an attribute sensor and a kinematic sensor, respectively:

$$\mathbf{z}_{1:k}^x = \{\mathbf{z}_1^x, \mathbf{z}_2^x, \dots, \mathbf{z}_k^x\}, \mathbf{z}_{1:k}^c = \{\mathbf{z}_1^c, \mathbf{z}_2^c, \dots, \mathbf{z}_k^c\}. \quad (8)$$

The general nonlinear system can be represented by:

$$\mathbf{x}_{k+1} = f^i(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{v}_k^i, \quad (9)$$

and

$$\mathbf{z}_k = h^i(\mathbf{x}_k) + \mathbf{w}_k^i, \quad (10)$$

where  $f^i$  is the state transition function for class  $i$ ;  $\mathbf{v}_k$  and  $\mathbf{w}_k$  are the process noise and measurement noise, respectively;  $\mathbf{u}_k$  is the model input. Let the estimated target state at time  $k$  be  $\hat{\mathbf{x}}_k$ . Given the system model (9) and (10), the state prediction conditioned on class at time  $k+1$  is

$$\hat{\mathbf{x}}_{k+1|k}^i = f^i(\hat{\mathbf{x}}_k) . \quad (11)$$

The predicted measurement is

$$\hat{z}_{k+1|k}^{x,i} = h(\hat{\mathbf{x}}_{k+1|k}^i) . \quad (12)$$

Note that predicted state and measurement above are all conditioned on the target class. If the true target class is  $i$ , the probability that the kinematic measurement  $\mathbf{z}_{k+1}^x$  at time  $k+1$  falls into the area around the predicted measurement of class  $i$  and should be larger than those of other classes. Let  $G_{k+1}^i$  be the tracking gate conditioned on class  $i$  at time  $k+1$  which is dependent on the predicted measurement  $\hat{z}_{k+1|k}^{x,i}$  of the class  $i$ , the descriptions above can be expressed as:

$$\Pr\{\mathbf{z}_{k+1}^{x,i} \in G_{k+1}^i | \mathbf{z}_{1:k}\} > \Pr\{\mathbf{z}_{k+1}^{x,j} \in G_{k+1}^j | \mathbf{z}_{1:k}\} \\ i, j = 1, \dots, s; i \neq j . \quad (13)$$

Based on the above discussion, the tracking gate we have proposed can be presented as:

$$G_{k+1}^i = \{z \in \mathfrak{R}^n : \zeta^i(z, \hat{z}_{k+1|k}^{x,i}) \leq \gamma^i\} , \quad (14)$$

where  $\gamma^i$  is the threshold of the new tracking gate, and  $\zeta^i$  is a performance index function.

#### 4 Tracking gate algorithm

Providing system models (9) and (10), the distribution of target state estimate conditioned by class at initial time is known, denoted as  $p(\mathbf{x}_0 | c=i, \mathbf{z}_0)$ , the recursive expressions for state prediction density and measurement prediction density conditioned by class at subsequent time period are given as below.

Class-conditioned state prediction density is

$$p(\mathbf{x}_{k+1} | c=i, \mathbf{z}_{1:k}) \\ = \int p(\mathbf{x}_{k+1} | c=i, \mathbf{x}_k) p(\mathbf{x}_k | c=i, \mathbf{z}_{1:k}) d\mathbf{x}_k , \quad (15)$$

and class-conditioned measurement prediction density is

$$p(\mathbf{z}_{k+1}^x | c=i, \mathbf{z}_{1:k}) \\ = \int p(\mathbf{z}_{k+1}^x | c=i, \mathbf{x}_{k+1}) p(\mathbf{x}_{k+1} | c=i, \mathbf{z}_{1:k}) d\mathbf{x}_{k+1} . \quad (16)$$

If  $p(\mathbf{z}_{k+1}^x | c=i, \mathbf{z}_{1:k})$  is Gaussian distributed (see

section 2), the optimal tracking gate for class  $i$  is an ellipsoidal gate. In a simple case of  $p(\mathbf{z}_{k+1}^x | c=i, \mathbf{z}_{1:k})$ , we can derive an analytic expression, and then find the position of the optimal tracking gate. If  $p(\mathbf{z}_{k+1}^x | c=i, \mathbf{z}_{1:k})$  is complicated, we need other numerical methods.

For any tracking gate, two critical points should be considered. One is to make the probability that the interested target falls into the gate as high as possible. The second is to let the amount that the gate contains as small as possible. Here the amount means area in terms of 2-dimensional space or volume in terms of 3-dimensional space. Generally speaking, the probability that the true measurement falls into the gate should be guaranteed not less than some predefined values (such as 0.95) by adjusting the threshold  $\gamma$ . Therefore, the volume of the tracking gate determines the performance of the gate. If the volume is small enough, the number of the clutter and the measurements originated from other targets falling in the gate will be reduced correspondently.

Whatever the probability distribution of the actual predicted measurement density function  $p(\mathbf{z}_{k+1}^x | c=i, \mathbf{z}_{1:k})$  is, for the ideal tracking gate  $G_{k+1}$ , the probability that measurements fall in the gate and the probability that measurements fall out the gate, should have the relationship:

$$p(\mathbf{z}_{k+1}^x | c=i, \mathbf{z}_{1:k}) \geq p(\tilde{\mathbf{z}}_{k+1}^x | c=i, \mathbf{z}_{1:k}) . \quad (17) \\ \mathbf{z}_{k+1}^x \in G_{k+1}, \tilde{\mathbf{z}}_{k+1}^x \notin G_{k+1}$$

The inequality above means that the function values for points within the gate should be bigger than those of outside the gate. According to this relationship, we assume a region exists in an  $n$ -dimensional measurement space:

$$\{\mathbf{z}_{k+1}^x \in \mathfrak{R}^n : p(\mathbf{z}_{k+1}^x | c=i, \mathbf{z}_{1:k}) \geq \gamma^i\} \quad (18)$$

satisfying the overage probability:

$$\Pr\{\mathbf{z}_{k+1}^x \in \mathfrak{R}^n : p(\mathbf{z}_{k+1}^x | c=i, \mathbf{z}_{1:k}) \geq \gamma^i\} = 1 - \alpha . \quad (19)$$

In most cases, the density function  $p(\mathbf{z}_{k+1}^x | c=i, \mathbf{z}_{1:k})$  does not have an analytic expression due to the nature of nonlinear system, so  $\gamma^i$  cannot be evaluated directly. Here, we use the following algorithm to estimate  $\gamma^i$ .

1) Draw  $N$  sample points from  $p(\mathbf{z}_{k+1}^x | c = i, \mathbf{z}_{1:k})$ , marked as  $\mathbf{z}^1, \mathbf{z}^2, \dots, \mathbf{z}^N$ ; Calculate the function values for these samples, i.e.,  $g^i$ , such as:

$$g^i = p(\mathbf{z}_{k+1}^x | c = i, \mathbf{z}_{1:k}) \Big|_{\mathbf{z}_{k+1}^x = \mathbf{z}^i}, \quad i = 1, \dots, N; \quad (20)$$

2) Sort the  $g^i$  satisfying  $g^{1,N} \leq g^{2,N} \leq \dots \leq g^{N,N}$ ;

3) Estimate  $\gamma^i$  using  $\hat{\gamma}^i = g^{N\alpha, N}$  so that:

$$G_{k+1}^i(\hat{\gamma}^i) = \{\mathbf{z}_{k+1}^x \in \mathfrak{R}^n : p(\mathbf{z}_{k+1}^x | c = i, \mathbf{z}_{1:k}) \geq \hat{\gamma}^i\}. \quad (21)$$

This algorithm only relies on simulation in estimating  $\gamma^i$ , and avoids the numerical integration. At the same time, the predicted measurement density needs not be normalized, which alleviates the computation requirement.

To determine whether a new observation  $\mathbf{z}_{k+1}^*$  in time  $k+1$  is falling in the gate, we just need to evaluate  $g^* = p(\mathbf{z}_{k+1}^x | c = i, \mathbf{z}_{1:k}) \Big|_{\mathbf{z}_{k+1}^x = \mathbf{z}_{k+1}^*}$  and compare it with  $\hat{\gamma}^i$ . If  $g^* \geq \hat{\gamma}^i$ , we think  $\mathbf{z}_{k+1}^*$  is a valid measurement.

If the target motion is Gaussian, we can get the Gaussian mixture system. The kinematic process and measurement process can be presented as:

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{G}\mathbf{u}_k + \mathbf{v}_k \quad (21)$$

$$\mathbf{z}_{k+1} = \mathbf{H}\mathbf{x}_{k+1} + \mathbf{w}_{k+1}, \quad (22)$$

where  $\mathbf{F}$  is a state transition matrix,  $\mathbf{G}$  is a control matrix,  $\mathbf{H}$  is a measurement matrix, and  $\mathbf{v}_k$  is Gaussian process noise,  $\mathbf{w}_{k+1}$  is Gaussian measurement noise. The input model  $\mathbf{u}_k$  depends on target class at time  $k$ ,

$$\mathbf{u}_k = m_k^{i,j} \in M^i. \quad (23)$$

Define the model transition probability for the  $i$  th class target:

$$\begin{aligned} p_{ij}^i &= \Pr\{\mathbf{u}_{k+1} = m_{k+1}^{i,j} | \mathbf{u}_k = m_k^{i,l}\} \\ &= \Pr\{m_{k+1}^{i,j} | m_k^{i,l}\}, \quad i = 1, \dots, s; l, j = 1, \dots, r(i) \end{aligned} \quad (24)$$

Denote the model probability conditioned on class at time  $k$  as:

$$\mu_{k|k}^{i,j} = \Pr\{m_k^{i,j} | c = i, \mathbf{z}_{1:k}\} \quad (25)$$

then the predicted model probability conditioned on class is

$$\begin{aligned} \mu_{k+1|k}^{i,j} &= \Pr\{m_{k+1}^{i,j} | c = i, \mathbf{z}_{1:k}\} \\ &= \sum_{l=1}^{r(i)} \Pr\{m_k^{i,l} | c = i, \mathbf{z}_{1:k}\} \Pr\{m_{k+1}^{i,j} | m_k^{i,l}, c = i, \mathbf{z}_{1:k}\}. \quad (26) \\ &= \sum_{l=1}^{r(i)} \mu_l^i(k | k) p_{lj}^i \end{aligned}$$

Define a merging probability as:

$$\begin{aligned} \eta_{k|k}^{i,l|j} &= \Pr\{m_k^{i,l} | m_{k+1}^{i,j}, \mathbf{z}_{1:k}\} \\ &= \frac{\Pr\{m_k^{i,l} | c = i, \mathbf{z}_{1:k}\} p_{lj}^i}{\Pr\{m_{k+1}^{i,j} | c = i, \mathbf{z}_{1:k}\}} = \frac{\mu_{k|k}^{i,l} \cdot p_{lj}^i}{\mu_{k+1|k}^{i,j}}. \quad (27) \end{aligned}$$

The state predictive density conditioned on class is

$$p(\mathbf{x}_{k+1} | c = i, \mathbf{z}_{1:k}) = \sum_{j=1}^{r(i)} p(\mathbf{x}_{k+1} | m_{k+1}^{i,j}, \mathbf{z}_{1:k}) \mu_{k+1|k}^{i,j} \quad (28)$$

and the measurement predictive density conditioned on class is

$$p(\mathbf{z}_{k+1}^x | c = i, \mathbf{z}_{1:k}) = \sum_{j=1}^{r(i)} p(\mathbf{z}_{k+1}^x | m_{k+1}^{i,j}, \mathbf{z}_{1:k}) \mu_{k+1|k}^{i,j}. \quad (29)$$

According to the Gaussian assumption, the measurement predictive density is a Gaussian mixture distribution, ie.:

$$p(\mathbf{z}_{k+1}^x | c = i, \mathbf{z}_{1:k}) = \sum_{j=1}^{r(i)} \mathbf{N}[\mathbf{z}_{k+1}^x; \hat{\mathbf{z}}_{k+1|k}^{x,j}, \mathbf{S}_{k+1|k}^{i,j}] \mu_{k+1|k}^{i,j}, \quad (30)$$

we get the combined position of our tracking gate

$$\hat{\mathbf{z}}_{k+1|k}^{x,i} = \sum_{j=1}^{r(i)} \mu_{k+1|k}^{i,j} \hat{\mathbf{z}}_{k+1|k}^{x,j} \quad (31)$$

and its combined covariance

$$\mathbf{S}_{k+1|k}^i = \sum_{i=1}^{r(i)} \mu_{k-1}^i \left[ \mathbf{S}_k^i + (\hat{\mathbf{z}}_{k|k-1}^x - \hat{\mathbf{z}}_{k|k-1}^{x,i})(\hat{\mathbf{z}}_{k|k-1}^x - \hat{\mathbf{z}}_{k|k-1}^{x,i})' \right]. \quad (32)$$

## 5 Simulation and analysis

This scenario represents a ground formation target tracking problem. It consists of four targets whose classes are assumed to be 1, 2, 1 and 2. The targets move from left with the speed of 25 m/s and separation 200 m as shown in Fig. 1.

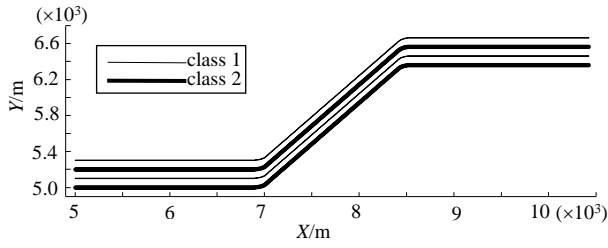


Figure 1. True formation target tracks.

The true state of the target at time  $k$  is

$$\mathbf{x}_k = [x_k, \dot{x}_k, y_k, \dot{y}_k]^T, \quad (33)$$

where  $x_k$  and  $y_k$  are the positions of the target and  $\dot{x}_k$  and  $\dot{y}_k$  are the velocities of the target in the X, Y coordinates, respectively. The kinematic part of the measurement vector is given by:

$$\mathbf{z}_k = [\xi_k, \eta_k]^T, \quad (34)$$

where  $\xi_k$  and  $\eta_k$  are the position measurements in X and Y coordinates. The system model is given by (21) and (22). The state transition matrix, the model input matrix and the measurement matrix are

$$\mathbf{F} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad (35)$$

where the sample time  $T=1s$ , process noise  $\mathbf{v}_k \sim N[0, 0.2^2]$  and measurement noise  $\mathbf{w}_k \sim N[0, 0.1^2]$ .

The corresponding model sets with the class information are  $\{0, 3g, -g\}$  and  $\{0, g, -g\}$  and the model transition probability for each set is

$$p^1 = p^2 = \begin{bmatrix} 0.95 & 0.05 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.05 & 0.95 \end{bmatrix}. \quad (36)$$

The initial model probabilities defined in  $M^i$  are assumed to be equal. The detection probability of the kinematic sensor is 0.96, and the clutter is assumed to be uniformly distributed with the average density 0.01 point/km<sup>2</sup> over the surveillance region. The class of clutter is selected with the

uniform probability between the two possible classes. In order to account for the uncertainty of the classifier output, we define a confusion matrix:

$$\mathbf{C} = [c_{ij}] = \begin{bmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{bmatrix}, \quad (37)$$

where  $c_{ij}$  is the likelihood of the true class being  $i$  when the classifier output is  $j$ . The tracking is based on the IMM filter. In the phase of data association, we use two kinds of gating technique separately. The estimated trajectories are shown in Fig. 2 to Fig. 4.

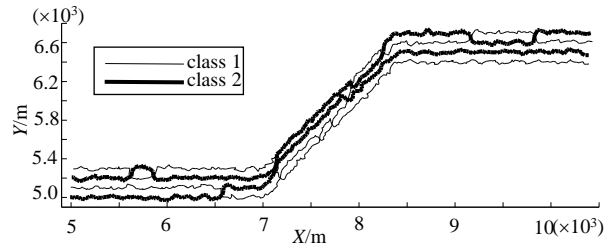


Figure 2. Estimated trajectories via the rectangular gate.

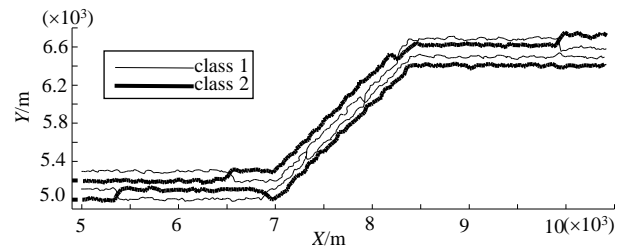


Figure 3. Estimated trajectories via the ellipsoidal gate.

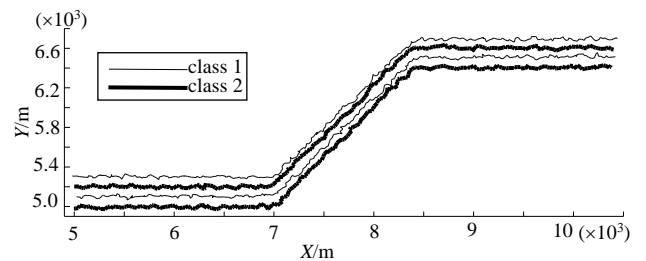


Figure 4. Estimated trajectories via the class-conditioned gate.

Fig. 2, 3 and 4 show that the measurements-to-tracks association using our tracking gate algorithm is significantly better than the rectangular gate and the ellipsoidal gate. In Fig. 2 and 3, there exist fault

associations to some extent because the other two gates failed to distinguish the target classes, and only used the “distance” between measurements and tracks to validate the true target measurement. Table 1 gives the probabilities that the measurements are associated with the correct trajectories for each tracking gate.

Table 1. Association probabilities

Tracking Gate	Association probabilities	
	Class 1	Class 2
Rectangular Gate	53%	61%
Ellipsoidal Gate	76%	79%
Class-conditioned Gate	100%	100%

## 6 Conclusion

In this paper we have developed an algorithm to construct tracking gate for general nonlinear systems with target class information. Most of traditional data association methods only use the target kinematic information, whereas the use of the target class information is usually left as post-processing units like target identification or recognition. In fact, target class information can also be used in data association to yield significant improvements to tracking accuracy and association purity. Because target motion process is determined by target class, the target state at any moment must match its corresponding model set. In detail, the new tracking gates are constructed by its individual target state equation and model input, which significantly reduces the errors caused by the detection uncertainty and the model input uncertainty.

According to our tracking gate algorithm, target kinematic process for each target class can be separately expressed. For nonlinear systems, a numerical method can be used for constructing the gates; For Gaussian mixture systems, we discussed the analytic expressions of the gate position and covariance. Simulation scenario indicated that our tracking gate can better associate measurements with the target tracks.

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